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A.Yu. KHODJAMIRIAN

DISPERSION SUM RULES FOR THE AMPLITUDES OF RADIATIVE TRANSITIONS IN QUARKONIUM

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А.Ю. ХОДЖАМИРЯН

ДИСПЕРСИОННЫЕ ПРАВИЛА СУММ ДЛЯ АМПЛИТУД РАДИАЦИОННЫХ ПЕРЕХОДОВ В КВАРКОНИИ

Предложен способ получения дисперсионных правил сумм для амплитуд радиационных переходов в кварконии ($c\bar{c}, b\bar{b}, \ldots$), основанный на асимптотической свободе на малых расстояниях в квантовой хромодинамике и на аналитичности. Эти правила сумм позволяют модельно-независимым образом оценить амплитуды радиационных переходов. Их вывод основан на вычислении на малых расстояниях треугольной вакуумной амплитуды, индуцированной токами тяжёлых кварков с соответствующими квантовыми числами J^{PC}, и на последующем использовании аналитических свойств этой амплитуды по двум кинематическим инвариантам. В качестве примера эти правила сумм получены для электрических дипольных переходов $^3P_0 \leftrightarrow {}^3S_1$. Применённые к чармонию, они находятся в неплохом согласии с имеющимися экспериментальными данными по переходам $\chi_0 \to J/\psi\,\gamma$, $\psi' \to \chi_0\gamma$. Получены также правила сумм для магнитно-дипольных переходов $^3S_1 \leftrightarrow {}^1S_0$, приводящие для чармония к противоречию с экспериментом, если отождествить $^1S_0$ -уровень с резонансом $X(2,83)$. Противоречие снимается, если $m(^1S_0) \simeq 3{,}0$ Гэв. Приводятся также оценки вероятностей переходов рассмотренных типов для b -кваркония.

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A dispersion approach is suggested to obtain sum rules for the amplitudes of radiative transitions in quarkonium ($c\bar{c}$, $b\bar{b}$,...) which is based on the asymptotic freedom at small distances in QCD and on the analyticity. These sum rules allow to estimate the amplitudes of radiative transitions in a model independent way. Their derivation consists in the calculation at small distances of the triangle vacuum amplitude induced by heavy quark currents with appropriate J^{PC} quantum numbers and in the subsequent application of the analytical properties of this amplitude on the kinematical variables. As an example the sum rules are derived for electric dipole transitions $^3P_0 \leftrightarrow {}^3S_1$. Applied to charmonium they agree with experimental data on $\chi_0 \to J/\psi\,\gamma$, $\psi' \to \chi_0\gamma$ transitions. The sum rules are obtained also for magnetic dipole transitions $^3S_1 \leftrightarrow {}^1S_0$. For charmonium they contradict the experiment if we identify the $^1S_0$-level with X(2,83).This contradiction disappears if $m(^1S_0) = 3{,}0$ GeV. The estimates are presented for the rates of radiative transitions of the considered types in b - quarkonium.

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A.Yu. KHODJAMIRIAN

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The method of dispersion sum rules in QCD developed in Ref. [1] has appeared extremely productive for the model independent calculation of various annihilation widths of heavy $q\bar{q}$-mesons or quarkonium levels ($q = c, b, t \ldots$) .

In this paper a certain simple generalisation of this technique is suggested which leads to dispersion sum rules for the amplitudes of radiative transitions between quarkonium levels.

Usually, these processes are treated in terms of nonrelativistic potential models so that the results are essentially dependent on the choice of the potential. In contrast with this treatment, here as in Ref. [1] we use only the very general principles of asymptotic freedom and analyticity.

To extract the matrix elements of radiative transitions between quarkonium levels with the given quantum numbers $(J^{PC})_1$ and $(J^{PC})_2$ we consider the triangle vacuum amplitude induced by the electromagnetic current j^{em}_{μ} corresponding to the real photon emission and by two q-quark currents j_1 and j_2

with quantum numbers $(J^{PC})_1$ and $(J^{PC})_2$ respectively.

The asymptotic freedom allows to calculate this amplitude at small distances. (In this paper we confine ourselves to the zeroth order on α_s and neglect the gluon corrections.) Then, the analytical properties on two kinematical variables $s_1 = p_1^2$ and $s_2 = p_2^2$ (p_1 and p_2 are the external momenta at j_1 and j_2 vertices), lead to dispersion sum rules relating the calculated bare amplitude to the j_μ^{em} matrix elements between physical $q\bar{q}$ - states with $(J^{PC})_1$ and $(J^{PC})_2$. Following Ref. [1] we extract in these relations the contribution of quarkonium levels which are below the thresholds of the "open" q -flavor production. Then we use the parton-like approximation for the remaining higher states contribution equating it to the corresponding bare quark one.

Finally, we obtain relations for amplitudes of those radiative transitions between lowest quarkonium levels which are detectable a have experimentally measurable rates.

