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THE RELATION BETWEEN THE PARAMETERS OF
AND MIT BAG MODEL

ԵՐԵՎԱՆ 1980 ԵՐԵՎԱՆ

In Ref. [1] we have hypothesized that the quantum chromodynamical vacuum, as considered in the perturbation theory, is not the true one and that the lower lying ground state is characterized by the non-zero strength of the gluon field, - the so-called "gluon condensate". This conclusion was based on the expression for the density of gluon field energy obtained in the one-loop approximation [1]:

$$\text{Re} \varepsilon = \frac{H^2}{2} + \frac{11C_2(G)}{48\pi^2} (gH)^2 \left[\ln \left(\frac{gH}{\mu_s} \right) - \frac{1}{2} \right], \quad (1)$$

where $H = \langle 0 | H | 0 \rangle$ is the magnetic field strength, μ_s is the normalization point [1,2]

$$\text{Re} \frac{\partial \varepsilon}{\partial H^2} \Big|_{gH = \mu_s^2} = \frac{1}{2}, \quad (2)$$

and $C_2(G)$ is the Casimir operator of the gauge group $G = SU_c(3)$, which should be replaced by $C_2(G) - \frac{4}{11} T(R)$ when the quark loops are taken into account. The minimum of (1) differs from the point $H = 0$ [1] and is at the point

$$(gH)_{\text{vac}}^2 = \mu_s^4 \exp \left\{ - \frac{96\pi^2}{11C_2(G)g^2} \right\} = \Lambda_s^4 \quad (3)$$

It is renormalization -group invariant because the expression for the energy density is renormalization invariant.

Using the result obtained with the help of the renormalization group [2]

$$Re \frac{\partial \mathcal{E}}{\partial H^2} = \frac{1}{2} \frac{g^2}{\bar{g}^2(t)}, \quad t = \ln \left(\frac{gH}{\mu_s^2} \right). \quad (4)$$

where the effective charge in the external field $g(t)$ is determined as

$$\frac{d\bar{g}}{dt} = \beta(\bar{g}), \quad \beta(g) = -\frac{11C_2(G)}{48\pi^2} g^3 + \dots \quad (5)$$

and assuming $\int \frac{dx}{\beta(x)} < \infty$, we find

$$(gH)_{vac}^2 = \mu_s^4 \exp \left\{ 2 \int \frac{dx}{\beta(x)} \right\}. \quad (6)$$

However, the total imaginary part of the effective Lagrangian in QCD is

$$2Im \mathcal{L} = \frac{1}{4\pi^2} \left[\pi (gf_1)^2 + (gf_1)(gf_2) \sum_{k=1}^{\infty} \frac{(-1)^k}{k \operatorname{sh} \left(\frac{k}{2} K \pi \right)} \right] \quad (7) \quad *)$$

where $f_1^2 = \mathcal{F} + (\mathcal{F}^2 + \mathcal{E}^2)^{1/2}$, $f_2^2 = -\mathcal{F} + (\mathcal{F}^2 + \mathcal{E}^2)^{1/2}$,

*) The formula (7) follows from the expression (3.18) of Ref.[3] when the integration contour in the complex range S is taken in the direction of $(f_1 - i f_2) S$. This result was obtained in collaboration with I.A.Batalin in 1978.

and $\mathcal{F} = \frac{1}{2} (E^2 - H^2)$, $\mathcal{E} = \vec{H} \cdot \vec{E}$,

is $2Im \mathcal{L} = (gH)^2 / 4\pi$ in the magnetic field [4] and is of the same order as that in the electric field [3] $2Im \mathcal{L} = (gE)^2 / 48\pi^2$. The imaginary part \mathcal{L} in the magnetic field is due to the unstable mode [4] $K_0^2 = K_n^2 - gH$, whose presence makes the level [1]

$$\mathcal{E}(H_{vac}^2) = -\frac{11C_2(G)}{192\pi^2} \Lambda_s = -\frac{11C_2(G)}{196\pi^2} (gH)_{vac}^2 \quad (8)$$

unstable and, hence, further lowering of the energy takes place.

