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ONCE MORE ON MULTIPARTICLE REGGE POLES

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I. Introduction

In the series of papers[1-3] by the two authors it was discovered* that besides the cuts, the Regge poles, generated by multiparticle states in t-channel exist in the field theories.

The obtained poles are obvious to correspond to the allowance for the t-channel unitarity of multiple bound states, whereas Mandelstam cuts correspond to the allowance for confi-

* The previous investigation of three particle and four particle Regge poles were given in[4], where they were shown to be situated to the right from the corresponding cuts.

gurations in which the particles, interacting in pairs, produce the two-particle bound states only.

As it is shown in[1-3], the intercept of such poles increases quadratically with the number of particles in t-channel. Qualitatively, the quadratic dependence of the intercept in the number of particles in t-channel is connected, as it's easy to see, with the great number of diagrams found in[1-4] having with the given order of g^2 the same asymptotics as the corresponding Mandelstam cuts (i.e. with the same number of particles in t-channel).

On the other hand, as one can see from[1,2], we deal with the effective ladders, coalesced along all the length of their ribs** so that if there is the quadratic dependence of the intercept of the multiparticle pole $\alpha^{(N,N)}(0)$ on N, found in[1,2]

$$\alpha^{(N,N)}(0) = -2N+1 + g^2(0)N^2$$

(here $g^2(0)$ is proportional to g^2 square coupling constant of φ^3- theory and is defined by a "transverse integral", corresponding to the shrinkage of all horizontal lines of the diagram into the points[2]), then $\simeq N^2 g^2(0) \ln(S/m^2)$ particles are generated in S-channel.

** In what follows, for simplicity, we'll consider the case with $n_1 = n_2 = N$, where $n_{1,2}$ is a number of particles in t-channel emitting (absorbing) rungs, leading to the maximum asymptotics of an amplitude at the given number of particles of t-channel.

However an arbitrary large number of particles in S-channel can't be emitted (absorbed) by 2N particles of t-channel without loosing ladder configurations, that in terms of asymptotics seem to be advantageous, as there is the effective repulsion between near-by rapidities rungs, emitted (absorbed) by one rib[5], connected with the decrease in the cross-section of interacting pair-particles with Δy rapidities difference going into zero. In other words, there is a definite minimum value of Δ rapidity in the ladder configurations, which renders it inexpedient to have lesser rapidity difference between the neighbouring ladders.

We'll see that due to this circumstance, the quadratic dependence of the intercept on N, beginning with the certain N (as it will be shown below sufficiently large ($\approx 1/g^2$)) is changed to a weaker one. It is the purpose of our work to investigate the influence of the above mentioned "repulsion" between particles-rungs with near-by rapidities, having common ribs in the ladders, on the main parameters of multiparticle Regge poles.

We'll show, that inspite of the fact, that the intercept dependence on N at $Ng^2_{(0)} > 1$ is weakened with regard for "repulsion", multiparticle poles are located to the right of the corresponding cuts.

Besides, here we discuss briefly the problem of how far to the right of $j=-1$ the multiparticle Regge poles can go.

2. Intercept and a slope of 2N-particle reggeon with regard for"repulsion" between rungs.

For convenience, let's begin with the discussion of a diagram with 2N particles in t-channel, leading to N-reggeon cuts.

In fig.1, corresponding to this diagram, only horizontal rungs (partons?) of the diagram for an elastic scattering amplitude are given for simplicity.

Let it be p rungs in each of N ladders. The asymptotics of this diagram for a forward scattering has the form (everywhere further we are interested only in S-dependence, omitting inessential factors):

$$\left(\frac{s}{m^2}\right)^{-2N+1}\left(\frac{g^2(0)\,\ln^p(s/m^2)}{p!}\right)^N. \qquad (1)$$

The repulsion allowance in (I) leads effectively to the fact, that at the summation over p

$$p \leq p_0 = \ln(S/m^2)/\Delta.$$

Now, let's find diagrams of the same asymptotics obtained from the diagram of fig.1 by interchanging rungs from different ladders.

As it follows from Ref.[2], these interchangings for maximum saturation of diagrams with factors $\ln(S/m^2)$ should be done so, that out of 2N t-channel particles N particles always emitted rungs, and N particles absorbed them**.

Now, if we want to take into account "the repulsion" between rungs, having at least one common rib, then the above mentioned interactions are obvious to be done only between particles-rungs from different ladders with near-by rapidities (within the rapidity range $\Delta y \simeq e/_p \ln(s/m^2)$ of one particle).

In this case it's important that after the interchange on every rib, the number of particle rungs be the same.

One of the possible diagrams with the asymptotics(I) when two rungs from ladders 1 and 2 are interchanged, is shown in fig.2 (these two rungs are marked in fig.1 with crosses).

As a result, on every rapidity level we can make (N! -I) interchanges, bringing to diagrams that are topologically different from the diagram of fig.1 and have the same asymptotics (I).