The plan of this paper is as follows.

First, the general outline of sum rules derivation is shown for the simplest electric dipole transitions between scalar ($^3P_0$) and vector ($^3S_1$) quarkonium levels which we denote S and V respectively. The obtained sum rules are then applied for the transitions of this type ($\chi_0(3415) \to J/\psi\,\gamma$, $\psi' \to \chi_0(3415)\,\gamma$) in charmonium and appear to be in agreement with the available experimental data.

Further, to demonstrate the variety of possibilities of our approach, we derive the analogous sum rules for magnetic dipole transitions between vector and pseudoscalar ($^1S_0$ or P) levels. Being applied to charmonium these sum rules allow to estimate the $J/\psi \to \eta_c \gamma$ transition rate which turns out to agree with the simplest nonrelativistic model prediction. In particular, both predictions drastically contradict the experimental upper bound if we identify η_c with the $X(2{,}83)$-resonance. Such contradiction disappears if m_{η_c} is somewhat larger ($\simeq 3{,}0$ GeV).

In conclusion we discuss briefly the application of the obtained sum rules to the radiative transitions in heavy quarkoniums and present crude estimates of the $S \leftrightarrow V$ and $V \leftrightarrow P$ transitions rates in b - quarkonium (the Υ -resonance family) .

Now we turn to derivation of the sum rules for radiative transitions between $^3P_0$ and $^3S_1$ (S and V) quarkonium levels. For the other types of transitions the general outline is the same .

Consider first a triangle amplitude which is the vacuum average of the T-product of three currents :

$$A_{\mu\nu}(k,p_1,p_2) = eQ\int dx\,dy\, e^{i(p_2x+ky)} \times$$
$$\times \langle 0 | T\{ j_1(0) j_\mu^{em}(x) j_{2\nu}(y)\} |0\rangle \equiv \qquad (1)$$
$$\equiv \left(\delta_{\mu\nu} - \frac{p_{2\mu}k_\nu}{(kp_2)}\right) A(s_1,s_2),$$

where Q is the q -quark charge; $j_\mu^{em} = \bar{q}^i \gamma_\mu q^i$ is the electromagnetic current, corresponding to the real photon emission with momentum K ; $j_1 = \bar{q}^i q^i$ and $j_{2\nu} = \bar{q}^i \gamma_\nu q^i$ are quark scalar and vector currents (sum over color indices); p_1 and p_2 are the external momenta at j_1 and j_2 vertices ($p_{1,2}^2 \equiv S_{1,2}$) respectively.

We suppose that j_1 and j_2 have nonvanishing matrix elements only between vacuum and those hadron states which contain pairs of heavy quarks $q\bar{q}$. Transitions into hadrons, containing only quarks of other flavors are negligible due to the well known OZI rule.

At small S_1 and S_2 ($S_{1,2} << m^2$, m is the quark mass) the amplitude $A(s_1,s_2)$ corresponds to highly virtual vacuum fluctuations taking place at distances $\sim 1/2m$ much smaller than the mean hadron size (the confinement length) . At such distances the strong interaction is weakened and is characterized in QCD by the quark gluon constant $\alpha_s \simeq 0{,}2$ (see for example Ref. [1]) . In zeroth order on α_s the amplitude A is determined by the triangle q-quark Feynman diagram shown in Fig. 1 . We denote by

$$A^{\circ}_{\mu\nu} \equiv \left(\delta_{\mu\nu} - \frac{p_{2\mu} K_\nu}{(K p_2)} \right) A^{\circ}(s_1,s_2)$$

the contribution of this diagram.

Since the $A(s_1,s_2) \simeq A^{\circ}(s_1,s_2)$ approximation is valid in a certain small but finite region near $S_1 = S_2 = 0$ the following equations take place:

$$\partial^k/\partial s_2^k \, \partial^n/\partial s_1^n A(0,0) \simeq \partial^k/\partial s_2^k \, \partial^n/\partial s_1^n A^{\circ}(0,0) \qquad (2)$$

at n and k not too large. If Eqs. (2) were true for all n and k it would mean the coincidence of $\mathcal{A}$ and $\mathcal{A}^{\circ}$ in the whole region of s_1 and s_2. This is in fact impossible since at too large positive values of s_1 and/or s_2 (at large distances) the asymptotic freedom is destroyed and the zeroth approximation on α_s is inapplicable.

To proceed, we use the analytical properties of $\mathcal{A}$ and $\mathcal{A}_0$ on two kinematical variables s_1 and s_2. The amplitude $\mathcal{A}$ defined by Eq.(1) corresponds to the physical vacuum fluctuations and its imaginary part on s_1 (s_2) is determined by the physical scalar (vector) intermediate states : resonances (the lowest and radially excited levels of quarkonium) and two- or many-particle states containing q-flavored hadron pairs.

