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THE GENERALIZED EFFECTIVE POTENTIAL IN NONLINEAR THEORIES OF THE 4-TH ORDER

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Н.С.АНАНИКЯН, Г.К.САВВИДИ

ОБОБЩЕННЫЙ ЭФФЕКТИВНЫЙ ПОТЕНЦИАЛ В НЕЛИНЕЙНЫХ ТЕОРИЯХ ЧЕТВЕРТОГО ПОРЯДКА

С помощью преобразований Лежандра, строится обобщенный эффективный потенциал $\Gamma(\varphi, G, H, S)$, зависящий от вакуумного среднего поля φ, двух и трехточечной связных функций Грина G, H и вакуумного среднего от классического действия $S = \langle 0 | S_{cl} | 0 \rangle$. Получено разложение $\Gamma(\varphi, G, H, S)$, аналогичное петлевому разложению эффективного действия $\Gamma(\varphi)$.

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By means of the Legendre transformations a generalized effective potential $\Gamma(\varphi, G, H, S)$ is constructed, which depends on φ , a possible expectation value of the quantum field; on G, H , possible expectation values of the 2- and 3-point connected Green functions and on $S = \langle 0 | S_{cl} | 0 \rangle$, a possible expectation value of the classical action. The expansion for the functional $\Gamma(\varphi, G, H, S)$ is obtained, which is similar to the loop expansion for the effective action $\Gamma(\varphi)$

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1. Introduction

Certain progress was achieved in understanding the QCD's vacuum structure in a number of works [1,2] In particular, a relation between B, a bag model constant, and average value of the gluon condensate field tensor in vacuum was obtained, which is in good agreement with the phenomenological data [3].

But it was difficult to prove the Lorentz-and-gauge invariance of the vacuum state. That is why it will be useful to develop such a formalism, in which order parameters are Lorentz and gauge invariant (for example. a possible expectation value of the classical action or Wilson correlator).

Since, by means of the functional Legendre transformations the problem of the vacuum expectation values calculations can be reduced to a variational one, it is interesting to construct a functional Γ , which depends on Lorentz-and-gauge invariant expectation values.

The motion equations for the functional Γ , which depends on S_{cl} , a possible expectation value of the classical action of the $g\varphi^3$ theory, were derived in our previous work [5]. In Ref. [5] we essentially used variational methods developed by Vasilev et al. [4]. Apart from usual sources we introduced in [5] a gauge-and-Lorentz-invariant source.

In this paper we constructed an analogous generalized effective potential in nonlinear theories of the 4-th order. In the second Section the functional Γ is determined and its motion equations ((2.7-8), (2.10-13)) are derived. in the third and fourth Sections these equations are solved by means of the iteration technique; as a result the expansion of Γ (4.3) is obtained, which is similar to the loop expansion of the effective action $\Gamma(\varphi)$ (4). In the end of the paper the expansion (4.3) is used as an illustration of the method to the anharmonic oscillator.

2. The Motion Equations for the Generalized Effective Potential

Let us determine the generating functional $Z(J, K, M, L)$ for the $g\varphi^4$ theory *):

$$\exp\left\{\frac{i}{\hbar} Z(J, K, M, L)\right\} = \tag{2.1}$$

$$= N^{-1}\int \mathcal{D}\varphi \exp\left\{\frac{i}{\hbar}\left[S(\varphi) + J(x)\varphi(x) + \frac{1}{2}K(x,y)\varphi(x)\varphi(y) + \frac{1}{3}M(x,y,z)\varphi(x)\varphi(y)\varphi(z) + L\cdot S\right]\right\}$$

In this expression $S(\varphi)$ is a classical action

$$S(\varphi) = \frac{1}{2}\, i\mathcal{D}^{-1}(x,y)\,\varphi(x)\,\varphi(y) + g\frac{\varphi^4(x)}{4!} \tag{2.2}$$

*) The results received in this work can be used in any non-linear theory of the 4-th order, in particular, in the non-abelian one.

