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THE GENERALIZED EFFECTIVE POTENTIAL AND ITS MOTION EQUATIONS

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ОБОБЩЕННЫЙ ЭФФЕКТИВНЫЙ ПОТЕНЦИАЛ И ЕГО УРАВНЕНИЯ ДВИЖЕНИЯ

С помощью преобразований Лежандра строится функционал $\Gamma(\varphi,G,S)$, зависящий от вакуумного среднего поля φ, двухточечной связной функции Грина G и вакуумного среднего от классического действия $S = \langle 0| S_{cl} |0\rangle$. Выводятся уравнения движения для функционала Γ на примере $g\varphi^3$ теории и предлагается итерационный метод их решения. Основное уравнение для Γ, которое решается посредством итераций, является не вариационным, как при обычных преобразованиях Лежандра, а обыкновенным. Развитый формализм легко обобщается на другие теории.

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By means of the Legendre transformations a functional $\Gamma(\varphi, G, S)$ is constructed which depends on φ - a possible expectation value of the quantum field, G - a possible expectation value of the 2-point connected Green function and $S = \langle 0 | S_{cl} | 0 \rangle$ - a possible expectation value of the classical action. The motion equations for the functional Γ are derived on the example of the $g\varphi^3$ theory and an iteration technique is suggested to solve them. A basic equation for Γ which is solved by means of iteration techniques is an ordinary and not a variation one, as it is the case at usual Legendre transformations. The developed formalism can be easily generalized as to other theories.

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1. Introduction

In the well-known works of Heisenberg and Euler /1/, Weisskopf /2/ and Schwinger /3/ the problem of polarization of vacuum by the external electromagnetic field was investigated. Schwinger obtained a general expression for the quantum correction to a classical action due to the vacuum polarization; this expression coincides with the one-loop expression for the effective action /4.5/. Together with the quantum correction the Lagrange function for the constant electromagnetic field is given in /1,2,3/.

In the work of Batalin, Matinyan and Savvidy the problem of vacuum polarization by a non-abelian gauge field, free of sources, was investigated *). In a particular case, when a field is covariant-constant /6/, just as in quantum electrodynamics, there was obtained the Lagrange function together with the quantum correction (see formula 3.18 in /6/). A real

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*) The restriction by such fields is due to the fact that in this case one can explicitely prove the gauge invariance of the effective action /6,7/

part of the Lagrangian (see formula 3.19 in /6/) for the magnetic field has the form /8/:

$$\mathcal{L} = -\frac{H^2}{2} - \frac{11 C_2(G)}{96\pi^2}(gH)^2\left[\ln\left(\frac{gH}{\mu^2}\right) - \frac{1}{2}\right] \qquad (1.1)$$

where $C_2(G)$ is a Kasimir's operator of the gauge group G (this result is valid also for the electric field if the substitution $H \to E$ is done). The energy density $\mathcal{E}(H) = -\mathcal{L}(H)$ has a minimum in the point /8/:

$$gH_{vac} = \mu^2 \exp\left\{-\frac{48\pi^2}{11 C_2(G) g^2}\right\} \qquad (1.2)$$

Using the result obtained by means of the renormalized group /9/ and assuming $\int_g \frac{dx}{\beta(x)} < \infty$ for the average field we have/8/:

$$gH_{vac} = \mu^2 \exp\left\{\int_g^\infty \frac{d\varkappa}{\beta(\varkappa)}\right\} \qquad (1.3)$$

However the total imaginary part of the Lagrangian (3.18,/6/)

$$2\,\mathrm{Im}\,\mathcal{L} = \frac{1}{4\pi^2}\left[\pi(gf_1)^2 + (gf_1)(gf_2)\sum_{K=1}^{\infty}\frac{(-1)^K}{K\,\mathrm{sh}\left(\frac{f_1}{f_2}K\pi\right)}\right. \qquad (1.4)$$

