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ANOMALIES IN SUPERSYMMETRIC GAUGE THEORIES

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Section 1

This paper deals with the question of whether the supersymmetry of the Lagrangian persists after the renormalization of supersymmetric gauge theories. In other words, whether there are anomalies in these theories (superanomalies), similar to Adler's anomalies in gauge theories [1], which violate the supersymmetry of renormalized theories. It is clear that this problem is directly connected with the possibility of constructing an invariant intermediate renormalization of supersymmetric gauge theories. A regularization procedure for such theories constructed in Refs. 2,3 is a supersymmet ric generalization of the regularization of gauge theories by the method of higher covariant derivatives. However, that procedure does not provide a complete regularization of the theory (neither it regularized gauge theories), but reduces the problem to the regularization of one-loop superdiagrams. However, that procedure was constructed for theories implying vectorlike interaction of the matter fields with the gauge-supermultiplet fields. In this paper the problem of superanomalies is considered for a general theory, when the matter fields are represented by arbitrary chiral multiplets. We have found that a general super-gauge theory has no superano-malies provided it does not contain gauge anomalies. The proof is based on the analysis of Ward indentities (WI) correspod-ing to gauge and supergauge invariances of the theory in the Wess-Zumino gauge.

In Sec.2 we specify the method employed to obtain the result, in Secs. 3 and 4 the gauge and supergauge identi-ties, respectively, are written out and analysed.

Section 2

Consider a general supersymmetric gauge theory in the Wess-Zumino gauge. The possibility of supersymmetric genera-lization of the regularization by the method of higher cova-riant derivatives reduces the problem of superanomalies in the general super-gauge theory to the investigation of one-loop diagrams only. Moreover, as it follows from the results of Ref. 3 , only one-loop diagrams with internal matter fields and external gauge-supermultiplet fields can contri-bute to superanomalies. Hence one has to consider only that part of the total Lagrangian which contributes to the above mentioned one-loop diagrams.

Thus, let us introduce the finite generating functional $W_R\,(B_\mu, \chi, \mathcal{D})$ for one-particle irreducible one-loop dia-grams with external fields B^a_μ, χ^a and $\mathcal{D}^a$ (B^a_μ, $\mathcal{D}^a$ and the Majorana spinor χ^a represent a gauge-supermultiplet).

$$
\begin{aligned}
W(B_\mu,\chi,\mathcal{D}) = \ln N^{-1}\int_R d\psi_q\, dA_q \exp\Big\{ i\int dx\Big[L & - \\
-\frac{1}{4}\delta Z_1 (F^a_{\mu\nu})^2 - \frac{1}{2}\delta Z_2 F^a_{\mu\nu}\hat{B}^{ac}_\mu B^c_\nu - \frac{1}{4}\delta Z_3 \hat{B}^{ac}_\mu B^c_\nu \hat{B}^{ad}_\mu B^d_\nu & + \\
+\frac{i}{4}\delta Z_4 \bar{\chi}^a \gamma_\mu \partial_\mu \chi^a + \frac{i}{4}\delta Z_5 \bar{\chi}^a \gamma_\mu \hat{B}^{ab}_\mu \chi^b + \frac{1}{8}\delta Z_6 \mathcal{D}^a\mathcal{D}^a & + \\
+\frac{1}{2} d_1 (\partial_\mu B^a_\mu)^2 + d_2 \mathcal{D}^a \partial_\mu B^a_\mu - \frac{1}{2} d_3 B^a_\mu B^b_\mu \partial_\lambda B^c_\lambda d^{abc}\Big]\Big\}, &
\end{aligned}
\tag{1}
$$