We confine ourselves to the case when there is one S --level (two V -levels) of quarkonium below the threshold of q-flavor production $s_1(s_2) < s_0$ with mass m_S ($m_V, m_{V'}$) so that $m_V < m_S < m_{V'}$. (Such situation takes place in the charmonium spectroscopy where S is the $\chi_0(3415)$ --meson and $V(V')$ is the $J/\psi(\psi')$ -meson .) The derivation presented further is easily modified for more complicated lay-out of the states below threshold.

Applying the unitarity condition twice, first on s_1 and then on s_2, we obtain the following representation of the double imaginary part $\rho(s_1,s_2) \equiv \mathrm{Im}_{s_2} \mathrm{Im}_{s_1} \mathcal{A}$ of the amplitude $\mathcal{A}$:

$$\rho(s_1,s_2)=\pi^2 e Q\{\mathcal{F}(S\to V\gamma)g_V m_V^2\delta(s_2-m_V^2)+ +\mathcal{F}(V'\to S\gamma)g_{V'}m_{V'}^2\delta(s_2-m_V'^2)\}g_S m_S^3\delta(s_1-m_S^2)+ +\rho^c(s_1,s_2). \tag{3}$$

where g and $\mathcal{F}$ determine the following matrix elements:

$$m_S^2 g_S=\langle 0|j_1|S\rangle\,,\quad V_\rho m_V^2 g_V=\langle 0|j_{2\rho}|V\rangle, \quad V'_\rho m_{V'}^2 g_{V'}=\langle 0|j_{2\rho}|V'\rangle \tag{4}$$

(V_ρ, V'_ρ are the wave functions of the vector mesons)

$$m_S V_\rho\left(\delta_{\mu\rho}-\frac{p_{2\mu}k_\rho}{(p_2k)}\right)\mathcal{F}(S\to V\gamma)=\langle S|j_\mu^{em}|V\rangle$$
$$m_S V'_\rho\left(\delta_{\mu\rho}-\frac{p_{2\mu}k_\rho}{(p_2K)}\right)\mathcal{F}(V'\to S\gamma)=\langle S|j_\mu^{em}|V'\rangle \tag{5}$$

so that $\mathcal{F}(S\to V\gamma)$ and $\mathcal{F}(V'\to S\gamma)$ are the invariant amplitudes of $S\to V+\gamma$ and $V'\to S+\gamma$ radiative transitions. The corresponding transition rates are

$$\Gamma(S\to V\gamma)=\frac{1}{2}\alpha Q^2|\mathcal{F}(S\to V\gamma)|^2 m_S\left(1-\frac{m_V^2}{m_S^2}\right) \tag{6}$$

$$\Gamma(V'\to S\gamma)=\frac{1}{6}\alpha Q^2|\mathcal{F}(V'\to S\gamma)|^2\frac{m_S^2}{m'_V}\left(1-\frac{m_S^2}{m'^2_V}\right). \tag{7}$$

Note that these amplitudes in terms of nonrelativistic potential models are proportional to the overlap integrals:

$$\mathcal{F}(S\to V\gamma) \sim \omega \int_0^\infty R_S(r) R_P(r)\, r^3 dr$$
$$\mathcal{F}(V'\to S\gamma) \sim \omega' \int_0^\infty R'_S(r) R_P(r)\, r^3 dr \, , \qquad (8)$$

where $\omega = m_S - m_V$, $\omega' = m_{V'} - m_S$; R_P and R_S (R'_S) are the radial wave functions of ${}^3P_0$ and ${}^3S_1(2^3S_1)$ -levels respectively. As for g_V , $g_{V'}$ and g_S , these constants are proportional to $R_S(0)$, $R'_S(0)$ and $R_P(0)$.

Lastly, the $\rho^c(s_1, s_2)$ in Eq. (3) denotes the total contribution of the intermediate states which are above the q-flavor production threshold into the double imaginary part of the amplitude A . The set of these states with quantum numbers 1^{--} (0^{++}) including both higher quarkonium levels (as V'' , S' etc. in our case) and continuum of states containing pairs of b - flavored hadrons is called continuum and denoted C_V (C_S) below.

Defined in this way, the spectral function $\rho^c(s_1, s_2)$ includes amplitudes of complicated and experimentally unobservable transitions of the following types : $C_S \to V(V') + \gamma$, $C_V \to S + \gamma$, $C_V \to C_S + \gamma$, $C_S \to C_V + \gamma$ (see Fig.2).