in the magnetic field is equal to /10,11/ $2\,\mathrm{Im}\,\mathcal{L}(H) = \frac{(gH)^2}{4\pi}$ and it is of the same order as the imaginary part in the electric field /6/ $2\,\mathrm{Im}\,\mathcal{L}(E) = \frac{(gE)^2}{48\pi^2}$. The imaginary part $\mathcal{L}$ in the magnetic field is due to availability of the unstable mode /10,11/ $K_0^2 = K_n^2 - gH$ which brings to the fact that the level /8/

$$\mathcal{E}(H_{vac}) = -\frac{11 C_2(G)}{192\pi^2}\mu^4 \exp\left\{-\frac{96\pi^2}{11 C_2(G) g^2}\right\} \qquad (1.5)$$

is unstable and, hence, the real vacuum lies lower *).

It seems to us that, although the covariant-constant field is not stable, nevertheless the average field value given by expression (1.2-3) is correct.

Therefore it is important to develop such a formalism which wouldn't be sensitive to a concrete configuration of a field. For this reason we want to construct a generalization of the effective potential $\Gamma(\varphi; G, S)$ which depends already not only upon $\varphi = \langle 0|\varphi|0\rangle$ but also upon a possible expectation value of the classical action $S = \langle 0|S_{c\ell}|0\rangle$ and which permits to calculate**) S_{vac}. The gauge- and Lorentz-invariance of this quantity makes it convenient for investigations.

To realize this program we have used the higher Legendre transformation technique developed by a number of authors /12-18/. The substantial change consists in introducing in

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*) Note, that in the field $\vec{A}_\mu = \vec{i}B_\mu + \vec{j}C_\mu + \vec{K}\mathcal{D}_\mu$, where B, C, and $\mathcal{D}$ are constant coefficients, there are unstable modes as well /19/.

**) As comparing S_{vac} with the vacuum expectation for the free theory, one can judge on the fundamental state displacement. In quantum mechanics with Lagrangian $\mathcal{L} = \dot{\varkappa}^2 - \omega^2 x^2$ there takes place the ratio $\langle \varkappa^2 \rangle = \langle \varkappa^2 \rangle = \frac{\hbar\omega}{4}$ so that $\langle \mathcal{L} \rangle = 0$ and by its difference from zero, when including non-linear terms, one can judge on the vacuum displacement.

the generating functional apart from usual sources of $J\cdot\varphi$ type, also the source $L\cdot S(\varphi)$, where L is the number*), and $S(\varphi)$ is a classical action.

In the second section the functional $\Gamma(\varphi, G, S)$ is determined and its motion equations (2.12-13), (216 -17) are derived. In the third section these equations are investigated, and it is found that one ordinary equation (3.13) is enough to construct Γ. In the fourth section the equation (3.13) is solved by means of the iteration technique. Finally, we obtain a generalized potential $\Gamma(\varphi, G, S)$ expansion which is analogous to the effective potential $\Gamma(\varphi)$ loop expansion.

2. Legendre transformation with the source $L\cdot S(\varphi)$ in the $g\varphi^3$ theory.

Let us determine the generating functional $Z(J,K,L)$ of connected Green's functions as

$$e^{\frac{i}{\hbar}Z(J,K,L)} = \mathcal{N}^{-1}\int \mathcal{D}\varphi \exp\frac{i}{\hbar}\Big\{S(\varphi) + J(x)\varphi(x) + \frac{1}{2}\varphi(x)K(x\cdot y)\varphi(y) + L\cdot S(\varphi)\Big\} \qquad (2.1)$$

In this expression $S(\varphi)$ is a classical action of the $g\varphi^3$ theory**)

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*) This source is gauge-invariant.

**) The developed formalism can be easily generalized as to the theories interesting from the physical point of view.

$$S(\varphi) = \frac{1}{2}\varphi(x)\, i\mathcal{D}^{-1}(x,y)\,\varphi(y) + \frac{g\,\varphi^3(x)}{3!} \qquad (2.2)$$

Here and in what follows, if not specially mentioned, the integration goes over repeated arguments. The operator $\mathcal{D}^{-1}$ is determined as:

$$i\,\mathcal{D}^{-1}(x-y) = -\left(\Box + m^2\right)\delta^4(x-y) \qquad (2.3)$$

and N^{-1} is a normalised constant.