where

$$
\begin{aligned}
L = \sum_q \Big[(\nabla_\mu(q) A_q)^+ \nabla_\mu(q) A_q + i\bar{\psi}_q \gamma_\mu \nabla_\mu(q)\psi_q & + \\
+ g\frac{q}{2}\mathcal{D}^a A^+_q \tau^a_q A_q - gq(\bar{\chi}^a A^+_q \tau^a_q \psi_q + \bar{\psi}_q \tau^a_q A_q \chi^a)\Big], & \\
F^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu, \quad \nabla_\mu(q) = \partial_\mu - ig\tau^a_q B^a_\mu\,, & \\
\hat{B}^{ac}_\mu = f^{abc} B^b_\mu\,, \quad d^{abc} = \sum_q \mathrm{Sp}\,\tau^c_q \{\tau^a_q, \tau^b_q\}\,. &
\end{aligned}
\tag{2}
$$

Spinor ψ_q and scalar A_q fields represent chiral matter supermultiplets (q denotes chirality and is equal to ± 1); matter fields are represented here by two, in gene-ral reducible, chiral supermultiplets, corresponding to two chiralities); τ_q are generators of corresponding reducible representations of matter fields, with τ_+ and τ_- differ-ing in general from each other; $\sum_q$ denotes a summation over chiralities; g is the coupling constant (for simplicity we will not write it down in what follows); the subscript

„R" here and below means a regularization of the theory by means of introducing a cut-off momentum Λ.

The summands in (1) proportional to δZ_i ($i = 1,2,\dots,6$) are counterterms which secure the convergence of the theory after the removal of the regularization ($\Lambda \to \infty$). The essence of the last summands in (1) (d_1, d_2 and d_3 are finite quantities) will be disclosed below.

The Lagrangian L is invariant under gauge transformations

$$\delta B^a_\mu = \partial_\mu \omega^a + f^{acb} B^c_\mu \omega^b, \quad \delta \chi^a = f^{acb} \chi^c \omega^b, \quad \delta \mathcal{D}^a = f^{acb} \mathcal{D}^c \omega^b, \tag{3}$$

$$\delta A_q = i\tau^a_q A_q \omega^a, \quad \delta A^+_q = -i\omega^a A^+_q \tau^a_q, \quad \delta\psi_q = i\tau^a_q \psi_q \omega^a,$$
$$\delta\bar\psi = -i\omega^a \bar\psi_q \tau^a_q,$$

($\omega^a(x)$ is an infinitesimal gauge parameter), and also under the global supersymmetry transformations [7]:

$$\delta B^a_\mu = \frac{1}{2}\bar\chi^a \gamma_\mu \gamma_5 \varepsilon, \quad \delta\mathcal{D}^a = i\partial_\mu \bar\chi^a \gamma_\mu \varepsilon + i\bar\chi^c \gamma_\mu f^{abc} B^b_\mu \varepsilon,$$
$$\delta\chi^a = \frac{1}{2}\mathcal{D}^a \varepsilon + \frac{i}{2}\gamma_5 \gamma_\mu \gamma_\nu F^a_{\mu\nu}\varepsilon + \frac{i}{2}\gamma_5\gamma_\mu\gamma_\nu f^{abc} B^b_\mu B^c_\nu \varepsilon, \tag{4}$$
$$\delta A_q = \bar\varepsilon \psi_q, \quad \delta A^+_q = \bar\psi_q \varepsilon, \quad \delta\psi_q = -i\frac{1+q\gamma_5}{2}\gamma_\mu \varepsilon \nabla_\mu A_q,$$
$$\delta\bar\psi_q = i(\nabla_\mu A)^+ \bar\varepsilon \gamma_\mu \frac{1-q\gamma_5}{2}$$

(ε is a Majorana spinor, an infinitesimal Grassmann parameter of super-gauge transformations).