N^{-1} is a normalized constant and the integration goes over repeated arguments. The operator $i\mathcal{D}^{-1}$ is determined as $i\mathcal{D}^{-1}(x,y) = -(\Box + m^2)\,\delta^4(x-y)$

Differentiating (2.1) with respect to the souces (J, K, M, L) we obtain $Z_{J(x)} = \langle \varphi(x) \rangle$, $Z_{K(x,y)} = \frac{1}{2} \langle \varphi(x)\varphi(y) \rangle$,

$Z_{M}(x,y,z) = \frac{1}{3!} \langle \varphi(x)\varphi(y)\varphi(z) \rangle$, $\quad Z_L = \langle S(\varphi) \rangle$

where $Z_{J(x)} \equiv \dfrac{\delta Z}{\delta J(x)}$ and so on. From (2.1) it follows that derivatives Z_J , Z_{JJ} , Z_K and Z_L are coupled by relations

$$Z_L = i\mathcal{D}^{-1}(x,y)\, Z_{K(x,y)} + \frac{g}{4!}\Big\{ \Big(\frac{\hbar}{i}\Big)^3 Z_{J(x)J(x)J(x)J(x)} + 3\Big(\frac{\hbar}{i}\Big)^2 Z^2_{J(x)J(x)} + 4\Big(\frac{\hbar}{i}\Big)^2 Z_{J(x)J(x)J(x)} Z_{J(x)} + 6\,\frac{\hbar}{i}\, Z_{J(x)J(x)} Z^2_{J(x)} + Z^4_{J(x)} \Big\} \tag{2.3}$$

The generating functional Z satisfies the Schwinger motion equation which expresses the property of the measure $\mathcal{D}\varphi$ transformation invariance in the integral (2.1)

$$(1+L)\Big\{ i\mathcal{D}^{-1}(x,y)\, Z_{J(y)} + \frac{g}{3!}\Big[Z^3_{J(x)} + 3\,\frac{\hbar}{i}\, Z_{J(x)} Z_{J(x)J(x)} + \Big(\frac{\hbar}{i}\Big)^2 Z_{J(x)J(x)J(x)} \Big] + J(x) + K(x,y)\, Z_{J(y)} + \frac{1}{2} M(x,y,z)\Big[Z_{J(y)} Z_{J(z)} + \frac{\hbar}{i} Z_{J(y)J(z)} \Big] = 0 \tag{2.4}$$

We can receive two more equations, connecting Z_K , Z_{JJ} , Z_M , using (2.1). But it is more convenient to receive them after Legendre transformation was performed, with the help of (2.9) [4.5] (see below).

Let us choose new independent variables $\varphi(x) \equiv Z_{J(x)}$

$iG(x,y) \equiv Z_{J(x)J(y)} = \frac{i}{\hbar}\left[2Z_{K(x,y)} - Z_{J(x)}Z_{J(y)}\right]$, $\quad i^2 H(x,y,z) \equiv Z_{J(x)J(y)J(z)}$, $\quad S \equiv Z_L$ and make the 4-th order Legendre transformation.

$$\Gamma(\varphi, G, H, S) = Z - J\cdot\varphi - \frac{1}{2}K<\varphi\varphi> - \frac{1}{3!}M<\varphi\varphi\varphi> - L\cdot S = \tag{2.5}$$

$$= Z - J\varphi - \frac{1}{2}K\varphi\varphi - \frac{\hbar}{2}KG - \frac{1}{3!}M\varphi\varphi\varphi -$$

$$- \frac{\hbar}{3!}MG\varphi - \frac{\hbar^2}{3!}MH - L\cdot S$$

Here and in what follows, if there is no necessity, the integrand arguments are omitted.

Varying $\Gamma(\varphi, G, H, S)$ over φ, G, H and S we obtain

$$\Gamma_\varphi = -J - K\varphi - \frac{1}{2}M\varphi\varphi - \frac{\hbar}{3!}MG; \qquad \Gamma_H = -\frac{\hbar^2}{3!}M \tag{2.6}$$

$$\Gamma_G = -\frac{\hbar}{2}K - \frac{\hbar}{3!}M\varphi; \qquad \Gamma_S = -L$$

To derive a complete system of equations, determining the generalized effective potential Γ, we rewrite (2.3-4) in the terms of

$$(1-\Gamma_S)\left\{i\mathcal{D}^{-1}_{(x,y)}\varphi_{(y)} + \frac{g}{3!}\left[\varphi^3_{(x)} + \frac{3\hbar}{i}\varphi_{(x)}\, iG(x,x) + \left(\frac{\hbar}{i}\right)^2 i^2 H(x,x,x)\right]\right\} = \Gamma_{\varphi(x)} \tag{2.7}$$

$$S - S(\varphi) - \frac{i\hbar}{2}\mathcal{D}^{-1}G - g\frac{\hbar^2}{4}G\varphi^2 - g\frac{\hbar^2}{8}G^2 - g\frac{\hbar^2}{3!}H\varphi = \frac{g}{4!}\left(\frac{\hbar}{i}\right)^3 Z_{JJJJ} \tag{2.8}$$

and use the relations

$$\frac{\delta J(x)}{\delta J(y)} = \delta(x-y), \quad \frac{\delta K(x,y)}{\delta J(z)} = 0; \quad \frac{\delta M(x,y,z)}{\delta J(t)} = 0; \quad \frac{\delta L}{\delta S} = 0 \tag{2.9}$$