Let us differentiate (2.1) with respect to the source We obtain

$$\mathcal{Z}_{J(x)} = \langle\varphi(x)\rangle,\ \mathcal{Z}_{K(x,y)} = \frac{1}{2}\langle\varphi(x)\,\varphi(y)\rangle,\ \mathcal{Z}_L = \langle S(\varphi)\rangle \qquad (2.4)$$

where

$$\mathcal{Z}_{J(x)} \equiv \frac{\delta \mathcal{Z}}{\delta J(x)} \qquad (2.5)$$

and so on.

The two-point connected Green's function $G(x,y)$ is determined as

$$\mathcal{Z}_{J(x)J(y)} = \frac{i}{\hbar}\left[\langle\varphi(x)\,\varphi(y)\rangle - \langle\varphi(x)\rangle\langle\varphi(y)\rangle\right] \equiv i\,G(x,y) \qquad (2.6)$$

From (2.1) it follows, that derivatives $\mathcal{Z}_J$, $\mathcal{Z}_{JJ}$. $\mathcal{Z}_K$ and $\mathcal{Z}_L$ are coupled by relations

$$\mathcal{Z}_{J(x)J(y)} + \frac{i}{\hbar}\mathcal{Z}_{J(x)}\mathcal{Z}_{J(y)} = \frac{2i}{\hbar}\mathcal{Z}_{K(x,y)} \qquad (2.7)$$

$$\mathcal{Z}_L = i\,\mathcal{D}^{-1}(x,y)\,\mathcal{Z}_{K(x,y)} + \qquad (2.8)$$

$$+\frac{g}{3!}\left\{\left(\frac{\hbar}{i}\right)^2\mathcal{Z}_{J(x)J(x)J(x)} + \frac{3\hbar}{i}\mathcal{Z}_{J(x)J(x)}\mathcal{Z}_{J(x)} + \mathcal{Z}^3_{J(x)}\right\}$$

The generating functional Z satisfies the Schwinger's motion equation, which expresses the property of the measure $\mathcal{D}\varphi$ transformation invariance in the integral (2.1):

$$(1+L)\left\{i\mathcal{D}^{-1}(\varkappa,y)Z_{J(y)}+\frac{g}{2}\left[Z_{J(\varkappa)}Z_{J(\varkappa)}-i\hbar Z_{J(\varkappa)J(\varkappa)}\right]\right\}+ \tag{2.9}$$

$$+J(\varkappa)+K(\varkappa,y)\,Z_{J(y)}=0$$

Let us choose new independent variables $S\equiv\langle S(\varphi)\rangle$, $G(\varkappa,y)\equiv\frac{1}{\hbar}\left[\langle\varphi(\varkappa)\varphi(y)\rangle-\langle\varphi(\varkappa)\rangle\langle\varphi(y)\rangle\right]$, $\varphi\equiv\langle\varphi(\varkappa)\rangle$ and make the third order Legendre transformation

$$\Gamma(\varphi,G,S)=Z-J\varphi-\frac{1}{2}K\varphi\varphi-\frac{\hbar}{2}GK-LS \tag{2.10}$$

Varying $\Gamma(\varphi,G,S)$ over φ, G and S we obtain

$$\Gamma_{\varphi(\varkappa)}=-J(\varkappa)-K(\varkappa\cdot y)\,\varphi(y)$$

$$\Gamma_{G(\varkappa\cdot y)}=-\frac{\hbar}{2}K(\varkappa\cdot y) \tag{2.11}$$

$$\Gamma_S=-L$$

Rewrite the Schwinger's equations (2.9) and (2.8) in new variables:

$$(1-\Gamma_S)\left\{i\mathcal{D}^{-1}(\varkappa\cdot y)\varphi(y)+\frac{g}{2}\left[\varphi^2(\varkappa)+\hbar G(\varkappa\cdot\varkappa)\right]\right\}=\Gamma_{\varphi(\varkappa)} \tag{2.12}$$

$$S-S(\varphi)-\frac{i\hbar}{2}\Delta^{-1}(\varkappa,y|y)\,G(\varkappa\cdot y)=\frac{g}{3!}\left(\frac{\hbar}{i}\right)^2 Z_{J(\varkappa)J(\varkappa)J(\varkappa)} \tag{2.13}$$

where $\Delta^{-1}(\varkappa,y|\varphi)$ is determined as

$$i\Delta^{-1}(x,y|\varphi)\equiv i\mathcal{D}^{-1}(x\cdot y)+g\,\varphi(x)\,\delta(x-y) \tag{2.14}$$