The generating functional $W(B,\chi,\mathcal{D})$, too, is formally invariant under infinitesimal transformations (3)

and (4) of gauge-supermultiplet fields: $W(B,\chi,\mathcal{D}) = W(B+\delta B, \chi+\delta\chi, \mathcal{D}+\delta\mathcal{D})$ (the corresponding transformations of matter fields in (3) and (4) are compensated by the shift of integration variables). Using this property we obtain the following formal WI for W corresponding to gauge transformations (3)

$$-\left(\delta^{ab}\partial_\mu + f^{abc} B^c_\mu(x)\right)\frac{\delta W}{\delta B^a_\mu(x)} + f^{acb}\mathcal{D}^c(x)\frac{\delta W}{\delta \mathcal{D}^a(x)} + f^{acb}\chi^c(x)\frac{\delta W}{\delta\chi^a(x)} = 0 \tag{5}$$

and supergauge transformations (4)

$$\frac{1}{2}\bar\chi^a(x)\gamma_\mu\gamma_5\varepsilon\frac{\delta W}{\delta B^a_\mu(x)} + i\left(\partial_\mu\bar\chi^a(x)\gamma_\mu\varepsilon + \bar\chi^c(x)\gamma_\mu f^{abc} B^b_\mu(x)\varepsilon\right)\frac{\delta W}{\delta\mathcal{D}^a(x)} +$$
$$+\left(\frac{1}{2}\mathcal{D}^a(x)\varepsilon + \frac{i}{2}\gamma_5\gamma_\mu\gamma_\nu F^a_{\mu\nu}(x)\varepsilon + \frac{i}{2}\gamma_5\gamma_\mu\gamma_\nu f^{abc} B^b_\mu(x) B^c_\nu(x)\varepsilon\right)\frac{\delta W}{\delta\chi^a(x)} = 0 \tag{6}$$

(in (6) integration over x is implied).

Let us now expand $W(B,\chi,\mathcal{D})$ in powers of the external fields:

$$W(B,\chi,\mathcal{D}) = \frac{1}{2}B^a_\mu \Pi^{ab}_{\mu\nu} B^b_\nu + \frac{1}{2}\bar\chi^a_\alpha \Gamma^{ab}_{\alpha\beta}\chi^b_\beta + B^a_\mu \Gamma^{ab}_\mu \mathcal{D}^b + \frac{1}{2}\mathcal{D}^a\Gamma^{ab}\mathcal{D}^b +$$
$$+\frac{1}{6}B^a_\mu B^b_\nu B^c_\lambda \Gamma^{abc}_{\mu\nu\lambda} + \frac{1}{2}B^a_\mu B^b_\nu \mathcal{D}^c \Gamma^{abc}_{\mu\nu} + \frac{1}{2}B^a_\mu \mathcal{D}^b\mathcal{D}^c\Gamma^{abc}_\mu + \tag{7}$$
$$+\frac{1}{6}\mathcal{D}^a\mathcal{D}^b\mathcal{D}^c\Gamma^{abc} + \frac{1}{2}B^a_\mu\bar\chi^b_\alpha\chi^c_\beta\Gamma^{abc}_{\mu,\alpha\beta} + \frac{1}{2}\bar\chi^a_\alpha\chi^b_\beta\mathcal{D}^c\Gamma^{abc}_{\alpha\beta}, +$$
$$+\frac{1}{24}B^a_\mu B^b_\nu B^c_\lambda B^d_\sigma \Gamma^{abcd}_{\mu\nu\lambda\sigma} + \dots$$

(subsequent terms are not written out explicitly because of their irrelevance).

Inserting the expansion (7) in (5), (6) and equating to zero the summands proportional to different powers of external fields we obtain a set of gauge and super-gauge identities interconnecting different vertex functions from the expansion (7). As will be shown below (Sections 3,4), these identities based on the formal invariance of W under transformations (3) and (4) on certain conditions hold in the final theory.