The next step is to postulate the validity of double dispersion relations with n (k) subtractions on s_1 (s_2) for the amplitude A at $s_1 = s_2 = 0$ and apply the representation (3). Then the following expression for the l.h.s. of Eq.(2) emerges:

$$\frac{1}{k!\,n!}\,\partial^k/\partial s_2^k\,\partial^n/\partial s_1^n\,A(0,0)=\frac{1}{\pi^2}\int\frac{ds_2}{s_2^{k+1}}\int\frac{ds_1}{s_1^{n+1}}\rho(s_1,s_2)=$$

$$=\frac{eQ\mathcal{F}(S\to V\gamma)g_V g_S}{m_V^{2k}m_S^{2n-1}}+\frac{eQ\mathcal{F}(V'\to S\gamma)g_{V'}g_S}{m_{V'}^{2k}m_S^{2n-1}}+ \tag{9}$$

$$+\int\frac{ds_2}{s_2^{k+1}}\int\frac{ds_1}{s_1^{n+1}}\rho(s_1,s_2).$$

The calculation of the r.h.s. of Eq.(2) determined by the triangle diagram in Fig.1 gives the following result which is also convinient to express in a form of double dispersion integral :

$$\frac{1}{k!\,n!}\,\partial^k/\partial s_2^k\,\partial^n/\partial s_1^n\,A^0(0,0)=\frac{1}{\pi^2}\int_{4m^2}^{\infty}\frac{ds_2}{s_2^{k+1}}\int_{4m^2}^{\infty}\frac{ds_1}{s_1^{n+1}}\rho^0(s_1,s_2),$$

$$\rho^0(s_1,s_2)=3/4\,eQm\,s_1\left[2v-(1-v^2)\ln\frac{1+v}{1-v}\right]\delta(s_1-s_2) \tag{10}$$

where $v\equiv\sqrt{1-4m^2/}$.

The origin of the δ - function in Eq.(10) is easy to understand if we proceed from the imaginary part of the triangle diagram on one of the variables s_1 or s_2 (when two internal quark lines are on the mass shell) and then do the analytical continuation on the other . The only singularity which emerges as a result of this continuation is the pole at $s_1 = s_2$ (corresponding to the third internal quark line on the mass shell) . The absence of any anomalous threshold for this triangle diagram is guaranteed by the fact that one of its external masses is equal to zero viz. the photon mass ($k^2=0$).

Finally, equating the r.h.s. of Eqs. (9) and (10) according to Eq.(2) , we obtain at various n and k a set of double dispersion sum rules relating the integrals over physical and bare quark states.

The practical use of these sum rules becomes possible if one can estimate the contribution of continuums C_V and C_S given by the integral from $\rho^c(s_1,s_2)$ in Eq. (9).

Following Ref.[1] we adopt the simplest hypothesis that this integral can be approximated by the integral from the bare spectral function $\rho^0(s_1,s_2)$ (corresponding to the free quark intermediate states) taken over the $s_1, s_2 > s_0$ region. Then the sum rules take the following final form:

$$
\begin{aligned}
&\mathcal{F}(S\to V\gamma) + \left(\frac{g_{V'}}{g_V}\right)\left(\frac{m_V}{m'_V}\right)^{2k}\mathcal{F}(V'\to S\gamma) = \\
&= \frac{m_V^{2k} m_S^{2n-1}}{g_V g_S}\int\limits_{4m^2}^{s_0}\frac{ds_2}{s_2^{k+1}}\int\limits_{4m^2}^{s_0}\frac{ds_1}{s_1^{n+1}}\,\rho^0(s_1,s_2) = \\
&= \frac{3}{4\pi^2 g_S g_V}\left(\frac{m_V}{2m}\right)^{2k}\left(\frac{m_S}{2m}\right)^{2n-1}\int\limits_0^{v_0} v\,dv\,(1-v^2)^{n+k-1}\left(2v-(1-v^2)\ln\frac{1+v}{1-v}\right).
\end{aligned}
\tag{11}
$$

where $v_0 \equiv \sqrt{1-4m^2/s_0}$.

Note that the parton-like approximation that we have used may be more crude than the analogous approximation applied in Ref.[1] . Strictly speaking, we equate integrals taken not only from different spectral functions ρ^c and

ρ^0 but also over different integration regions. The reason is that some of the complicated diagrams of $C_V \to C_S + \gamma$, $C_S \to C_V + \gamma$ transitions contributing to the spectral function $\rho^c(s_1, s_2)$ (see Fig.2d) may have anomalous thresholds at $s_1 < s_0$ or $s_2 < s_0$. The detailed discussion of these diagrams is however out of scope of this paper.

Now we turn to applying the obtained sum rules (11) for the main $S \leftrightarrow V$ radiative transitions in charmonium $\chi_0(3415) \to J/\psi + \gamma$, $\psi' \to \chi_0(3415) + \gamma$.