Using relations

$$\frac{\delta J(x)}{\delta J(y)} = \delta(x-y), \quad \frac{\delta K(x,y)}{\delta J(z)} = 0, \quad \frac{\delta L}{\delta J(x)} = 0 \tag{2.15}$$

one can obtain (see Appendix A):

$$\delta(x-y) = i G(x,z)\, Q_{\varphi(x)\varphi(y)} + \frac{2i}{\hbar} G(x,z)\, \Gamma_{G(z,y)} + \frac{1}{i} Z_{J(x)J(z)J(t)}\, Q_{\varphi(y)G(z,t)} \tag{2.16}$$

$$0 = i G(x,y)\, Q_{\varphi(y)G(z,t)} + \frac{1}{i} Z_{J(x)J(u)J(v)}\, Q_{G(u,v)G(z,t)} \tag{2.17}$$

where

$$Q_{\varphi(x)\varphi(y)} \equiv \Gamma_{\varphi(x)s} \Gamma_{ss}^{-1} \Gamma_{s\varphi(y)} - \Gamma_{\varphi(x)\varphi(y)}$$

$$Q_{\varphi(x)G(z,t)} \equiv \Gamma_{\varphi(x)\varrho} \Gamma_{ss}^{-1} \Gamma_{sG(z,t)} - \Gamma_{\varphi(x)G(z,t)} \tag{2.18}$$

$$Q_{G(z,t)G(u,v)} \equiv \Gamma_{G(z,t)s} \Gamma_{ss}^{-1} \Gamma_{sG(u,v)} - \Gamma_{G(z,t)G(u,v)}$$

For convenience, in Eqs.(2.13) and (2.16-17) the quantity Z_{JJJ} is preserved which can be expressed through derivatives Γ by means of (2.17) and substituted in (2.13). (2.16). Eqs.(2.12-13) and (2.16-17) completely determine the functional Γ.

3. Extraction of invariants.

The Schwinger's equation (2.12) is linear inhomogeneous and its general solution is a sum of a particular solution and a general solution of the homogeneous part. As the particular solution $\Gamma = S$ may be taken, then the general solution will have the form

$$\Gamma = S + F \tag{3.1}$$

where F is the solution of the equation

$$F_S\left[i\mathcal{D}^{-1}(x\cdot y)\varphi(y) + \frac{g}{2}\left(\varphi^2(x) + \hbar G(x,x)\right)\right] + F_{\varphi(x)} = 0 \tag{3.2}$$

Writing out for (3.2) the characteristic equation we obtain that F depends on the invariant $S - S\langle\varphi\rangle - \frac{g\hbar}{2} G\varphi$ and variable G. However, from (2.13) follows that it is more convenient to choose the invariant in the form:

$$\tilde{S} = S - S(\varphi) - \frac{i\hbar}{2}\Delta^{-1}(x\cdot y|\varphi)\; G(x,y) \tag{3.3}$$

Taking this fact into account, we calculate operators $Q_{\varphi\varphi}$, $Q_{\varphi G}$, Q_{GG} in (2.18) (in what follows the differentiation with respect to $\tilde{S}$ is denoted as F'):

$$Q_{\varphi(x)\varphi(y)} = F'\, i\, \Delta^{-1}(x\cdot y/\varphi) \tag{3.4}$$

$$Q_{\varphi(x)G(z,t)} = F'\,\frac{g\hbar}{2}\,\delta(x-z)\,\delta(x-t) \tag{3.5}$$

$$Q_{G(z,t)G(u.v)} \equiv Q(z,t/u,v) = F'_{G(z,t)}\,F''^{-1}\,F'_{G(u.v)} - F_{G(z.t)G(u.u)} \tag{3.6}$$