To find the lower state it is necessary to introduce a dynamical variable allowing for the excitation of the unstable mode with the Higgs type Lagrangian [5]. Such a hidden Higgs mechanism leads to the production of magnetic domains and the reduction of the energy density by the value C which is of the order of unity [6]

$$\frac{H^2}{2} \longrightarrow \frac{\langle H^2 \rangle}{2} (1-C) \quad (9)$$

We shall consider two kinds of ansatz: the Nielsen-Ni-nomiya ansatz [6] with $C_1^{-1} = 1.18$, and the Ambjorn-Olesen one [6] with $C_2^{-1} = 1.16$ *). The first one corresponds to the

*) These numbers are analytically expressed [6] as:
 $C_1^{-1} = \left(\sum_{n=-\infty}^{+\infty} e^{-\pi n^2} \right)^2$, $C_2^{-1} = \sqrt{\frac{3}{2}} \sum_{n=-\infty}^{\infty} e^{-\frac{\pi\sqrt{3}}{2}(n_1^2 + n_2^2) - i\pi n_1 n_2}$

production of rectangular domains, while the second one to the production of more stable, -the hexagonal domains.

Thus, both the effects, - the vacuum polarization [1,2,3] and the hidden Higgs mechanism [5,6] stipulated by the non-stable mode, yield that the energy density be in reality the sum of (9) and the second summand in (1). However, contrary to (1) such an expression is not renormalization group invariant and hence, it must be written in the form:

$$\mathcal{E} = \frac{\langle H^2 \rangle}{2} (1-C) + \frac{11C_2(G)}{48\pi^2} g^2 \langle H^2 \rangle \left[(1-C) \ln \frac{\sqrt{g^2 \langle H^2 \rangle}}{\Lambda_s^2} - \frac{1}{2} \right] \quad (10)$$

The minimum of the latter expression is in the point

$$g^2 \langle H^2 \rangle_{vac} = \Lambda_s^4 \exp \left\{ \frac{1}{C^{-1} - 1} \right\} \quad (11)$$

and is equal to

$$\begin{aligned} \mathcal{E}(\langle H^2 \rangle_{vac}) &= - \frac{11C_2(G)}{192\pi^2} \Lambda_s^4 (1-C) \exp \left\{ \frac{1}{C^{-1} - 1} \right\} \quad (12) \\ &= - \frac{11C_2(G)}{48} \frac{d_s}{\pi} \langle H^2 \rangle_{vac} (1-C) \end{aligned}$$

where $d_s = \frac{g^2}{4\pi}$. Assuming now that, just as in the bag model [7], the difference of energy densities has a meaning of the constant $B = 145 \text{ MeV}^4$ and that under Lorentz transformations $\langle H^2 \rangle \rightarrow \frac{1}{2} \langle G_{\mu\nu}^a G_{\mu\nu}^a \rangle$ we obtain

$$B = \frac{11C_2(G)}{96} \frac{d_s}{\pi} \langle G_{\mu\nu}^a G_{\mu\nu}^a \rangle \cdot (1-C) \quad (13)$$

The numerical value for $\frac{d_s}{\pi} \langle G^2 \rangle = 0.012 \text{ GeV}^4$ was obtained from generalized sum rules for the charmonium [8]. Substituting to the right-hand side of (13) the numerical values for $\langle G^2 \rangle$ and $C_2(G) \rightarrow 33 - 2N_f$, where N_f is the number of flavours

which we take to be three, and for $C_{1,2}$ we have

$$B_1 = 151 \text{ MeV}^4, \quad B_2 = 147 \text{ MeV}^4 \quad (14)$$

just in a good agreement with the phenomenological value for B . Note that the effective constant depending on the field (5) [2] in vacuum (11) is

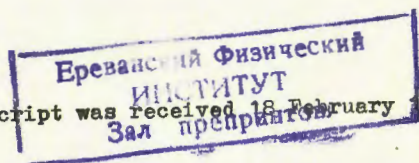
$$\frac{\bar{g}^2}{4\pi} = \frac{6\pi}{11C_2(G)} \frac{4}{\ln(g^2 \langle H^2 \rangle / \Lambda_s^4)} = \frac{6\pi}{11C_2(G)} \cdot 4 \cdot (C^{-1}) \approx 0.45 \quad (15)$$

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