Free propagators $\mathcal{D}_q^{ik}(K)$ and $S_q^{ik}(K)$ of the fields A_q^i and ψ_q^i respectively, are given in the momentum space by

$$\mathcal{D}_q^{ik}(K)=\frac{i\delta^{ik}}{K^2}\,,\quad S_q^{ik}(K)=iP_q\frac{\hat{K}\delta^{ik}}{K^2}\,,\quad P_q=\frac{1+q\gamma_5}{2}\,,\quad \hat{K}=K_\mu\gamma_\mu . \tag{8}$$

The vertex functions of the expansion (7) that are used in the following are given (in the momentum-integral representation) by

$$\Pi_{\mu\nu}^{ab}(P)=\delta^{ab}\Big[\tau\int_R\frac{dr}{(2\pi)^4}\frac{P_\mu P_\nu+2(rp-p^2)g^{\mu\nu}}{r^2(r-p)^2}+i\delta Z_1(P_\mu P_\nu-g^{\mu\nu}p^2)+id_1P_\mu P_\nu\Big], \tag{9}$$

$$\Gamma_{\alpha\beta}^{ab}(P)=\delta^{ab}\Big[-\tau\int_R\frac{dr}{(2\pi)^4}\frac{\hat{r}-\hat{p}}{r^2(r-p)^2}+\frac{i}{2}\delta Z_4\hat{P}\Big]_{\alpha\beta}\,, \tag{10}$$

$$\Gamma_\mu^{ab}(P)=\delta^{ab}\Big[\frac{\tilde{\tau}}{2}\int_R\frac{dr}{(2\pi)^4}\frac{(2r-P)_\mu}{r^2(r-p)^2}-d_2P_\mu\Big]=-\delta^{ab}P_\mu\Big(\frac{i\pi^2\tilde{\tau}}{4(2\pi)^4}+d_2\Big), \tag{11}$$

$$\Gamma^{ab}(P)=\delta^{ab}\Big[\frac{\tau}{4}\int_R\frac{dr}{(2\pi)^4}\frac{1}{r^2(r-p)^2}+i\frac{1}{4}\delta Z_6\Big], \tag{12}$$

$$\Gamma_{\mu\nu}^{abc}(P,K,q)=-\frac{1}{2}A^{abc}\Big[\int_R\frac{dr}{(2\pi)^4}\frac{(2r-p)_\mu(2r+K)_\nu}{r^2(r+K)^2(r-p)^2}-g^{\mu\nu}\int_R\frac{dr}{(2\pi)^4}\frac{1}{(r+K)^2(r-p)^2}\Big], \tag{13}$$

$$\begin{aligned}\Gamma_{\mu,\alpha\beta}^{abc}(P,K,q)=&-\frac{1}{2}\int_R\frac{dr}{(2\pi)^4}\frac{1}{r^2(r+q)^2(r-K)^2}\Big\{i\tau f^{abc}\gamma_{\lambda,\alpha\beta}\big[(r+q)_\mu K_\lambda-\\&-(r-K)_\mu q_\lambda+(r-K,r+q)g_{\mu\lambda}\big]+A^{abc}(\gamma_5\gamma_\lambda)_{\alpha\beta}\big[(r+q)_\mu(2r-K)_\lambda+\\&+(r-K)_\mu(2r+q)_\lambda-(r-K,r+q)g^{\mu\lambda}\big]-i\varepsilon^{\sigma\mu\nu\lambda}(r-K)_\sigma(r+q)_\nu\times\\&\times\big[A^{abc}\gamma_\lambda+i\tau f^{abc}\gamma_\lambda\gamma_5\big]_{\alpha\beta}\Big\}+\frac{1}{2}\delta Z_5f^{abc}\gamma_{\mu,\alpha\beta}\,,\end{aligned} \tag{14}$$

