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EXCITATION OF NON-LINEAR OSCILLATIONS IN PLASMA BY A FINITE ELECTRON BEAM

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The excitation of non-linear oscillations in plasma by charged particle beams is of certain interest from the point of view of the possible use of characteristics of such oscillations in order to determine the physical parameters of the passing particles as well as the plasma itself.

The excitation and stability of plasma non-linear waves produced by uniform electron beams are considered in the work [1]. In present paper we consider the excitation of non-linear oscillations in cold plasma by a monoenergetic electron beam having limit size in the direction of beam motion and analyse the dependence of the characteristics of the excited oscillations on the beam parameters.

Let a non-relativistic electron beam, infinite in the directions of the axes x and y and having a thickness z_0 moves along the axis z with a velocity $V_0 \ll c$. If the beam density is equal to n_b then at small thicknesses z_0 one may assume that the magnitude $\sigma = e n_b z_0$ is the beam charge surface density. Therefore, one may consider such a beam as a charged "plane" and write the charge density in the form $\rho = -\sigma \delta (z - V_0 t)$. Thus, with these assumptions the problem under consideration is essentially one-dimension problem and the self-consistent system of equations discribing the beam - plasma interaction in the hydrodynamic approximation for non - relativistic beam and plasma particle velocities takes the following form:

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial z}(nu)=0\,, \qquad \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial z}=-\frac{e}{m}E\,,$$

$$\frac{\partial E}{\partial t}=4\pi e n u+4\pi V_0\sigma\,\delta(z-V_0t)\,, \tag{1}$$

$$\frac{\partial E}{\partial z}=4\pi e(n_0-n)-4\pi\sigma\,\delta(z-V_0t)\,,$$

where $n=n(z,t)$ and $u=u(z,t)$ are the density and velocity of the plasma electrons, n_0 is the equilibrium density equal to the density of ions in rest, e is the absolute magnitude of electron charge, respectively.

Though we have a one-dimension problem, nevertheless the system of equations (1) is complicated and therefore we shall limit ourselves studying the stationary solutions of this system when all the functions depend on z and t in the form of combination of these variables $\tilde z = z - V_0 t$. One may show that in the one-dimensional case under consideration the obtained stationary solutions coincide with the solutions having the form of simple waves. To obtain the stationary solutions of the system (1) one may rewrite them introducing the potential φ instead of E $\left(E=-\dfrac{\partial\varphi}{\partial z}\right)$:

$$\frac{d}{d\tilde z}\,n(u-V_0)=0$$

$$\frac{d}{d\tilde{z}}\left(m(u-V_0)^2/2 - e\varphi\right) = 0\,,$$
$$\frac{d^2\varphi}{d\tilde{z}^2} = 4\pi e(n-n_0) + 4\pi\sigma\delta(\tilde{z}). \qquad (2)$$

From the first two equations of the system (2) it follows $n(u-V_0) = \text{const.}$ and $(u-V_0)^2 - \frac{2e}{m}\varphi = \text{const.}$ To determine the integration constants entering into these current and energy conservation laws let us choose the potential φ equal to zero in the point where $\tilde{z} = 0$ and also assume $u(0)=0$ and $n(0) = n_0$ (for an another choice of $U(0)$ we should have a constant flux of plasma electrons). Then we obtain:

$$n = \frac{n_0 V_0}{V_0 - u}\,, \quad \frac{m(u-V_0)^2}{2} - e\varphi = \frac{mV_0^2}{2}\,,$$

or substituding $\chi \equiv 1 + e\varphi / \frac{mV_0^2}{2}$

$$n = n/\sqrt{\chi}\,, \quad V_0 - u = V_0\sqrt{\chi}\,. \qquad (3)$$

Writing the last expression we have taken into account the fact that the root square has sign "plus" since the electron density n has to be a certain positive magnitude.

Substituding n from (3) into the last equation (2) and using dimensionless variables we obtain

$$\frac{d^2\chi}{dx^2} - \left(\frac{1}{\sqrt{\chi}} - 1\right) = 2\sqrt{2}\,\frac{V_\sigma}{V_0}\,\delta(x) \qquad (4)$$

where

$$x \equiv \frac{\sqrt{2}\,\omega_p\tilde{z}}{V_0}\,, \quad V_\sigma = \sqrt{\frac{\pi\sigma^2}{mn_0}}\,, \quad \omega_p = \sqrt{\frac{4\pi n_0 e^2}{m}}$$