Taking from Ref. [1] the values of $g_V = g_{J/\psi} = 0,13$, $g_V' = g_{\psi'} = 0,075$, $g_S = g_{\chi^0} = 0,105$ and $m = m_c = 1,25 \div 1,30$ GeV along with $m_V = m_{J/\psi} = 3,095$ GeV, $m_V' = m_{\psi'} = 3,685$ GeV and charm threshold $\sqrt{s_0} = 3,7$ GeV, we obtain from Eq.(11) the sum rules for amplitudes $\mathcal{F} = \mathcal{F}(\chi_0 \to J/\psi\,\gamma)$ and $\mathcal{F}' = \mathcal{F}(\psi' \to \chi_0 \gamma)$ in numerical form. They are presented in Table 1 at $n,k \leqslant 4$.
Recall that the value of the c-quark mass which we have used [1] is in fact the mass at small distances or the current quark mass. The values of $g_{J/\psi}$ and $g_{\psi'}$ according to their definition (4) are determined by the J/ψ and ψ' leptonic widths:

$$\Gamma(V \to e^+e^-) = \frac{4\pi\alpha^2 Q^2}{3} g_V^2 \qquad (12)$$

while the value of g_{χ^0} is estimated [1] from the diangle sum rules for the $\langle j_1 j_1 \rangle$ product since this constant has no direct physical meaning.

Making use of the available experimental data [2] : $B(\chi_0 \to J/\psi\gamma) = 3,3 \pm 1,0$ % , $B(\psi' \to \chi_0\gamma) = 7 \pm 2$ %, $\Gamma_{tot}(\psi') = 228 \pm 56$ keV and of the theoretical estimate[1] $\Gamma_{tot}(\chi_0) \cong 5$ MeV, we obtain from Eqs. (6)-(7) the following experimental values of amplitudes

$$|\mathcal{F}| = 0,41 \pm 0,20 \quad , \quad |\mathcal{F}'| = 0,255 \pm 0,18 \tag{13}$$

which are to be compared with those estimated from our sum rules. The last values obtained by the solution of the relations of Table 1 at definite n and $k,k+1$ are presented in Table 2 .

In spite of the approximate character of these estimates their agreement with experiment is rather good (within experimental errors) at $n,k \leq 4$ and is violated for higher moments. This last circumstance is natural since as it has been noted earlier the zeroth order on α_s is inapplicable at large n and k . At the same time, the parton-like approximation of the continuum is improved for large moments since it gives decreasing relative contribution into the total integral over physical states while $n + k$ increases . For example, in the case considered above this contribution is equal to $\cong$55,30, 20,10,5% at $n + k = 2,3,4,5,6$ respectively. Therefore, we may assume that at certain optimal n and k the sum rules are most reliable. In our case we expect it to be at $n,k = 2,3$ or $n + k = 4,5,6$. However this assumption is difficult to test since experimental errors in (13) are still large.

Note the high sensitivity of the obtained sum rules to the c - quark mass value. The best agreement with experiment seems to be achieved at $m_c = 1{,}30$ GeV . We also draw attention to the different signs of the $\mathcal{F}$ and $\mathcal{F}'$ at $n + k \geqslant 4$. This fact maybe is not occasional but corresponds in terms of the nonrelativistic potential approach to the different signs of the overlap integrals (see Eq.(8)).

Now we shall derive the analogous sum rules for the magnetic dipole transitions between $^3S_1$ and $^1S_0$ (V and P) quarkonium levels. The other types of radiative transitions will be considered elsewhere.

We assume that below the q - flavor threshold there are two pseudoscalar resonances P and P' with masses m_P and $m_{P'}$ so that $m_P < m_V < m_{P'} < m_{V'}$ and four $V \leftrightarrow P$ transitions are possible.

We proceed from a triangle amplitude

$$A^5_{\mu\nu}(K, p_1, p_2) = \varepsilon_{\mu\nu\alpha\beta} K_\alpha p_{1\beta} A^5(s_1, s_2)$$

analogous to that defined in Eq. (1) .This amplitude is induced by quark currents $j^{em}_\mu(K)$, $j^5_1(p_1) = i\bar{q}^i\gamma_5 q^i$ and $j_{2\nu}(p_2) = \bar{q}^i\gamma_\nu q^i$ (the corresponding external momenta are in parentheses). Applying to the amplitude A^5 the above described procedures of calculation at small distances, analytical continuation on $s_1 = p_1^2$ and $s_2 = p_2^2$ and parton-like approximation of the continuum, we obtain the following sum rules :

$$\mathcal{F}(V\to P\gamma) + \left(\frac{g_{P'}}{g_P}\right)\left(\frac{m_P}{m_P'}\right)^{2K+1}\mathcal{F}(P'\to V\gamma) +$$

$$+\left(\frac{g_{V'}}{g_V}\right)\left(\frac{m_V}{m_{V'}}\right)^{2n}\left\{\mathcal{F}(V'\to P\gamma) + \left(\frac{g_P'}{g_P}\right)\left(\frac{m_P}{m_P'}\right)^{2K+1}\mathcal{F}(V'\to P'\gamma)\right\} =$$