$$\begin{aligned}\Gamma_{\mu\nu\lambda}^{abc}(P,K,q)=&\frac{1}{3}\Big\{\Big[\int_R\frac{dr}{(2\pi)^4}\frac{1}{r^2(r+K)^2(r-p)^2}\Big(-i\tau f^{abc}(2r-p)_\mu(2r+K)_\nu(2r-p+K)_\lambda+\\&+\frac{i}{2}\tau f^{abc}Sp(\gamma_\mu\hat{r}\gamma_\nu(\hat{r}+\hat{K})\gamma_\lambda(\hat{r}-\hat{p}))-\frac{1}{2}A^{abc}Sp(\gamma_5\gamma_\mu\hat{r}\gamma_\nu(\hat{r}+\hat{K})\gamma_\lambda(\hat{r}-\hat{p}))\Big)\Big]+\\&+\begin{bmatrix}p\leftrightarrow q\\a\leftrightarrow c\\\mu\leftrightarrow\lambda\end{bmatrix}+\begin{bmatrix}K\leftrightarrow q\\b\leftrightarrow c\\\nu\leftrightarrow\lambda\end{bmatrix}\Big\}+\delta Z_2f^{abc}\big[(p-q)_\nu g^{\mu\lambda}+(q-K)_\mu g^{\nu\lambda}+(K-p)_\lambda g^{\mu\nu}\big]+\varkappa_{\mu\nu\lambda}^{abc},\end{aligned}$$

$$\begin{aligned}\varkappa_{\mu\nu\lambda}^{abc}=&d^{abc}\Big\{g^{\mu\nu}\Big[q_\lambda d_3+\int_R\frac{dr}{(2\pi)^4}\frac{(2r-q)_\lambda}{r^2(r-q)^2}\Big]+\\&+g^{\mu\lambda}\Big[K_\nu d_3+\int_R\frac{dr}{(2\pi)^4}\frac{(2r-K)_\nu}{r^2(r-K)^2}\Big]+g^{\nu\lambda}\Big[P_\mu d_3+\int_R\frac{dr}{(2\pi)^4}\frac{(2r-P)_\mu}{r^2(r-p)^2}\Big]\Big\}=\\&=d^{abc}\Big(d_3-\frac{i\pi^2}{2(2\pi)^4}\Big)\big(g^{\mu\nu}q_\lambda+g^{\mu\lambda}K_\nu+g^{\nu\lambda}P_\mu\big)\,,\end{aligned} \tag{15}$$

$$\Gamma^{abcd}_{\mu\nu\lambda\sigma}(p,k,q,t)=\sum_q\Big\{\Big[\frac{1}{4}Sp\left(\tau_q^a\tau_q^b\tau_q^c\tau_q^d\right)\int_R\frac{dr}{(2\pi)^4}\,\frac{1}{r^2(r+t)^2(r-q)^2(r+p+t)^2}\times$$

$$\times\Big((2r+2t+p)_\mu(2r-2q-k)_\nu(2r-q)_\lambda(2r+t)_\sigma-Sp\gamma_\mu\frac{1+q\gamma_5}{2}(\hat r+\hat p+\hat t)\gamma_\nu\times$$

$$\times(\hat r-\hat q)\gamma_\lambda\hat r\gamma_\sigma(\hat r+\hat t)\Big)-g^{\mu\nu}Sp\left(\tau_q^a\tau_q^b\tau_q^c\tau_q^d\right)\int_R\frac{dr}{(2\pi)^4}\,\frac{(2r+t)_\sigma(2r-q)_\lambda}{r^2(r+t)^2(r-q)^2}+\tag{16}$$

$$+\frac{1}{2}g^{\mu\nu}g^{\lambda\sigma}Sp\left(\tau_q^a\tau_q^b\tau_q^c\tau_q^d\right)\int_R\frac{dr}{(2\pi)^4}\,\frac{1}{(r+t)^2(r-q)^2}\Big]+$$

$$+\text{ symmetrizations over legs}\Big\}+i\delta Z_3\Big[g^{\mu\nu}g^{\lambda\sigma}\left(f^{bde}f^{eca}+f^{ade}f^{ecb}\right)+$$

$$+g^{\mu\lambda}g^{\nu\sigma}\left(f^{cde}f^{eba}+f^{ade}f^{ebc}\right)+g^{\mu\sigma}g^{\nu\lambda}\left(f^{dce}f^{eba}+\right.$$

