индекс 3624

А.П.ГАРЯКА

ЭНЕРГЕТИЧЕСКИЙ СПЕКТР НУКЛОНОВ В АТМОСФЕРЕ ПРИ ЛОГАРИФМИЧЕСКОМ РОСТЕ СЕЧЕНИЯ НЕУПРУГОГО ВЗАИМОДЕЙСТВИЯ

(на английском языке)

Ереванский физический институт

Тех.редактор А.С.Абрамян

Заказ 367 ВФ-03946 Тираж 299

Подписано к печати 27/IX-78г. Формат издания 60x84/16

0,5 уч.изд.л. Ц. 3 к.

Издано Отделом научно-технической информации Ереванского физического института, Ереван-36, пер.Маркаряна 2

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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

ЕФИ-327(52)-78

A.P.GARYAKA

ENERGY SPECTRUM OF NUCLEONS IN ATMOSPHERE IN THE CASE OF LOGARITHMIC RISE OF INELASTIC INTERACTION CROSS-SECTION

ԵՐԵՎԱՆ ЕРЕВАН
1978

At present the rise of the nucleon-nucleon and nucleon-nucleus inelastic cross-section is established for energies up to 10^3 GeV in direct accelerator [1] and cosmic rays experiments [2] .

Earlier, in cosmic rays an indication was received of the rise of inelastic cross-section on the basis of data on the increase of the index of energy spectra at mountain altitudes in comparison with that for primary spectra [3] , and on the change of attenuation length of ionization bursts [4] .

Recently, in a number of cosmic rays experiments the lower bound on the nucleon attenuation cross-section in air up to 30 TeV was measured [5,6] Available data don't contradict to the parametrization in the form:

$$\sigma_{NAir} = \sigma_0 (1 + g \ln E) . \qquad (1)$$

The energy spectra of nucleons in atmosphere in the case of cross-section rise have been investigated in a number of works [7,8,9] .

The case of logarithmic rise was carefully analyzed in N.L.Grigorov's paper for $g \ll 1$.

Up to now it was not proposed the exact analytic solution for arbitrary values of g .

In the present paper the exact solution of the diffusion equation for nucleons in the atmosphere is given for the case of $E^{-\gamma}$ primary spectrum with the logarithmic rise of inelastic cross section under the assumption that the scaling law took place for secondary nucleons spectrum in nucleon-nucleus interactions.

The diffusion equation of vertical nucleon flux in atmosphere is written as follows

$$\frac{\partial I(z,E)}{\partial z} = -\frac{I(z,E)}{\lambda(E)} + \int_0^1 \frac{I(z,E/u)\,w(u)}{\lambda(E/u)}\frac{du}{u}\,, \qquad (2)$$

where z, $w(u)$ and $\lambda(E)$ are the depth of atmosphere in g/cm , the scale-invariant secondary nucleon spectrum, and the nucleon mean free path in air, respectively, $u = E/E_0$ is the elasticity of the nucleon.

For constant cross-section solution is known to have the form

$$I(z,E) = I_0 E^{-\gamma} \exp\left(-\frac{z}{\lambda}\left(1 - \langle u^{\gamma-1}\rangle\right)\right)$$

$$\langle u^{\gamma-1}\rangle = \int_0^1 du\cdot u^{\gamma-1} w(u)\,, \quad I(0,E) = I_0 E^{-\gamma}.$$

For the case $\lambda(E) = \lambda_0 (1 + g \ln E)^{-1}$ we shall try to find the solution in the form

$$I(z,E) = I(0,E)\exp\left(-\frac{\chi_1(z)}{\lambda(E)} + \chi_2(z)\right), \qquad (3)$$

Substituting (3) into (2) we get

$$\frac{\partial I(z,E)}{\partial z} = -I(z,E)\left(\frac{1 - \langle u^{\gamma-1+\frac{g}{\lambda_0}\chi_1}\rangle}{\lambda(E)} + \frac{g}{\lambda_0}\langle u^{\gamma-1+\frac{g}{\lambda_0}\chi_1}|\ln u|\rangle\right) \qquad (4).$$

Differentiating the left -hand side of (4) and dividing by $I(z,E)$ we get:

$$-\frac{\chi_1'(z)}{\lambda(E)} + \chi_2'(z) = -\frac{1 - \langle u^{\gamma-1+\frac{g}{\lambda_0}\chi_1}\rangle}{\lambda(E)} + \frac{g}{\lambda_0}\langle u^{\gamma-1+\frac{g}{\lambda_0}\chi_1}|\ln u|\rangle$$

Equating separately the energy dependent and independent terms we get

$$\chi_1'(z) = 1 - \int_0^1 u^{\gamma-1+\frac{g}{\lambda_0}\chi_1} w(u)\,du \qquad (5)$$

$$\chi_2'(z) = \frac{g}{\lambda_0}\int_0^1 u^{\gamma-1+\frac{g}{\lambda_0}\chi_1(z)} w(u)\,|\ln u|\,du\,. \qquad (6)$$