(see Appendix A in Ref. 5),

$$\delta(x-y)=\frac{2i}{\hbar}\Gamma_{G(x,z)}G(z,y)+\frac{3i}{\hbar}\Gamma_{H(x,z,t)}H(z,t,y)+iQ_{\varphi(x)\varphi(z)}G(z,y)+ \tag{2.10}$$
$$+iQ_{\varphi(x)G(z,t)}H(z,t,y)-Q_{\varphi(x)H(z,t,u)}Z_{J(z)J(t)J(u)J(y)}$$

$$0=\frac{3i}{\hbar}\Gamma_{H(x,y,t)}G(t,z)+iQ_{G(x,y)\varphi(t)}G(t,z)+iQ_{G(x,y)G(t,u)}H(t,u,z)-Q_{G(x,y)H(u,v,\omega)}Z_{J(u)J(v)J(\omega)J(t)} \tag{2.11}$$

$$0=iQ_{H(x,y,z)\varphi(u)}G(u,t)+iQ_{H(x,y,z)G(u,v)}H(u,v,t)-Q_{H(x,y,z)H(u,v,\omega)}Z_{J(u)J(v)J(\omega)J(t)} \tag{2.12}$$

where

$$Q_{AB}=\Gamma_{AS}\,\Gamma_{SS}^{-1}\,\Gamma_{SB}-\Gamma_{AB} \tag{2.13}$$

In Eqs. (2.7-8) and (2.10-12) the quantity Z_{JJJJ} is preserved, which can be expressed through derivatives Γ by Eq.(2.12). Eqs. (2.7-8) and (2.10-12) completely determine the functional Γ .

3. Extraction of Invariants

In the same way as in Ref. [5] we can write using (2.7) $\Gamma=S+F$, where F depends on the invariant

$$S-S(\varphi)-\frac{i\hbar}{2}\mathcal{D}^{-1}G-\frac{g\hbar^2}{3!}H\varphi-\frac{g\hbar^2}{4}G\varphi^2$$

and variables G, H . However, from (2.8) follows that it is more convenient to choose the invariant in the form

$$\tilde{S}=S-S(\varphi)-\frac{i\hbar}{2}\mathcal{D}^{-1}G-\frac{g\hbar}{4}G\varphi^2-\frac{g\hbar^2}{8}G^2-\frac{g\hbar^2}{3!}H\varphi \tag{3.1}$$

Rewriting operators Q_{AB} in the terms of $F(\tilde{S}, G, H)$ and substituting them in (2.10-2.12) we have

$$1 = \frac{2i}{\hbar} F_G G + \frac{3i}{\hbar} F_H H - \frac{g\hbar^2}{3!} F' Z_{JJJJ} \tag{3.2}$$

$$0 = \frac{3i}{\hbar} F_H G + \frac{ig\hbar^2}{4} F' H + i Q_{GG} H - Q_{GH} Z_{JJJJ} \tag{3.3}$$

$$0 = \frac{ig\hbar^2}{3!} F' G + i Q_{GH} H - Q_{HH} Z_{JJJJ} \tag{3.4}$$

$$\tilde{S} = \frac{g}{4!} \left(\frac{\hbar}{i} \right)^3 Z_{JJJJ} \tag{3.5}$$

where

$$Q_{\tilde{A}\tilde{B}} = F'_{\tilde{A}} \; F''^{-1} \; F'_{\tilde{B}} - F_{\tilde{A}\tilde{B}} \tag{3.6}$$

and F' denotes the differentiation with respect to $\tilde{S}$.

Eqs.(3.2-5) now completely determine the functional F and arguments ($x, y \dots$) in these equations are written in the same order as it was in (2.10-2.12).

Our next purpose is to solve Eqs.(3.2-5) by means of iteration techniques. The iteration is essentially facilitated owing to the fact that F depends on $\frac{\tilde{S}^2}{G^4}$, $\frac{H^2}{G^3}$ only. Whence it follows that in order to find F it is sufficient from Eqs.(3.2-5) to extract one equation of the form $F_G = \dots$, where on the right side are derivatives F of a higher order.