$$\left.+f^{dbe}f^{eca}\right)\Big].$$

The following notations were introduced:

$$\sum_q Sp\,\tau_q^a\tau_q^b=\delta^{ab}\tau\ ,\quad \sum_q q\,Sp\,\tau_q^a\tau_q^b=\tilde\tau\,\delta^{ab},$$

$$A^{abc}=\sum_q q\,Sp\,\tau_q^a\left\{\tau_q^b,\tau_q^c\right\}.$$

Terms proportional to π^2 are surface terms which are generated by the translation of integration variables in the linearly divergent integrals and are given in the limit of the removed renormalization ($\Lambda\to\infty$).

Note that in vectorlike theories (when τ_+ and τ_- are

the same) the quantities $\tilde\tau$ and A^{abc} are identically zeros.

Let us now turn to the direct checking of WI using the expressions of vertex functions given above.

Section 3

First we write out and analyse gauge WI considering only those identities which contain integrals with worse than logarithmic divergences (in unregularized theory) since the origin of the WT violating anomalies is the nonzero surface term arising from the translation of integration variables in such integrals.

Substituting (7) into the generalized identity (5) and differentiating the latter with respect to external fields appropriate number of times (with a subsequent equating to zero of external fields), the following set of a gauge identities in the momentum space is obtained:

$$P_\mu\,\Pi^{ab}_{\mu\nu}(p)=0\ ,\tag{17}$$

$$P_\mu\,\Gamma^{ab}_\mu(p)=0\ ,\tag{18}$$

$$iP_\mu\Gamma^{abc}_{\mu\nu\lambda}(p,k,q)=f^{abe}\,\Pi^{ec}_{\nu\lambda}(q)-f^{abe}\,\Pi^{ec}_{\nu\lambda}(k),\tag{19}$$

$$P_\mu \Gamma^{abc}_{\mu\nu}(p,K,q) = 0\,, \tag{20}$$

$$i(p+K)_\mu \Gamma^{abc}_{\mu,\alpha\beta}(q,K,p) = f^{abe}\Gamma^{ec}_{\alpha\beta}(K) - f^{abe}\Gamma^{ec}_{\alpha\beta}(-p)\,, \tag{21}$$

$$-ip_\mu \Gamma^{abcd}_{\mu\nu\lambda\sigma}(p,K,q,t) = f^{dae}\Gamma^{ebc}_{\sigma\nu\lambda}(p+t,K,q) +$$

$$+ f^{cae}\Gamma^{ebd}_{\lambda\nu\sigma}(p+q,K,t) + f^{bae}\Gamma^{ecd}_{\nu\lambda\sigma}(p+K,q,t)\,. \tag{22}$$

Substituting (9) in the identity (17) which is the polarization operator $\Pi^{ab}_{\mu\nu}(p)$ transversality condition, and (11) into (18) we find that the latter hold provided

$$d_1 = \frac{\pi^2\tau}{2(2\pi)^4}\,, \quad d_2 = -\frac{i\pi^2\tilde{\tau}}{4(2\pi)^4}\,, \quad \left(\Gamma^{ab}_\mu(P) \equiv 0\right)\,. \tag{23}$$

This investigation of identities (17) and (18) sheds light on the necessity for introduction of additional finite counterterms in the Lagrangian (1). This is due to the fact that the adopted regularization procedure is not gauge invariant and cannot ensure the automatic fulfilment of the renormalized WI (in the absence of anomalies) in the limit of removed regularization. The introduction of specially chosen finite counterterms compensates this drawback of the regula-

rization.