$$= \frac{3}{2\pi^2 g_P g_V}\left(\frac{m_V}{2m}\right)^{2n-2}\left(\frac{m_P}{2m}\right)^{2K+1}\int_0^{V_0} v\,dv\,(1-v^2)^n \,\ln\frac{1+v}{1-v}\,, \qquad (14)$$

where $m_P^2 g_P = \langle 0|j_1^5|P\rangle$, $m_{P'}^2 g_{P'} = \langle 0|j_1^5|P'\rangle$,

$$m_P^{-1}\varepsilon_{\mu\nu\alpha\beta}V_\nu K_\alpha P_{2\beta}\,\mathcal{F}(V\to P\gamma) = \langle P|j_\mu^{em}|V\rangle \qquad (15)$$

etc. The transition rates are determined by the amplitudes $\mathcal{F}(V\to P\gamma)$ etc. as follows :

$$\Gamma(V\to P\gamma) = \frac{\alpha Q^2}{24}\left|\mathcal{F}(V\to P\gamma)\right|^2\frac{m_V^3}{m_P^2}\left(1-\frac{m_P^2}{m_V^2}\right)^3 \qquad (16)$$

$$\Gamma(P'\to V\gamma) = \frac{\alpha Q^2}{8}\left|\mathcal{F}(P'\to V\gamma)\right|^2 m_P'\left(1-\frac{m_V^2}{m_{P'}^2}\right)^3. \qquad (17)$$

The considered lay-out of the lowest pseudoscalar levels is expected for charmonium where P is the η_c and P' is the η_c' . However, the reliable information on these levels and on the magnetic dipole transitions relating them to J/ψ and ψ' is yet absent.

Recall that the simplest nonrelativistic model gives:
$\mathcal{F}(\psi'\to\eta_c\gamma) = \mathcal{F}(\eta_c'\to J/\psi\gamma) = 0, \quad \mathcal{F}(\psi'\to\eta_c'\gamma) = \mathcal{F}(J/\psi\to\eta_c\gamma)$
(see for example Ref[3]). At least we can suppose that the first three amplitudes do not exceed the last one .

Then noticing that these three amplitudes enter the l.h.s. of Eq. (13) with coefficients less than unit (in fact $g_{\psi'} < g_{J/\psi}$ and we can assume that $g_{\eta_c'}/g_{\eta_c} \simeq g_{\psi'}/g_{J/\psi}$). we may leave there only the amplitude $\mathcal{F}(J/\psi \to \eta_c \gamma)$ and estimate its order of value. Here two possibilities are worth discussing:

1. $m_{\eta_c} = 2{,}83$ GeV i.e. the η_c is identified with the mysterious X(2,83) - resonance which has been observed in $e^+e^- \to J/\psi \to 3\gamma$. [4]. In this case the diangle sum rules obtained in Ref. [1] lead to the following estimate $g_{\eta_c} \simeq$ $\simeq 0{,}09$. With this value of g_{η_c} Eq.(13) gives at n,k=2,3 the prediction for the $\mathcal{F}(J/\psi \to \eta_c\gamma)$ in the following limits:

$$\mathcal{F}(J/\psi \to \eta_c\gamma) = 4{,}3 \div 5{,}4$$

corresponding to the width

$$\Gamma(J/\psi \to \eta_c\gamma) = 42 \div 60 \text{ keV}$$

which is much larger than the experimental upper bound [5]

$$\Gamma(J/\psi \to X\gamma) < 1{,}7\% \cdot \Gamma_{tot}(J/\psi) = 1{,}2 \text{ keV}, \quad (18)$$

2. $m_{\eta_c} = 3{,}0$ GeV . Then we have [1] $g_{\eta_c} \cong 0{,}13$ and our prediction is

$$\mathcal{F}(J/\psi \to \eta_c\gamma) = 4{,}2 \div 5{,}7$$

which is close to that obtained at $m_{\eta_c} = 2{,}83$ GeV but gives a much smaller width

$$\Gamma(J/\psi \to \eta_c\gamma) = 1{,}7 \div 3{,}3 \text{ keV}$$

due to a reduced phase space in Eq.(16). The last value is of the same order as the experimental bound (18).

It is interesting to note that both estimates of the amplitude $\mathcal{F}(J/\psi \to \eta_c \gamma)$ are close to the prediction of the simplest nonrelativistic model of magnetic dipole transition (see for example Ref. [1]).

Recall that the conclusion about m_{η_c} being close to 3,0 GeV was also obtained in Ref. [6] as a result of detailed analysis of certain diangle sum rules with the account of gluon corrections.