This non-uniform equation may be replaced by an uniform one with an additional condition for the discontinuity of the first derivative of the function χ in the points $x = 0$:

$$\left.\frac{d\chi}{dx}\right|_{-\epsilon}^{\epsilon} = 2\sqrt{2}\,\frac{V_\sigma}{V_0}\,, \qquad \epsilon \to +0\,. \tag{5}$$

The general solution of the uniform equation has been considered in the work [2]. Multiplying equation (4) (without right hand side) by $d\chi/dx$ one may carry out first integration. Determining the integration constant from the condition (5) taking into account the equations $\chi(0)=1$, $\chi'(+0)=\sqrt{2}\,\frac{V_\sigma}{V_0}$ and $\chi'(-0)=-\sqrt{2}\,\frac{V_\sigma}{V_0}$ one ontains:

$$\left(\frac{d\chi}{dx}\right)^2 + 2\left(\sqrt{\chi}-1\right)^2 = 2\,\frac{V_\sigma^2}{V_0^2} \tag{6}$$

From this equation it follows immediately:

$$\sqrt{\chi_{min}} \leq \sqrt{\chi} \leq \sqrt{\chi_{max}}\,,$$

where (7)

$$\sqrt{\chi_{min}} = 1-\frac{V_\sigma}{V_0}\,, \quad \sqrt{\chi_{max}} = 1+\frac{V_\sigma}{V_0}$$

Since $\sqrt{\chi_{min}}$ is a positive magnitude one obtains $V_\sigma < V_0$. Let us also note that, as it is seen from (3) and (7) , the magnitude V_σ is the maximum velocity of plasma electrons $-V_\sigma \leq u \leq V_\sigma$ and

$$\frac{n_0 V_0}{V_0+V_\sigma} \leq n \leq \frac{n_0 V_0}{V_0-V_\sigma}\,.$$

Thus, the stationary state in cold plasma with ions in rest and in the presence of a "charged plane" type beam is possible only when the condition $V_\sigma < V_0$ is satisfied. According to its physical meaning V_σ determines the maximal kinetic energy of plasma electrons which corresponds to the electrostatic energy per electron of the "charged plane" field $\left((2\pi\sigma)^2/8\pi n_0 = = mV_\sigma^2/2\right)$.

Integrating equation (6) we obtain the following implicit expression for $\chi(x)$:

$$\sqrt{2}\,x = \pm 2\left\{-\sqrt{\frac{V_\sigma^2}{V_0^2}-\left(\sqrt{\chi}-1\right)^2}+\frac{V_\sigma}{V_0}+\arcsin\frac{\sqrt{\chi}-1}{V_\sigma/V_0}\right\} \quad (8)$$

This relation together with the expressions (3) determines the dependence of n, u and other characteristics of the non - linear stationary waves excited in plasma on the electron beam parameters V_σ and V_0.

The dependence of the magnitudes χ, $\frac{n}{n_0}$, $\frac{u}{V_0}$ and $E/2\pi\sigma = \pm\sqrt{1-\frac{V_0^2}{V_\sigma^2}\left(\sqrt{\chi}-1\right)^2}$ on the dimensionlees variable X for two values of the ratio $V_\sigma/V_0 = 0.5$ and $V_\sigma/V_0 \approx 1$ is given schematically in Fig.1 - 4.

Now let us consider the problem of the excitation of longitudinal non-linear stationary oscillations in relativistic cold plasma by a relativistic electron beam having the above discussed shape. Using the complete system of hydrodynamics relativistic equations and Maxwell's equations one may obtain the following equation for the longitudinal component of the dimensionless momentum $\rho_z = \bar{u}_z/\sqrt{1-\bar{u}_z}$, $\bar{u}_z = u_z/c$, $\beta = V_0/c$:

$$\frac{d^2}{d\tilde{z}^2}\left(\beta\rho_z-\sqrt{1+\rho_z^2}\right)+\frac{\omega_p^2}{c^2}\,\frac{\rho_z}{\beta\sqrt{1+\rho_z^2}-\rho_z}=0 \tag{9}$$

with the boundary condition at the points $\tilde{z}=0$:

$$\frac{d}{d\tilde{z}}\left(\beta\rho_z-\sqrt{1+\rho_z^2}\right)\Big|_{-\epsilon}^{\epsilon}=-\frac{4\pi\sigma e}{mc^2},\quad \epsilon\rightarrow+0. \tag{10}$$