Immediate examination of the identity (19) shows that it is true only upon the condition that

$$\frac{2}{3}\frac{i\pi^2}{(2\pi)^4}A^{abc}\varepsilon^{\beta\alpha\lambda\nu}K_\alpha q_\beta + d^{abc}\left(d_3 - \frac{i\pi^2}{2(2\pi)^4}\right)\left(P_\nu q_\lambda + P_\lambda K_\nu + g_{\nu\lambda}P^2\right) + \tag{24}$$

$$+ if^{abc}\left[(q_\nu q_\lambda - g_{\nu\lambda}q^2) - (K_\nu K_\lambda - g_{\nu\lambda}K^2)\right]\left[\left(\frac{\pi^2\tau}{6(2\pi)^4} + \delta Z_2\right) - \left(\delta Z_1 + \frac{\tau\pi^2}{2(2\pi)^4}\right)\right] = 0.$$

whence it follows that

$$d_3 = \frac{i\pi^2}{2(2\pi)^4}\,; \qquad \delta Z_2 = \delta Z_1 + \frac{\tau\pi^2}{3(2\pi)^4}\,. \tag{25}$$

As for the first summand in (24), it is just the anomalous term which violates the identity (19) and thus reproduces the well-known situation in gauge theories. The absence of anomalies in this identity (and as it turns out, in a whole theory) is guaranteed on the condition that

$$A^{abc} = 0. \tag{26}$$

All investigation in what follows will be carried out on the condition (26). It is worth noting that additional finite counterterms in (1) had been written out with account of (26). Now the identity (20) holds trivially (see (13)).

The analyses of remaining gauge identities lead to the relations:

$$\delta Z_5 = \delta Z_4 + \frac{\pi^2\tau}{2(2\pi)^4}\,; \qquad \delta Z_3 = \delta Z_2 + \frac{\pi^2\tau}{6(2\pi)^4}\,. \tag{27}$$

From (25) and (27) it follows that the constants δZ_1 ,

δZ_2 and δZ_3 differ from each other and δZ_4 differs from δZ_5 by finite quantities, i.e. their divergent parts satisfy the equations $\delta Z_{1\,div} = \delta Z_{2div} = \delta Z_{3div}$; $\delta Z_{4div} = \delta Z_{5div}$. Note that these equations being taken into account, the divergent parts of the counterterms in (1) combine into gauge-invariant structures proportional to $(G^a_{\mu\nu})^2$ and $\bar{\chi}^a(\partial_\mu\delta^{ab} + \hat{B}^{ab}_\mu)\chi^b$ respectively. Thus the requirement of gauge invariance of renormalized theory brings to a number of relations and we shall take them into account in analysing superidentities in the next section.

Section 4

To obtain superidentities one has to substitute (7) in the general superidentity and equate to zero the summands proportional to different powers of external fields. The following set of superidentities ensues (again we have written out only those superidentities in which integrals diverging worse than logarithmically are present)

$$\int dp\, \bar{\chi}^a_\alpha(p)\,\mathcal{D}^b(-p)\left[\frac{1}{2}\Gamma^{ab}_{\alpha\beta}(p) - (\hat{p})_{\alpha\beta}\Gamma^{ab}(p)\right]\varepsilon_\beta = 0\,, \qquad (28)$$

$$\int dp\, \bar{\chi}^a_\alpha(p)\, B^b_\nu(-p)\left[(\gamma_\mu\gamma_5\varepsilon)_\alpha \Pi^{ab}_{\mu\nu}(p) + p_\mu \Gamma^{ab}_{\alpha\beta}(p)(\gamma_5[\gamma_\mu,\gamma_\nu]\varepsilon)_\beta\right] = 0, \qquad (29)$$

$$\begin{aligned}\int dp\,dk\,dq\,\delta(p+k+q)\,\bar{\chi}^a_\alpha(-p)\,B^b_\nu(-k)\,B^c_\lambda(-q)\Big[\tfrac{1}{2}(\gamma_\mu\gamma_5\varepsilon)_\alpha \Gamma^{abc}_{\mu\nu\lambda}(p,k,q) + \\ + i f^{dbc}(\gamma_5\gamma_\nu\gamma_\lambda\varepsilon)_\beta \Gamma^{ad}_{\alpha\beta}(p,k+q) - (\gamma_\mu\varepsilon)_\alpha p_\mu \Gamma^{cba}_{\lambda\nu}(q,k,p) - \\ - k_\mu \Gamma^{cab}_{\lambda,\alpha\beta}(q,p,k)(\gamma_5[\gamma_\mu,\gamma_\nu]\varepsilon)_\beta\Big] = 0\,.\end{aligned} \qquad (30)$$