In conclusion we will briefly discuss the application of the obtained results to the heavy quarkoniums (q = b,t,...). On general grounds we expect that the sum rules in the form obtained above are less reliable in these cases. The reason is that while the quark mass increases, the threshold of quark pair production and that of the physical continuum ($4m^2$ and S_0 respectively) get closer. Hence, the relative contribution of the continuum into the dispersion integrals increases. For example at $m = m_b \simeq 4,5$ GeV the parton-like estimates of this contribution into the sum rules (11) and (14) give 80,65,60 % and 40,35,30 % at n + k = = 4,5,6 respectively. So the possible deviation of these estimates from a true value of the continuum contribution may be considerable.

To reach some independence of the accuracy of the continuum estimates we need to consider higher moments of the sum rules (n,k >> 4) for which the zeroth approximation on α_s is already inapplicable and relevant gluon corrections are to be accounted. Recall that for heavy quarks there are specific "Coulomb" gluon corrections (see Ref. [7]) .

Here we confine ourselves to the use of the sum rules in the zeroth order on α_s given by Eq.(11) and (14) . The estimates of transition amplitudes obtained in this way may be only of a correct order of value.

In particular we will estimate the amplitudes of the main $S \leftrightarrow V$ transitions in b - quarkonium. We use the b - -quark mass value $m = m_b = 4{,}65$ GeV obtained in Ref.[7] and the following parameters of Υ and Υ'-resonances :

$m_\Upsilon = 9{,}46$ GeV , $m_{\Upsilon'} = 10{,}01$ GeV , $g_\Upsilon = 0{,}07$, $g_{\Upsilon'} = 0{,}04$ (the values of constants $g_{\Upsilon,\Upsilon'}$ are determined by means of Eq.(12) from the Υ , Υ' leptonic widths; the experimental information on these resonances is taken from Ref.[8]. Assuming that $m_{\chi_b} \cong 9{,}7$ GeV , $g_{\chi_b} \simeq g_\Upsilon$ (χ_b is the lowest $^3P_0$ -level of b-quarkonium) and $\sqrt{S_0} = 11{,}0$ GeV(the b-flavor threshold) and neglecting the contribution of Υ'' we obtain from Eq.(11) at n,k = 2,3 :

$$\mathcal{F}(\chi_b \to \Upsilon\gamma) \cong 0{,}04 , \quad \mathcal{F}(\Upsilon' \to \chi_b\gamma) \cong 0{,}025$$

These amplitudes according to Eqs.(6),(7) correspond to the following widths:

$$\Gamma(\chi_b \to \Upsilon\gamma) \cong 2{,}7 \text{ keV}, \quad \Gamma(\Upsilon' \to \chi_b\gamma) \cong 0{,}4 \text{ keV}$$

We may estimate also the rate of the main magnetic dipole transition $\Upsilon \to \eta_b\gamma$ (where η_b is the b-quark analogue of the η_c) leaving in Eq. (14) only the amplitude $\mathcal{F}(\Upsilon \to \eta_b\gamma)$ and assuming that $m_{\eta_b} = 9{,}40$ GeV, $g_{\eta_b} \simeq g_\Upsilon$ Then $\Gamma(\Upsilon \to \eta_b\gamma) \simeq 10^{-3}$ keV .

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Table 1 . The sum rules for the amplitudes $\mathcal{F}$ and $\mathcal{F}'$ of the $\chi_0 \to J/\psi\,\gamma$ and $\psi' \to \chi_0 \gamma$ transitions.

k	The r.h.s. of Eq.(11)	The l.h.s. of Eq.(11) at m_c = 1,25 GeV (m_c = 1,30 Gev)			
	n	1	2	3	4
1	$\mathcal{F} + 0{,}41\,\mathcal{F}'$	0,45	0,535	0,64	0,825
		(0,36)	(0,41)	(0,46)	(0,55)
2	$\mathcal{F} + 0{,}29\,\mathcal{F}'$	0,44	0,53	0,68	0,89
		(0,34)	(0,38)	(0,45)	(0,555)
3	$\mathcal{F} + 0{,}20\,\mathcal{F}'$	0,43	0,56	0,73	0,995
		(0,31)	(0,37)	(0,46)	(0,57)
4	$\mathcal{F} + 0{,}14\,\mathcal{F}'$	0,46	0,60	0,82	1,13
		(0,31)	(0,375)	(0,47)	(0,62)

Table 2. The amplitudes $\mathcal{F}$ and $\mathcal{F}'$ obtained by the solution of Table 1 relations at various n and k, and at m_c = 1,25 Gev (m_c = 1,30 Gev)

n	K = 1,2		K = 2,3		K = 3,4	
	$\mathcal{F}$	$\mathcal{F}'$	$\mathcal{F}$	$\mathcal{F}'$	$\mathcal{F}$	$\mathcal{F}'$
1	0,41	0,08	0,41	0,11	0,53	-0,5
	(0,29)	(0,17)	(0,24)	(0,33)	(0,31)	(-0,05)
2	0,36	0,42	0,63	-0,33	0,70	-0,67
	(0,31)	(0,25)	(0,41)	(0,11)	(0,37)	(-0,08)
3	0,77	-0,33	0,84	-0,55	1,03	-1,50
	(0,43)	(0,08)	(0,48)	(-0,11)	(0,49)	(-0,17)
4	1,05	-0,54	1,16	-1,20	1,44	-2,25
	(0,57)	(-0,04)	(0,60)	(-0,17)	(0,74)	(-0,83)