After the first integration assuming $\rho_z(0)=0$ and using the condition (10) we obtain the following equation:

$$\frac{d}{d\tilde{z}}\left(\beta\rho_z-\sqrt{1+\rho_z^2}\right)=\frac{\sqrt{2}\,\omega_p}{c}\left[\sqrt{1+\rho_\sigma^2}-\sqrt{1+\rho_z^2}\right]^{1/2} \tag{11}$$

where

$$\rho_\sigma=\rho_{z\,max}=\sqrt{\left(1+\frac{\pi\sigma^2}{2n_0mc^2}\right)^2-1}\;;\quad -\rho_\sigma\leqslant\rho_z\leqslant\rho_\sigma. \tag{12}$$

The integration of (11) results in the following formula for the longitudinal component of the velocities $\bar{u}_z$ as functions of $\tilde{z}=z-v_0t$:

$$\frac{\sqrt{2}\,\omega_p\tilde{z}}{c}=\pm\int_0^{\bar{u}_z}\frac{(\beta-\bar{u}_z)\,d\bar{u}_z}{(1-\bar{u}_z^2)^{3/2}\left[(1-\bar{u}_{z\,max}^2)^{-1/2}-(1-\bar{u}_z^2)^{-1/2}\right]^{1/2}} \tag{13}$$

where $\bar{u}_{z\,max}=\rho_\sigma/\sqrt{1+\rho_\sigma^2}$ and ρ_σ are determined by the relation (12).

In the general case the integral in (13) may be expressed by elliptic functions [3, 4]. The space period $\tilde{Z}_\lambda$ (wavelength) and the oscillation frequency ω are given by the expressions

$$\tilde{Z}_\lambda=\frac{4\sqrt{2}\,V_0}{\omega_p}\left(1+\left(\sqrt{1-\bar{U}_{z\,max}^2}\right)^{-1}\right)^{1/2}\left[E-\frac{\sqrt{1-\bar{U}_{z\,max}^2}}{1+\sqrt{1-\bar{U}_{z\,max}^2}}K\right], \tag{14}$$

$$\omega=2\pi V_0/\tilde{Z}_\lambda$$

where K and E are the total elliptic integrals of <u>the first</u> and second kind with module $\kappa=\left[\frac{1-\sqrt{1-\bar{U}_{z\,max}^2}}{1+\sqrt{1-\bar{U}_{z\,max}^2}}\right]^{1/2}$.

In the two limit cases $\bar{U}_{z\,max}\ll 1$ and $1-\bar{U}_{z\,max}\ll 1$ the oscillation frequency is given by the following simple expressions:

$$\omega=\omega_p\left(1-\frac{3}{16}\bar{U}_{z\,max}^2\right),\quad \bar{U}_{z\,max}\ll 1 \tag{15}$$

$$\omega=\frac{\pi\omega_p}{2\sqrt{2}}\left(1-\bar{U}_{z\,max}^2\right)^{1/4},\quad 1-\bar{U}_{z\,max}\ll 1 \tag{16}$$

Let us also give the expressions for the density n and field E_z as functions of U_z (It is not difficult to obtain them using the initial system of equations of motion, continuity and Maxwell's equations):

$$n=\frac{n_0\beta}{\beta-\bar{u}_z},\quad E_z=\pm\frac{mc}{e}\sqrt{2}\,\omega_p\left\{\left(\sqrt{1-\bar{U}_{z\,max}^2}\right)^{-1}-\left(\sqrt{1-\bar{U}_z^2}\right)^{-1}\right\}^{1/2} \tag{17}$$

The formulae (13), (14) and (17) determine all the characteristics of the longitudinal non-linear oscillations in dependence of the electron beam parameters σ and V_o. However, it is necessary to note that in the relativistic case the one-dimensional motion is impossible and velocity components u_x and u_y appear due to the presence of magnetic fields in the initial equations. The relation between the longitudinal and transversal oscillations results in a fact that the solutions of the equations for the velocity components are non-stable due to the terms proportional to $\beta^2 (\beta < 1)$ in the equations for the transversal components [3].

As the analysis shows the application of an external magnetic field does not remove the solution non-stability though changes its character.

Figure Captions

Fig. 1. The dependence of $\chi(x)$ on X for $V_\sigma/V_o = 0.5$ and $V_\sigma/V_o \approx 1$. The maximum, inflexion, and minimum take place at $X_{max} = \frac{1}{\sqrt{2}}\left(2\frac{V_\sigma}{V_o} + \pi\right)$; $X_{inf} = \frac{2}{\sqrt{2}}\left(2\frac{V_\sigma}{V_o} + \pi\right)$; $X_{min} = \frac{1}{\sqrt{2}}\left(2\frac{V_\sigma}{V_o} + 3\pi\right)$

Fig. 2. The dependence of $n(x)/n_o$ on X. The maximum and minimum take place at $X_{min} = \frac{1}{\sqrt{2}}\left(2\frac{V_\sigma}{V_o} + \pi\right)$, $X_{max} = \frac{1}{\sqrt{2}}\left(2\frac{V_\sigma}{V_o} + 3\pi\right)$.

