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EXOTIC ATOMS PRODUCTION IN eN AND μN INTERACTIONS

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1. Introduction

In recent years some theoretical /1 , 2/ and experimental /3/ investigations were performed , devoted to hydrogen-like exotic atoms A_{eh} consisting of lepton ℓ and unstable hadron h . These considerations are interesting , for example for pion charge radius determination by measuring the Lamb shift in $(\mu\pi)$-atom. The fine structure and Lamb shift of the $(\mu\pi)$-atom energy levels were calculated in Ref./1/ in one- and two-photon exchange approximation including hadronic vacuum polarization effects. The contribution of the pion size to the full shift is of the order of 1%.

The $(\mu\pi)$-atoms were discovered for the first time at the Brookhaven National Laboratory /3/. They were produced in $K_L^0 \to A_{\mu\pi} + \nu$ decay. The theoretical consideration of this generation mechanism of $(\mu\pi)$-atoms was examined in Ref./2/. The branching ratio for this rare decay was found to be of the order of 10^{-7}.

In this paper we consider another mechanism of the exotic atoms production, the mechanism in which the ℓh -atoms are generated in two-particle eN and μN interactions:

$$\ell(K) + N(p) \to A_{\ell h}(K_1) + h'(p_1') \tag{1}$$

The quantities in brackets represent the corresponding momenta of particles (atom), and

2. The Cross Section

The process (1) in the lowest order in electromagnetic interaction is described by the diagram of Fig.1.

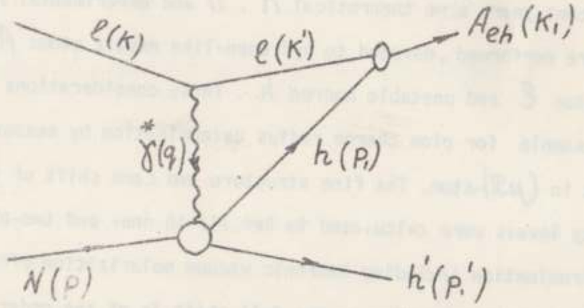

Fig.1

The corresponding invariant amplitude

$$\mathcal{M} = \frac{\overset{*}{\psi}_n(0)}{\sqrt{2\mu}} \bar{u}_{K'} \gamma_\mu u_K \frac{4\pi\alpha}{-q^2} < h(p) h'(P_1') | j^\mu | N(P) > , \quad (2)$$

where $\mu = \frac{m_e m_h}{m_e + m_h}$ is reduced mass of the A_{eh} , $m_1 = m_e + m_h$, $K' = \frac{m_e}{m_1} K_1$, $P_1 = \frac{m_h}{m_1} K_1$, $q = K - K'$ is momentum transfer, $\psi_n(q)$ is the atom Schrodinger wave function, n is the principal quantum number. Since $\psi_{n\ell}(r) \sim r^\ell$ at $r \to 0$, the atoms are produced in the s -state, for which

$$|\psi_n(0)|^2 = \frac{\alpha^3 \mu^3}{\pi n^3} \quad (3)$$

In the incident lepton-nucleon c.m.s. ($\vec{K} + \vec{P} = 0$) for the spin averaged cross section of atoms production in the n-th excited state we have;

$$\frac{d\sigma_{(n)}(s,\theta)}{d\Omega} = \frac{\alpha^5 \mu^2 |\vec{K}_1|}{8\pi n^3 s |\vec{P}| q^4} \ell_{\mu\nu} W^{\mu\nu} , \quad (4)$$

where

$$\ell_{\mu\nu} = 2\left(K_\mu K'_\nu + K_\nu K'_\mu + \frac{q^2}{2} g_{\mu\nu} \right),$$

$$W_{\mu\nu} = \frac{1}{2} \sum < N(P) | j^\mu | h(P_1) h'(P_1') >< h(P_1) h'(P_1') | j^\nu | N(P) >,$$

$s = (K+P)^2$, θ is the scattering angle.

We introduce as usual /4/ the virtual photoproduction cross sections in the $\gamma^* N$ c.m.s. ($\overset{*}{\vec{P}} + \overset{*}{\vec{q}} = 0$)

$$\frac{d\sigma_{ab}(W,\overset{*}{\theta})}{d\Omega^*} = \frac{\alpha |\overset{*}{\vec{P}}_1|}{16\pi |\vec{P}| W} \overset{*}{\varepsilon}{}^\mu_a(q) \varepsilon^\nu_b(q) \; W_{\mu\nu} \quad (6)$$

where $\varepsilon^\mu_a(q)$ is the virtual photon polarization vectors ($a = +, 0, -$). $W = (P+q)^2$, θ^* is the scattering angle*.

Using the well-known properties of $\frac{d\sigma_{ab}}{d\Omega^*}$ following from the P -invariance and hermiticity of electromagnetic interaction , for the cross section of the process (1) we have

$$\frac{d\sigma_{(n)}(s,\theta)}{d\Omega} = \Gamma_n \left\{ \frac{d\sigma_{++}(W,\overset{*}{\theta})}{d\Omega^*} + \varepsilon \frac{d\sigma_{+-}(W,\theta^*)}{d\Omega^*} + \right.$$
$$\left. + \left[\varepsilon + \frac{2m_e^2}{q^2}(1-\varepsilon)\right] \frac{d\sigma_{00}(W,\theta^*)}{d\Omega^*} + 2\sqrt{\varepsilon\left[\varepsilon + 1 + \frac{4m_e^2}{q^2}(1-\varepsilon)\right]} \, \mathrm{Re} \frac{d\sigma_{+0}(W,\theta^*)}{d\Omega^*} \right. \quad (7)$$

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* Troughout this paper the quantities in the $\gamma^* N$ c.m.s. are labelled by $*$, the ones in the lab.frame - by $\sim$, and the ones in ℓN c.m.s. are unlabelled.