In Eq.(3.2) we take $x = y$, integrate over x and substitute Z_{JJJJ} from (3.5)

$$tr\,\hat{1} = \frac{2i}{\hbar} F_G G + \frac{3i}{\hbar} F_H H + \frac{4i}{\hbar} F'\tilde{S} \tag{3.7}$$

From (3.7) it follows that F has the form

$$F(\tilde{S}, G, H) = \frac{\hbar}{2i}\, tr \ln G + \Theta\left(\frac{\tilde{S}^2}{G^4}, \frac{H^2}{G^3}\right) \tag{3.8}$$

In the $g\varphi^3$ -theory, in contrast to $g\varphi^4$, the functional Θ depends on one variable $\frac{\tilde{S}^2}{G^3}$ [5].

Multiplying (3.2) by G and (3.3) by H , then subtracting (3.2) from (3.3) and substituting the expression received from (3.4-3.5) for F' we obtain

$$2F_G = -i\hbar G^{-1} H Q_{GG} H G^{-1} - \frac{3}{2}\hbar G^{-1} H \underbrace{Q_{HG} H G^{-1}} H G^{-1} + \tag{3.9}$$

$$+2i\hbar G^{-1} Z_{JJJJ} Q_{HG} H G^{-1} - \hbar G^{-1} Z_{JJJJ} Q_{HH} Z_{JJJJ} G^{-1} -$$

$$- \frac{3\hbar i}{2} G^{-1} H \underbrace{Q_{HH} Z_{JJJJ} G^{-1}} H G^{-1}$$

The sign $\underbrace{\qquad}$ stands for integration over one argument. In other cases the arguments are integrated in the succession they are written.

Z_{JJJJ} can be determined from (3.4-3.5)

$$Z_{JJJJ} = -i\,\frac{4!\tilde{S}}{g\hbar^3}\,\frac{Q^{-1}G}{tr(Q^{-1}G)} - i\,tr\left\{Q^{-1}Q_{HG}H\right\}\left[\frac{Q^{-1}G}{tr(Q^{-1}G)} - \frac{Q^{-1}Q_{HG}H}{tr\left\{Q^{-1}Q_{HG}H\right\}}\right] \tag{3.10}$$

and Q^{-1} is defined as

$$Q_{H(x,p,q)H(u,v,\omega)}\, Q^{-1}_{(u,v,\omega|s,t,v)} = \frac{1}{6}\left[\delta(x-s)\delta(p-t)\delta(q-v) + \cdots\right] \tag{3.11}$$

where

$$\mathrm{tr}\, Q^{-1}G = \int Q^{-1}(x,x,x|y,y,y)\, G(x,y)\, d^4x\, d^4y$$

$$\mathrm{tr}\{Q^{-1}Q_{HG}H\} = \int Q^{-1}(x,x,x|u,v,t)\, Q_{H(u,v,t)G(p,q)} H(p,q,x)\, d^4(x,u,v,t,p,q) \tag{3.12}$$

Eq.(3.9) is solved by the iteration technique.

First we take $F = \frac{\hbar}{2i}\mathrm{tr}\ \ln\ G$ (see 3.8-9). The only item in (3.9) which is not equal to zero has the form $G^{-1}HQ_{GG}H\ G^{-1}$, and Q_{GG} is to be calculated using (3.6). Substituting $\frac{\hbar}{2i}\mathrm{tr}\,\ln G$ for F we have

$$Q_{GG} = -F_{GG} = \frac{\hbar}{2i}\ G^{-2} \tag{3.13}$$

and from (3.9) obtain

$$F^{(0)} = \frac{\hbar}{2i}\,\mathrm{tr}\,\ln G + \frac{i\hbar^2}{12} H G^{-3} H \tag{3.14}^{*)}$$

This expression we shall take as a zero approximation for in our iteration. Then it is necessary to calculate Q_{GG} , Q_{HG} , Q_{HH} (see 3.14) and Q^{-1} (see 3.11) in the same approximation and substitute them into the right part of Eq.(3.9). Integrating it over G we obtain $F^{(1)}$ and so on. In the next section the process of iteration is given in de-

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*) Although $Q^{(0)}_{HH}$ has no inverse (see (3.11)), it is not a serious difficulty, since the first step of iteration can be done using Eqs.(3.2-6), then Q_{HH} has an inverse operator (see (4.1)).

tail.