Substituting them into Eqs.(2.16-17) we obtain:

$$\delta(x-y) = \frac{2i}{\hbar}G(x,z)F_{G(z,y)} + \frac{1}{i}Z_{\mathcal{J}(x)\mathcal{J}(y)\mathcal{J}(y)}\,\frac{g\hbar}{2}F' \tag{3.7}$$

$$0 = -G(x,z)\,\frac{g\hbar}{2}F'\delta(z-t) + Z_{\mathcal{J}(x)\mathcal{J}(u)\mathcal{J}(y)}\,Q(u,y/z,t) \tag{3.8}$$

Rewrite now (2.13) in new variables

$$\tilde{S} = \frac{g}{3!}\left(\frac{\hbar}{i}\right)^2 Z_{J(\varkappa)J(\varkappa)J(\varkappa)} \qquad (3.9)$$

In Eq.(3.7) we take $x = y$, integrate over $\varkappa$ and substitute Z_{JJJ} from (3.9):

$$\operatorname{tr}\hat{1} = \frac{3i}{\hbar}\tilde{S}F' + \frac{2i}{\hbar} G F_G \qquad (3.10)$$

here $\operatorname{tr}\hat{1} \equiv \int \delta(\varkappa - \varkappa)\, d^4x$ The general solution (3.10) has the form:

$$F(\tilde{S}, G) = \frac{\hbar}{2i}\operatorname{tr}\ln G + \Theta\left(\frac{\tilde{S}^2}{G^3}\right) \qquad (3.11)$$

whence it follows that in order to find Γ it is sufficient from Eqs.(3.7-9) to extract one equation of the form $F_G = \ldots$, where on the right side are derivatives F of a higher order.

Substituting Z_{JJJ} from (3.8) into (3.7), (3.9) we obtain:

$$2F_{G(x,y)} = -i\hbar G^{-1}(\varkappa, y) + \frac{1}{\hbar}\left(\frac{3!\tilde{S}}{g\hbar}\right)^2 \frac{Q^{-1}(\varkappa\varkappa/yy)}{\operatorname{tr}^2(GQ^{-1})} \qquad (3.12)$$

$$F' = \frac{1}{\hbar}\left(\frac{3!}{g\hbar}\right)^2 \frac{\tilde{S}}{3\operatorname{tr} GQ^{-1}} \qquad (3.13)$$

where

$$\operatorname{tr} GQ^{-1} \equiv \int G(\varkappa, y)\, Q^{-1}(y, y/\varkappa, \varkappa)\, d^4\varkappa\, d^4y \qquad (3.14)$$

and

$$Q(z,t/u,v)\, Q^{-1}(v,v/\varkappa, y) = \frac{1}{2}\left[\delta(z-\varkappa)\delta(t-y) + \delta(z-y)\delta(t-\varkappa)\right] \qquad (3.15)$$

$$(3.15)$$

Eq.(3.12) is solved by the iteration technique /16-17/. As a zero approximation for F' is taken (see (3.11) or (3.12))

$$F^{(0)} = \frac{\hbar}{2i} \, tr \, \ln G \tag{3.16}$$

Then it is necessary to calculate Q^{-1} in the same approximation and substitute it to the right part of Eq.(3.12). Integrating it over G we obtain $F^{(1)}$ and so on. However, it is convenient to iterate Eq.(3.13), since it is an ordinary one. In the next section the process of iteration of this equation is given in detail.