In expression (7)

$$\Gamma_n = \frac{4\alpha^4\mu^2 W|\overset{*}{\vec{p}}||\vec{K}_1|}{-q^2 n^3 s|\overset{*}{\vec{p}}_1||\vec{p}|(1-\varepsilon)} \tag{8}$$

ε is the virtual photon polarization parameter

$$\varepsilon = \left(1 - \frac{q^2}{2\tilde{K}_\perp^2}\right)^{-1} \tag{9}$$

where $\tilde{K}_\perp$ is a transverse component of $\tilde{\vec{K}}$ (with respect to $\tilde{\vec{q}}$) in the lab. frame ($\tilde{\vec{p}} = 0$).

Note, that all structure functions $\frac{d\sigma_{ab}(W,\overset{*}{\theta})}{d\Omega^*}$ are measured in $\ell+N \to \ell+h+h'$ processes.

3. Numerical Results at Low Energies

Some simple numerical estimates can be done if we take into account that at $q^2 \to 0$

$$\frac{d\sigma_{++}(W,\overset{*}{\theta})}{d\Omega^*} \to \frac{d\sigma^\gamma(W,\overset{*}{\theta})}{d\Omega^*}, \quad \frac{d\sigma_{a,0}(W,\overset{*}{\theta})}{d\Omega^*} \to 0, \tag{10}$$

where $\frac{d\sigma^\gamma(W,\overset{*}{\theta})}{d\Omega^*}$ is the unpolarized cross section for the ordinary photoproduction:

$$\gamma + N \longrightarrow h + h' \tag{11}$$

In ℓN -collisions $-q^2 < 10^{-2}\frac{\text{Gev}^2}{c^2}$ for $\tilde{K}_e^0 < 2{,}5\,\text{Gev}$ and all scattering angles. From $m_e << m_\pi, m_K$ it follows that $|\vec{K}'| << |\vec{K}|$ and $q \simeq K$, hence the eN and $\overset{*}{\gamma}N$ c.m.s. coincide. It is easy to show that in the above mentioned energy range $\varepsilon \simeq 0$, and we have the following simple expression for the cross section of the process $e+N \to A_{eh} + h'$:

$$\frac{d\sigma_{(n)}(s,\theta)}{d\Omega} = \frac{4\alpha^4 m_e^2}{-q^2 n^3} \quad \frac{d\sigma^\gamma(s,\theta)}{d\Omega} \tag{12}$$

We calculated the $(e\pi)$ -atoms production cross section in the ground state using the expression (12) and experimental data from /5/*).

The predicted magnitude of the cross section for $e+p \to A_{e\pi} + n$ process in energy range $0{,}3\,\text{Ge} < \tilde{K}_e^0 < 0{,}6\,\text{Gev}$ and θ = 0, 60 , 90 are presented in Table 1.

Table 1

The cross section of $e+p \to A_{e\pi} + n$ process

$10^{40}\cdot\frac{d\sigma_0}{d\Omega}$ in cm²/ster \ $\tilde{K}_e^0$ in Gev	0.3	0.35	0.4	0.45	0.5	0.55	0.6
θ = 0	3.2	7.3	7.5	7.9	7.5	7.7	8.5
θ = 60	2.0	1.5	0.9	0.64	0.5	0.46	0.47
θ = 90	1.5	0.9	0.43	0.27	0.21	0.17	0.19

In order to estimate $\mu+p \to A_{\mu\pi} + n$ process cross section we note that at $\theta = 0$, $\varepsilon = 0$ and $-q^2 \sim 10^{-2}\frac{\text{Gev}^2}{c^2}$ for $0{,}6\,\text{Gev} < \tilde{K}_\mu^0 < 1\,\text{GeV}$. Thus we again can determine $\frac{d\sigma_{ab}(W,\overset{*}{\theta})}{d\Omega^*}$ from experimental data on photoproduction. Then, taking into account (10) from (7) and (8) we have:

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*) The experimental errors are not taken into account in numerical estimations.

$$\frac{d\sigma_{(1)}^{\mu+p\to A_{\mu\pi}+n}}{d\Omega}\simeq(6\div7)\cdot10^{-38}\frac{cm^2}{ster}\qquad \text{at}\quad 0{,}6\,Gev<\tilde{K}^{\circ}_{\mu}<1\,Gev$$

Using K^+ photoproduction experimental data /6/ we obtain from Eq.(12)

$$\frac{d\sigma_{(1)\;(\theta\simeq30^{\circ})}^{e+p\to A_{eK}+\Lambda^{\circ}}}{d\Omega}\simeq(2\div2{,}4)\cdot10^{-42}\frac{cm^2}{ster}\qquad \text{at}\; 1\,Gev<\tilde{K}^{\circ}_{e}<2{,}2\,Gev$$

$$\frac{d\sigma_{\;(\theta\simeq30^{\circ})}^{e+p\to A_{eK}+\Sigma^{\circ}}}{d\Omega}\simeq(0{,}4\div1{,}8)\cdot10^{-42}\frac{cm^2}{ster}\quad \text{at}\; 1{,}3\,Gev<\tilde{K}^{\circ}_{e}<2{,}2\,Gev$$

One can conclude that the extreme small values of obtained cross-sections and the large probability of dissociating of fast hydrogen-like system in the target material /7/ show that the considered mechanism of exotic atom formation cannot be investigated experimentally at contemporary e - and μ -beams intensities.

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ОБРАЗОВАНИЕ ЭКЗОТИЧЕСКИХ АТОМОВ В eN И μN
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(на английском языке)

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