4. Iteration Solution of Equation (3.9)

As it was mentioned above, Eq.(3.9) can be solved by means of iteration techniques.

The iteration goes with respect to the orders of G^{-1}. We take $F^{(0)}$ (3.14) as a zero approximation and calculate operators Q_{GG}, Q_{GH}, Q_{HH}, Q^{-1} (3.6), (3.11)

$$Q_{GG} = -i\hbar H^2 G^{-5} \qquad Q_{GH} = -\frac{\hbar^2}{2i} H G^{-4} \qquad (4.1)$$

$$Q_{HH} = -\frac{i\hbar^2}{6} G^{-3} \qquad Q^{-1} = \frac{6i}{\hbar^2} G^3$$

Substituting (4.1) into the right side of Eqs.(3.9-10), integrating over G and preserving the terms not exceeding the 4-th order in G^{-1} we obtain:

(4.2)

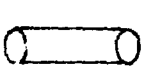
$F^{(1)} = \frac{i\hbar}{48}\left(\frac{4!}{g\hbar^2}\right)^2 \tilde{S}^2$ / 

where  $= \int G^4(x,y)\, d^4x\, d^4y$.

Following the iteration process in the same way we receive the expansion of F in orders of G^{-1}.

The expansion up to the 6-th order has the form:

$F = \frac{\hbar}{2i} \operatorname{tr} \ln G + \frac{i\hbar^2}{12}$  $+ \frac{i\hbar}{48}\left(\frac{4!\tilde{S}}{g\hbar^2}\right)^2 1$ /  $- 3i \frac{\tilde{S}}{g}$  /  $+$

$+ \frac{3i\hbar^3}{16}$ ( $)^2$ /  $+ \frac{i\hbar^3}{6}$  $- \frac{5i\hbar^3}{24}$  $- \frac{i\hbar}{2\cdot 4!}\left(\frac{4!\tilde{S}}{g\hbar^2}\right)^3$  / ( $)^3$ (4.3) where $\overset{x \qquad y}{\overline{\qquad\quad}} \equiv G(x,y)$, $\underset{y \quad z}{\overset{x}{\lambda}} \equiv H(\alpha\beta\gamma)G^{-1}(\alpha x)G^{-1}(\beta y)\,G^{-1}(\gamma,z)$,

$$\underset{z \quad t}{\overset{x \quad y}{\times}} \equiv \delta(x-y)\,\delta(x-z)\,\delta(x-t)$$

Note, that when excluding the source L , which is equivalent to condition $\Gamma_S = 0$ (see (2.6)), one can find S as a function of φ, G, H . Substituting $S = S(\varphi, G, H)$ into Γ (4.3) we obtain the known expansion for the functional $\Gamma(\varphi, G, H)$ [4].

Let us regard $\Gamma(\varphi, G, H, S)$ in the Hartree-Fock approximation [6] $G = \mathcal{D}$, where we take $\varphi = 0$. The generalized effective potential (see (3.1), (4.3)) then has the form:

$$\Gamma = S + \frac{\hbar}{2i}\,\mathrm{tr}\,\ln\mathcal{D} + \frac{i\hbar^2}{12}\ominus + \frac{i\hbar}{48}\left(\frac{4!}{g\hbar^2}\right)^2 \frac{\left(S - \frac{i\hbar}{2}\,\mathrm{tr}\,\hat{1} - \frac{g\hbar^2}{2}\,\mathcal{D}^2\right)^2}{\int \mathcal{D}^4(x,y)\,d^4x\,d^4y} \tag{4.4}$$

where only the terms not exceeding the 4-th order in G^{-1} are preserved.

From equations $\Gamma_H = 0$, $\Gamma_S = 0$ (2.6) we can find the stationary points of the functional Γ (4.4). The first equation gives $H = 0$ and the second gives

$$S_{vac} = \frac{i\hbar}{2}\,\mathrm{tr}\,\hat{1} + \frac{g\hbar^2}{8}\,\mathcal{D}^2 + i\left(\frac{g\hbar^3}{24}\right)\mathcal{D}^4 + \cdots \tag{4.5}$$