4. Iteration solution of equation (3.13).

Let us calculate the operator Q in a zero approximation, substituting $F^{(0)}$ (3.16) into (3.6)

$$Q^{(0)}(z,t/u,v) = \frac{\hbar}{2i} \frac{1}{2}\left[G^{-1}(z,u)\, G^{-1}(t,v) + G^{-1}(z,v)\, G^{-1}(t,u)\right] \tag{4.1}$$

and obtain the inverse (3.15)

$$Q^{(0)^{-1}}(u,v/x,y) = \frac{2i}{\hbar} G(u,x)\, G(v,y) \tag{4.2}$$

From (3.14) we have

$$tr \, G \, Q^{(0)-1} = \frac{2i}{\hbar} \int G^3(x,y)\, d^4x\, d^4y \tag{4.3}$$

Substituting (4.3) into (3.13) and integrating over $\tilde{S}$ we obtain:

$$F^{(1)} = \frac{i}{12} \left(\frac{3!}{g\hbar}\right) \frac{\tilde{S}^2}{\Theta} \tag{4.4}$$

where

$$\Theta = \int G^2(\varkappa \cdot y)\, d^4\varkappa\, d^4 y \tag{4.5}$$

In what follows, in diagrams, to the line •——• corresponds propagator $G(\varkappa, y)$, and the integration goes along the closed lines.

Proceeding with the iteration process we substitute $F^{(0)} + F^{(1)}$ into determination Q (3.6):

$$Q^{(1)}(z,t/u,v) = i\left(\frac{3!}{g\hbar}\right)^2\left(\frac{\tilde{S}}{\Theta}\right)^2 G(u,v)\left[\delta(v-t)\delta(u-z)+\delta(v-z)\delta(u-t)\right] \tag{4.6}$$

and find the inverse to Q

$$\left[Q^{(0)} + Q^{(1)}\right]^{-1} = Q^{(0)-1} - Q^{(0)-1}Q^{(1)}Q^{(0)-1} + \tag{4.7}$$

Preserving in (4.7) the terms not exceeding the second order in $\tilde{S}$ we obtain for (3.14):

$$\mathrm{tr}\, G Q^{-1} = \frac{2i}{\hbar}\,\Theta - \frac{i}{2}\left(\frac{2i}{\hbar}\right)^2\left(\frac{3!\tilde{S}}{g\hbar}\right)^2 \frac{\text{☮}}{\Theta^2} \tag{4.8}$$

Having integrated Eq.(3.13) we have:

$$F^{(2)} = -\frac{i}{24\hbar}\left(\frac{3!\tilde{S}}{g\hbar}\right)^4 \frac{\text{☮}}{\Theta^4} \tag{4.9}$$

and so on.

Let us write out a final expression for the functional F , presented as a series in $\tilde{S}^2$:

$$\Gamma = S + \frac{\hbar}{2i}\,\mathrm{tr}\ln G + \frac{i}{12}\left(\frac{3!\tilde{S}}{g\hbar}\right)^2 \frac{1}{\Theta} - \tag{4.10}$$

$$- \frac{i}{24\hbar}\left(\frac{3!\tilde{S}}{g\hbar}\right)^4 \frac{\text{☮}}{\Theta^4} +$$

Note, that when excluding the source L , which is equivalent to condition (see 2.11)

$$\Gamma_{\rho} = 0$$

one can find S as a function of φ and G .

Substituting S as a function of φ and G into $\Gamma(\varphi, G, S)$ we obtain the known expansion for the functional $\Gamma(\varphi, G)$ /16-18/.

The performed consideration permits one to hope that an analogous investigation will be successfully applied to non-abelian theories, which will be described in our further publications.

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APPENDIX A

In the terms of φ, G, S
the ratio (2.15) has the form:

$$1 = \frac{\delta \mathcal{J}}{\delta \varphi}\frac{\delta \varphi}{\delta \mathcal{J}} + \frac{\delta \mathcal{J}}{\delta G}\frac{\delta G}{\delta \mathcal{J}} + \frac{\delta \mathcal{J}}{\delta \wp}\frac{\delta S}{\delta \mathcal{J}} \tag{A.1}$$

$$0 = \frac{\delta K}{\delta \varphi}\frac{\delta \varphi}{\delta \mathcal{J}} + \frac{\delta K}{\delta G}\frac{\delta G}{\delta \mathcal{J}} + \frac{\delta K}{\delta \wp}\frac{\delta S}{\delta \mathcal{J}} \tag{A.2}$$

$$0 = \frac{\delta L}{\delta \varphi}\frac{\delta \varphi}{\delta \mathcal{J}} + \frac{\delta L}{\delta G}\frac{\delta G}{\delta \mathcal{J}} + \frac{\delta L}{\delta \wp}\frac{\delta \wp}{\delta \mathcal{J}} \tag{A.3}$$