Analogous interchanges should be made on all p rapidity levels so, that the total number of all diagrams with the asymptotics (I) at the given p will be equal to:

$$\sum_{K=0}^{p} C_K^p (N!-1)^K = (N!)^p \qquad (2)$$

and for the asymptotics of the sum of all such diagrams with the fixed number of rungs Np we'll have

$$\left(\frac{s}{m^2}\right)^{-2N+1} \left(\frac{(g^2\gamma_0)\ln(s/m^2))^p}{p!} \right)^N (N!)^p . \qquad (3)$$

Using Stirling formula we can put the expression (3) in the form:

$$\left(\frac{S}{m^2}\right)^{-2N+1}\left(\frac{g^2(0)\,N\ln(S/m^2)}{p}\right)^{pN} \qquad (3')$$

which shows that it can have the maximum at

$$p = p_{max} \equiv \frac{Ng^2(0)}{e}\ln(S/m^2).$$

Now, if $p_{max} < p_0 = \ln(S/m^2)/\Delta$ (i.e. $Ng^2(0) < e/\Delta$) then "the repulsion"between the rungs is still unessential and the asymptotics of the elastic forward scattering amplitude will be defined by p_{max} . For it we'll have:

$$\left(\frac{S}{m^2}\right)^{-2N+1}\left(\frac{g^2(0)\,N\ln(S/m^2)}{p_{max}}\right)^{Np_{max}} = \left(\frac{S}{m^2}\right)^{-2N+1+\frac{N^2g^2(0)}{e}} \qquad (4)$$

~~But if~~ $\cancel{p_{max} > p_0}$ ~~(i.e.~~ $\cancel{Ng^2(0) >}\ e/\Delta$) then " the repulsion" effectively comes into play, and for the forward amplitude asymptotics we'll get:

$$\left(\frac{S}{m^2}\right)^{-2N+1}\left(\frac{g^2(0)\,N\ln(S/m^2)}{p_0}\right)^{Np_0} = \left(\frac{S}{m^2}\right)^{-2N+1+\frac{N}{\Delta}\ln(g^2(0)N\Delta)} \qquad (5)$$

So, for the intercept of the multiparticle Regge pole*** we have two regimes (coinciding at $Ng^2(0)=e/\Delta$):

$$\alpha^{(N,N)}(0) = -2N+1+N^2g^2(0)/e\,, \quad Ng^2(0)<e/\Delta$$

$$\alpha^{(N,N)}(0) = -2N+1+\frac{N}{1}\ln(g^2(0)N\Delta), \quad Ng^2(0)>e/\Delta \tag{6}$$

We see that at large N ($N > \approx 5/g^2(0)$) the quadratic intercept dependence is substituted for the dependence $\sim N\ln N$.

However, for a very wide interval of N ($N \gg 1/g(0)$) the quadratic law for $\alpha^{(N,N)}(0)$, obtained in[1-3] isn't changed if we take into account "the repulsion".

It's obvious, that in this range of N for $q^2 \ll N^2m^2$ (q is a momentum transfer), the tragectories $\alpha^{(N,N)}(q)$ are straight line and their slope is equal to a two-particle ladder slope[6].

In the range of larger N ($Ng^2(0)>e/\Delta$), in analogy with[6], one can obtain the following expression for a trajectory of the pole at $q^2 \ll N^2m^2$:

*** Unfortunately, the complexity of the problem doesn't allow us to give the strict proof for the pole character of this singularity, but this statement was proved at $N=2$ in Ref[4], and in Ref.[2] some physical arguments (remaining in force with regard for repulsion too) pointing out to the fact, that in the general case we deal with a pole, were given.

$$\alpha^{(N,N)}(q) = -2N+1+\frac{N}{\Delta}\ln\left[Ng^2(0)\Delta\left(1-\beta\, q^2/N^2m^2\right)\right] \quad (7)$$

(β is a constant)

whence, the dependence $\alpha'^{(N,N)}(0) \sim 1/N$, is similar to 2N-particle cut follows for 2N particle pole slope.

3. Concluding remarks.

In conclusion we'll make some remarks. One can see from the formula (6), that in principal, with the increase of N, one can approach $\alpha^{(N,N)}(0) = 1$ value and even pass it at $N > e^{2\Delta/g^2(0)\Delta}$. However, one should take into account, that in this range of values for $N \sim 1/g^2(0)$, the ribs of the ladders begin essentially screening each other (even in the limits of one rapidity "level").

In fact, all 2N of ribs in the impact parameter space are located inside the circle of the finite radius $R \simeq \gamma/m$ [2]. The probability of interaction of a rung with one of 2N ribs will be $W_N \simeq g^2/NR^2m^2$.
At $N \simeq c/g^2$ W_N can become of an order of unity, and "a flow" of rungs will be absorbed intensively already by the nearest ribs. It'll lead to decrease in the number of effective interchanges $\sim\left(\frac{C}{g^2}\right)!$ at $N > N_c' = c/g^2$ instead of $N!$ and all this will depend on how C is large.

Unfortunately, we can't calculate C. It's reasonable to suppose that C is determined by S-unitarity condition, and at $C = e^{2\Delta}/\Delta$ we would have a pomeron.

However another situation when $C < e^{2\Delta}/\Delta$ and the 2N particles pole doesn't reach $j = 1$ is quite possible.

In this situation at $N > N_c$ it's more advantageous for rungs to be generated of multiparticle poles with $N \leq N_c$ number of particles in t-channel.

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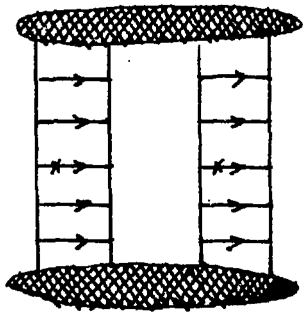

Fig.1

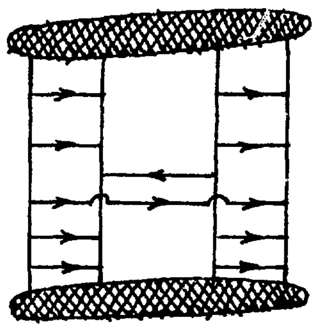

Fig.2

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