Substituting the explicit expressions of vertex functions in the first two equations we find that they hold provided relations

$$\delta Z_1 = \delta Z_4\,, \quad \delta Z_6 = \delta Z_4 + \frac{\pi^2 \tau}{2(2\pi)^4}\,. \qquad (31)$$

are satisfied.

As for the third identity, it is true on the condition

$$\delta Z_2 = \delta Z_4 + \frac{\pi^2\tau}{3(2\pi)^2} \qquad (32)$$

which is correlated with (25) and (31) and also with the first of the relations (27).

Thus the requirement of the superinvariance of the theory in question leads to new relations between the renormalization constants. Combining (25), (27), (31) we find that

$$\delta Z_1 = \delta Z_2 - \frac{\pi^2\tau}{3(2\pi)^4} = \delta Z_3 - \frac{\pi^2\tau}{2(2\pi)^4} = \delta Z_4 = \delta Z_5 - \frac{\pi^2\tau}{2(2\pi)^4} = \delta Z_6 - \frac{\pi^2\tau}{2(2\pi)^4}\,. \qquad (33)$$

Note that these relations being taken into account, the divergent parts of counterterms in (1) unite in a supersymmetric gauge-invariant structure.

It is worth reminding that in analysing superidentities the relations of Sec.3, which followed from the requirement of gauge-invariance, were supposed to be satisfied. The relation (32) which is needed for the validity of the superidentity (30) is added to the ones obtained in Sec.3. In particular, the violation of (26) will invalidate the identity (30).

As it was pointed out above, anomalies can arise only i identities which contain integrals diverging worse than logarithmically (in unregularized theory), while in identities comprising at worst logarithmically divergent integrals translations of integration variables are permitted (no surface terms appear). The formal proof of the latter identities in unregularized theory is correct in regularized theory as well. In particular, superidentities with logarithmically divergent quantities will evidently hold after renormalization. It is worth noting, however, that we are investigating the question of whether the theory that is manifestly gauge-invariantly renormalizable is at the same time supersymmetrically renormalizable. In the renormalization procedure we used the requirement of gauge-invariance led to the necessity of adding finite counterterms. Hence, if the superidentities with logarithmically divergent quantities contain vertices which have acquired additional finite counterterms, generally speaking, one must verify that these extra terms do not violate the identity. However, it turns out that in the case under discussion (with $A^{abc} = 0$) superidentities with logarithmically divergent quantities do not contain such vertices. Thus from the analyses of superidentities one can deduce that superanomalies do not exist. It is important to note that this statement is true only on the condition (26), i.e. if gauge identities are free of anomalies.

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REFERENCES

1 S.L.Adler, Phys.Rev. 177, 2426, 1969;
J.S.Bell and R.Jackiw, Nuovo Cimento 60A, 47, 1969.

2 Ferrara S., Piguet O., Nucl.Phys. 363, 261, 1975.

3 Krivoschekov V.K., Teor. Mat. Fiz. 36, 291, 1978.

4 Slavnov A.A., Nucl.Phys. B31, 301, 1971; Teor. Mat. Fiz. 13, 174, 1972.

5 Slavnov A.A., Teor. Mat. Fiz. 33, 210, 1977.

6 Wess J., Zumino B., Nucl.Phys. B78, 1, 1974.

7 De Wit B., Freedman D.Z., Phys.Rev. D12, 2286, 1975.

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АНОМАЛИИ В СУПЕРСИММЕТРИЧНЫХ КАЛИБРОВОЧНЫХ ТЕОРИЯХ

(на английском языке)

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