From (A.3) we express $\frac{\delta \wp}{\delta \mathcal{J}}$ and substitute into (A.1), (A.2):

$$1 = \frac{\delta \mathcal{J}}{\delta \varphi}\frac{\delta \varphi}{\delta \mathcal{J}} + \frac{\delta \mathcal{J}}{\delta G}\frac{\delta G}{\delta \mathcal{J}} + \frac{\delta \mathcal{J}}{\delta \wp}\left[-\frac{\delta L}{\delta \varphi}\frac{\delta \varphi}{\delta \mathcal{J}}\left(\frac{\delta L}{\delta S}\right)^{-1} - \frac{\delta L}{\delta G}\frac{\delta G}{\delta \mathcal{J}}\left(\frac{\delta L}{\delta \wp}\right)^{-1}\right] \tag{A.4}$$

$$0 = \frac{\delta K}{\delta \varphi}\frac{\delta \varphi}{\delta \mathcal{J}} + \frac{\delta K}{\delta G}\frac{\delta G}{\delta \mathcal{J}} + \frac{\delta K}{\delta \wp}\left[-\frac{\delta L}{\delta \varphi}\cdot\frac{\delta \varphi}{\delta \mathcal{J}}\left(\frac{\delta L}{\delta \wp}\right)^{-1} - \frac{\delta L}{\delta G}\frac{\delta G}{\delta \mathcal{J}}\left(\frac{\delta L}{\delta \wp}\right)^{-1}\right] \tag{A.5}$$

Using the ratios (2.4) and (2.11) write expressions for variables participating in (A.4) and (A.5):

$$\frac{\delta \varphi}{\delta \mathcal{J}} = iG\,;\quad \frac{\delta G}{\delta \mathcal{J}} = iZ_{\mathcal{J}\mathcal{J}\mathcal{J}}\,;\quad \frac{\delta \mathcal{J}}{\delta G} = -\Gamma_{\varphi G} + \frac{2}{\hbar}\Gamma_{GG}\cdot\varphi$$

(A.6)

$$\frac{\delta \mathcal{J}}{\delta \varphi} = -\Gamma_{\varphi\varphi} + \frac{2}{\hbar}\Gamma_{G} + \frac{2}{\hbar} + \frac{2}{\hbar}\Gamma_{G\varphi}\cdot\varphi$$

$$\frac{\delta \mathcal{J}}{\delta \wp} = -\Gamma_{\varphi\wp} + \frac{2}{\hbar}\Gamma_{G\wp}\cdot\varphi\,;\qquad \frac{\delta K}{\delta \varphi} = -\frac{2}{\hbar}\Gamma_{G\varphi}$$

$$\frac{\delta K}{\delta G} = -\frac{2}{\hbar}\Gamma_{GG}\,;\qquad \frac{\delta K}{\delta \wp} = -\frac{2}{\hbar}\Gamma_{G\wp}\,.$$

Finally, substituting (A.6) into (A.1) and (A.2) we obtain the coupling equations in the terms $\Gamma(\varphi, G, S)$

$$1 = iG\,Q_{\varphi\varphi} + \frac{2i}{\hbar} G\,\Gamma_G + \frac{1}{i} Z_{\mathcal{J}\mathcal{J}\mathcal{J}}\,Q_{G\varphi} \tag{A.7}$$

$$0 = iG\,Q_{\varphi G} + \frac{1}{i} Z_{\mathcal{J}\mathcal{J}\mathcal{J}}\,Q_{GG} \tag{A.8}$$

The form of the operators $Q_{\varphi\varphi}$, $Q_{\varphi G}$ and Q_{GG} is given in (2.18).

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