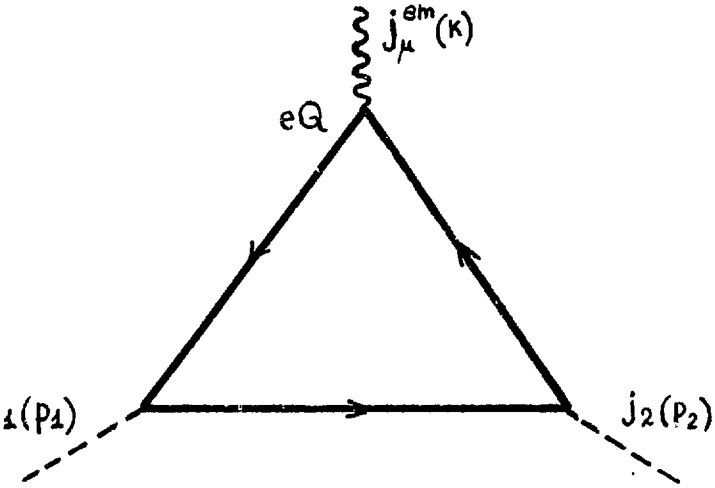

Fig. 1

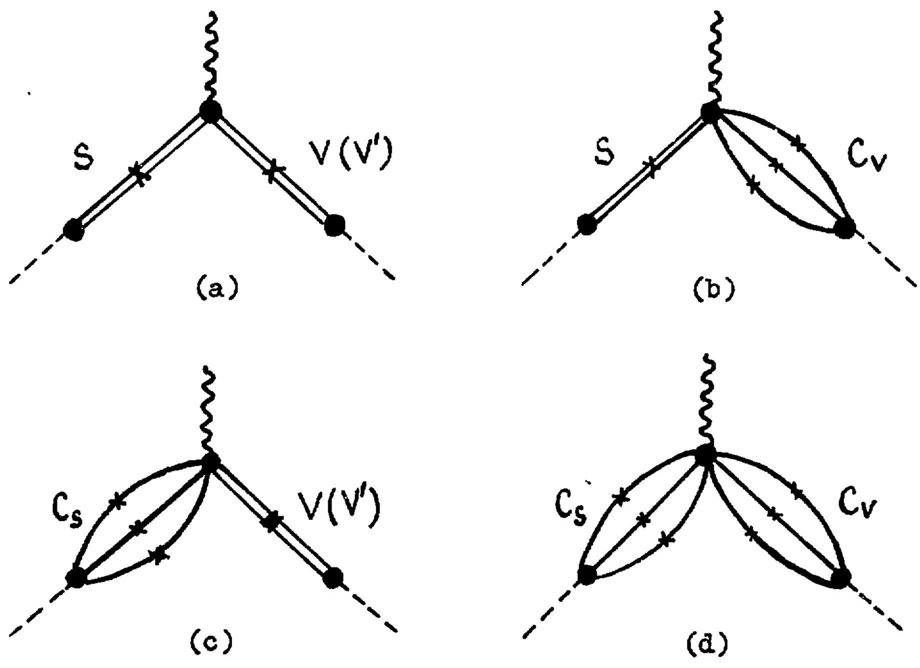

Fig.2

FIGURE CAPTIONS

Fig.1 The Feynman diagram corresponding to the zeroth approximation on α_s of the amplitude (1) .

Fig.2 The graphical representation of the r.h.s. of Eq.(3): the diagram (a) ((b) - (d)) corresponds to the resonance contributions (to the continuum contribution $\rho^c(s_1, s_2)$).

R E F E R E N C E S

1. V.A.Novikov et al, Phys.Rev.Lett. 38,626 (1977) ; Phys. Lett. 67B, 409 (1977) ; Phys.Reports 41C , 1 (1978)
2. Review of Particle Properties, Phys.Lett.75B , 1 (1978)
3. L.B.Okun, A.Yu.Khodjamirian, Pisma v ZhETP, 23,46 (1976)
4. W.Braunschweig et al, Phys.Lett. 67B, 243 (1977)
5. C.J.Biddick et al, Phys.Rev.Lett. 38,1324 (1977)
6. M.A.Shifman et al, Phys.Lett. 77B, 80 (1978)
7. M.B.Voloshin, Yad.Phys. 29,1368 (1979)
8. H.Spitzer, DESY 78/56 (1978)

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А.Ю.ХОДЖАМИРЯН

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РАДИАЦИОННЫХ ПЕРЕХОДОВ В КВАРКОНИИ

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