The energy difference of the full ($g \neq 0$) and free ($g = 0$) theories is given by formula [4]:

$$-\mathcal{E}(g)\int dt = \Gamma + J\varphi + \frac{1}{2}K\varphi\varphi + \frac{\hbar}{2}KG + \frac{1}{3!}M\varphi\varphi\varphi + \tag{4.6}$$

$$+\frac{\hbar}{3!}MG\varphi + \frac{\hbar^2}{3!}MH + LS - \frac{i\hbar}{2}\,\mathrm{tr}\,\hat{1} - \frac{\hbar}{2i}\,\mathrm{tr}\,\ln\mathcal{D}$$

which can be written in the stationary point $J = K = L = M = 0$ as

$$-\mathcal{E}(g)\int dt = \Gamma_1\Big|_{S=S_{vac}} - \frac{i\hbar}{2}\,\mathrm{tr}\,\hat{1} - \frac{\hbar}{2i}\,\mathrm{tr}\,\ln \mathcal{D} \tag{4.7}$$

Substituting (4.5) into (4.4) and (4.7) we have

$$\mathcal{E}(g) = -\frac{g\hbar^2}{8}\mathcal{D}^2 - \frac{ig^2\hbar^3}{48}\ominus \tag{4.8}$$

For the anharmonic oscillator in the quantum mechanics $\left(\mathcal{L} = \frac{m\dot{x}^2}{2} - \frac{m\omega^2 x^2}{2} + \frac{g x^4}{4!}\right)$ the Green function $\mathcal{D}(T)$ is equal to $e^{i\omega T}/2m\omega$, that is why from (4.5) and (4.8) we have

$$S_{vac} \approx \frac{g\hbar^2}{32m^2\omega^2}\ ; \quad \mathcal{E} \approx -\frac{g\hbar^2}{32m^2\omega^2}\,, \qquad g < 0 \tag{4.9}$$

It can be shown that the Legendre transformation examined in this work is equivalent to the Legendre transformation with respect to the coupling constant. This means that we consider the coupling constant as a source. Indeed, the exponent in the right side of (2.1) can be written in the form $\exp\frac{i}{\hbar}\left\{J\varphi + \frac{1}{2}K\varphi\varphi + \frac{1}{3!}M\varphi\varphi\varphi + \frac{L\varphi^4}{4!}\right\}$ that will result in the following changes in the theory:

i) the stationary conditions (2.6) are

$$\Gamma_\varphi = i\mathcal{D}^{-1}\varphi \qquad \Gamma_H = 0$$

$$\Gamma_G = \frac{\hbar}{2i}\mathcal{D}^{-1} \qquad \Gamma_S = -g$$

ii) $S \longrightarrow S = \left\langle \frac{\varphi^4}{4!} \right\rangle\,, \quad S(\varphi) \longrightarrow S(\varphi) = \frac{\varphi^4}{4!}$

and the invariant (3.1) has the form

$$\tilde{S} \rightarrow \tilde{S} = S - S(\varphi) - \frac{\hbar}{4}G\varphi^2 - \frac{\hbar^2}{8}G^2 - \frac{\hbar^2}{3!}H\varphi$$

iii) $$\Gamma \to \Gamma = F(\tilde{S}, G, H)\Big/_{g=1}$$

If proceeding from the symmetry properties of the theory $\langle \varphi \rangle = 0$ (as in the spinor case), then:

$$\Gamma = \frac{\hbar}{2i} \operatorname{tr} \ln G + \frac{i\hbar}{48} \left(\frac{4!\tilde{S}}{\hbar^2} \right) \frac{1}{\quad} - \tag{4.10}$$

$$- \frac{i\hbar}{2 \cdot 3!} \left(\frac{4!\tilde{S}}{\hbar^2} \right)^3 \frac{\quad}{(\quad)^3} + \dots$$

where $\tilde{S} = S - \frac{\hbar^2}{8} G^2$, and the stationary conditions are $\Gamma_G = \frac{\hbar}{2i} \mathcal{D}^{-1}$, $\Gamma_S = -g$.

Moreover, when φ is N -plet, this Legendre transformation can be treated as the Legendre transformation with respect to N . The question how it can help in the solving of the strong coupling constant problem and $1/N$ -expansion in Yang-Mills theories is to be investigated in the future.

We hope to use in our further publications the developed formalism in non-abelian gauge theories and develop the Legendre transformation techniques with the gauge-invariant source which generates the Wilson correlator.

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ОБОБЩЕННЫЙ ЭФФЕКТИВНЫЙ ПОТЕНЦИАЛ В НЕЛИНЕЙНЫХ ТЕОРИЯХ ЧЕТВЕРТОГО ПОРЯДКА

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