Fig. 3. The dependence of $\frac{U(x)}{V_c}$ on X. The maximum and minimum take place at the same values of X as in Fig. 2.

Fig. 4. The dependence of $\frac{E(x)}{2\pi\sigma}$ on x; $-1 \leq \frac{E(x)}{2\pi\sigma} \leq 1$.

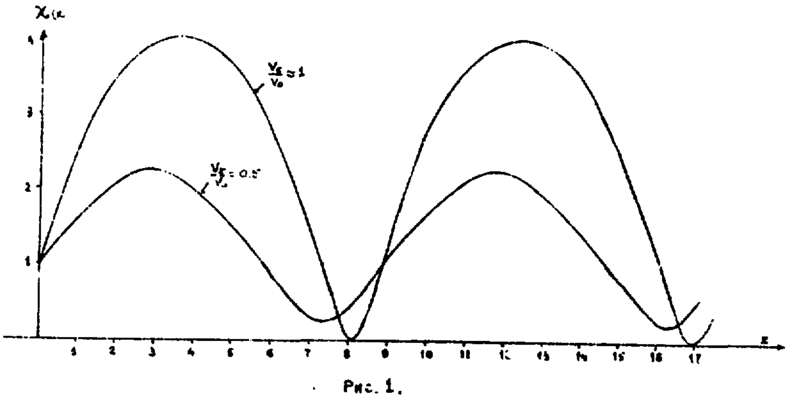

Рис. 1.

Fig.1

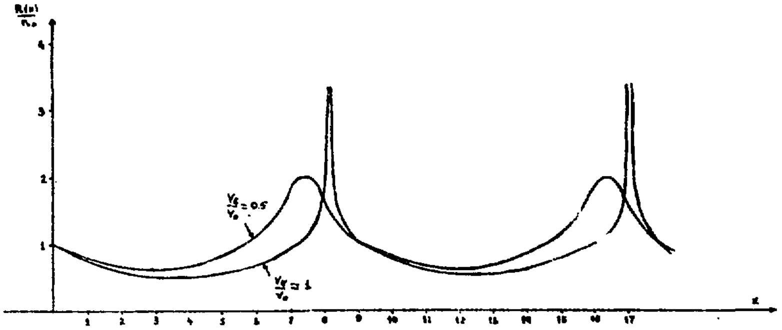

Рис. 2

Fig.2

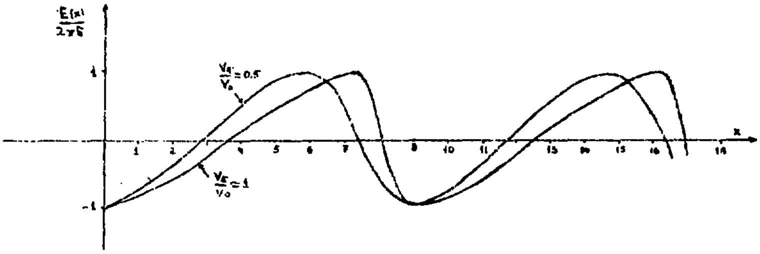

Рис 4

Fig.3

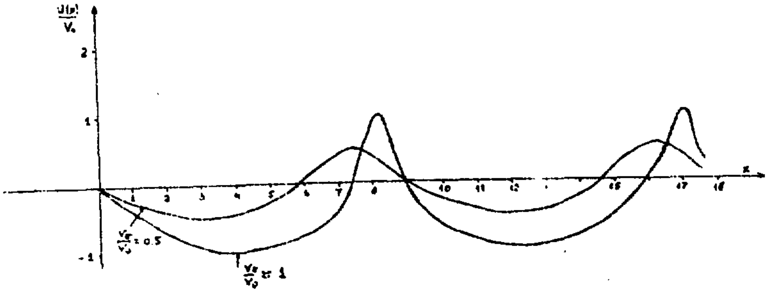

Рис 3.

Fig.4

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ВОЗБУЖДЕНИЕ НЕЛИНЕЙНЫХ КОЛЕБАНИЙ В
ПЛАЗМЕ ОГРАНИЧЕННЫМ ЭЛЕКТРОННЫМ ПУЧКОМ

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