Under the boundary condition $\chi_1(0) = 0$ the solution of equation (5) is as follows:

$$z = \int_0^{\chi_1} \frac{dx}{1 - \langle u^{\gamma-1+\frac{g}{\lambda_0}x}\rangle} \qquad (7)$$

The solution of equation (6) is

$$\chi_2 = \frac{g}{\lambda_0}\int_0^{\chi_1} d\chi\, \frac{\langle u^{\gamma-1+\frac{g}{\lambda_0}\chi}|\ln u|\rangle}{1 - \langle u^{\gamma-1+\frac{g}{\lambda_0}\chi}\rangle}\,. \qquad (8)$$

As was expected, χ_2 is not an independent function of z , it depends on χ_1 . For any specific form of function $w(u)$, we shall find, integrating the left-hand side of (7) $z = f(\chi_1)$ and, respectively $\chi_1 = f^{-1}(z)$ (f^{-1} - the inverse function of f).

Thus, we show that the solution of (2) in the form (3) exists and gives the procedure for the obtaining of functions $\chi_1(z)$ and $\chi_2(z)$. Reverting to equation (4) it is easy to see that the solution can be written in the form

$$I(z,E) = I_0 E^{-\gamma}\exp\left(-\frac{z}{\lambda(E)} + \int_0^z dz' \int_0^1 du\, w(u)\frac{u^{\gamma-1+\frac{g}{\lambda_0}\chi_1(z')}}{\lambda(E/u)}\right) \qquad (9)$$

which is analogous to the solution for the constant λ.

There is the obvious transition to the solution for constant λ and to the approximate one $I^0(z,E) = I(0,E)\,e^{-z/\lambda(E)}$ for $g \ll 1$.

The expression (9) may be also written in the form

$$I(z,E) = I(0,E)\exp\left(-z/L(z,E)\right) =$$
$$= I(0,E)\exp\left[-z\left(\frac{1}{\lambda(E)} - \int_0^1 dt \int_0^1 du\, \frac{w(u)u^{\gamma-1+\frac{g}{\lambda_0}\chi_1(tz)}}{\lambda(E/u)}\right)\right]. \qquad (9')$$

By analogy, we name $L(z,E) = \left(\frac{1}{\lambda(E)} - \int_0^1 dt \int_0^1 du\, \frac{w(u)u^{\gamma-1+\frac{g}{\lambda_0}\chi_1(tz)}}{\lambda(E/u)}\right)$

an attenuation length of nucleons with energy E up to depth z. As soon as $\chi_1(z)$ is a rising function of its argument, the attenuation length decreases with the increase of atmospheric depth due to the rise of the index of energy spectrum with the depth.

The obtained solution is correct for any g, and at any z/λ.

As it was shown in Ref.[7], for $g \ll 1$ an approximate solution is sufficiently precise for practical applications. The exact solution may be of great importance in two cases. First, beginning from some energy interval, the cross-section may begin change with the energy as

$$\sigma = \sigma_0 (1 + G \ln E)$$

, with $G \geq 1$. Second, the cross-section may have the form (which doesn't contradict to the available data)

$$\sigma = \sigma_0 (1 + g(1 + h \ln E) \ln E).$$

, with $h \ll 1$.

In the first case our solution appear to be the most acceptable for the analysis of nucleon spectra, and in the second case, it may serve as good approximate solution with. $G = g(1 + h \ln E)$

In Fig. 1 we present χ_1 as a function of z/λ_0 for various values of g = 0,04, 0.1, 0.5, 1.0 and the ratio $I(z,E)/I^0(z,E)$ where $I^0(z,E)$ is the approximate solution [7]. We took $W(u) = 1$ to describe the secondary nucleon spectrum.

The author would like to thank Profs. A.Ts.Amatuni, S.G.Matinyan and E.A.Mamidzhanyan for helpful discussions and valuable comments.

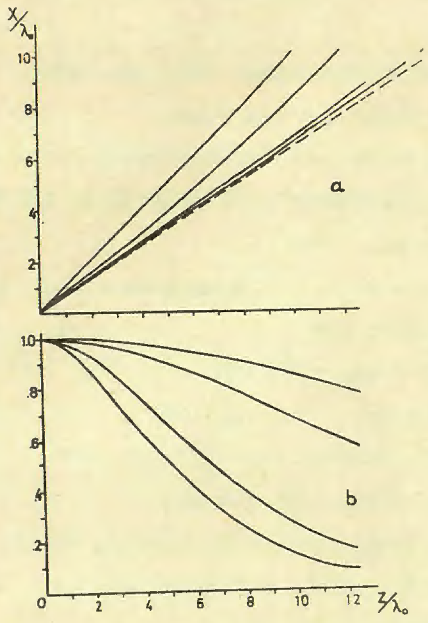

FIGURE CAPTIONS

a) χ_1 as a function of z/λ_0 at g = 0.04, 0.1, 0.5, 1

b) Ratio $I(z,E)/I^0(z,E)$ as a function of z/λ_0.

$I(z,E)$ - exact solution,

$I^0(z,E)$ - approximate solution

g = 0.04, 0.1, 0.5, 1.

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The manuscript was received 6 June 1978