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A. YU. KHODJAMIRIAN

RADIATIVE TRANSITIONS IN QUARKONIUM AND QUANTUM CHROMODYNAMICS

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1. Introduction

The dispersion treatment of the heavy quarkonium levels ($q\bar{q}$-resonances, where $q=c,b,\ldots$) developed in Ref. [1] turned out to be one of the most fruitful applications of the quantum chromodynamics (QCD).

Based on the very general principles of the theory, such as asymptotic freedom, analyticity and unitarity, this approach consists in the derivation of dispersion sum rules for certain amplitudes (vacuum averages) induced by the q-quark currents. The sum rules connect the QCD calculated amplitudes in deep virtual region with the $q\bar{q}$-resonance contributions to the physical imaginary parts of these amplitudes. Due to this connection the properties of the lowest quarkonium levels sited below the q-flavor threshold turn out to be distinctly fixed. So one is able to predict various parameters of these levels, such as their couplings to the quark currents, their annihilation widths and even their masses.

The purpose of this work is to calculate in a similar way the widths of radiative transitions between the quarkonium levels.

Usually these processes are treated in the framework of nonrelativistic potential models. Contrary to that the approach suggested here is purely relativistic. It does not contain such ingredients as interaction potential, radial wave functions or overlap integrals.

In our approach the matrix elements of radiative transitions are represented by the ordinary Lorentz-invariant three-particle vertices (with all the particles on the mass shell) built out of four-momenta and polarization vectors of the two resonances and the photon which participate in a given transition.

As it will be seen further, our approach is a rather natural generalization of the method developed in Ref. [1] and is closely related to this method both in basic elements and in some technical details . The generalization consists in the use of double dispersion relations (double imaginary parts or Mandelstam type spectral functions) instead of usual one-dimensional dispersion relations used in Ref. [1] .

The double dispersion relations are in fact most suitable for our purposes. Really, if we write down such a relation for the triangle vacuum amplitude induced by three quark currents j_1, j_2 and j^{em} (see Fig.1) , then the resonance part of the physical double spectral function will contain exactly the matrix elements of $1 \to 2+\gamma$, $2 \to 1+\gamma$ type radiative transitions, where 1 (2) is the $q\bar{q}$-resonance with the same J^{PC} quantum numbers as those of the $j_1(j_2)$ current. In derivation of double dispersion relations we shall use the analyticity on s_1 and s_2

variables which are the squares of the external four-momenta in j_1 and j_2 vertices, respectively.

To obtain a set of sum rules as complete as possible we shall differentiate the dispersion relations not only at $s_1 = 0$, $s_2 = 0$ (following Ref.[1]), but also at $s_1 = -\infty$, $s_2 = -\infty$ .The last procedure, as it will be shown below, will help us to obtain information on the transitions between resonances sited above the lowest quarkonium states. The expansion at $s = -$ in one-dimensional dispersion relations for the c-quark polarization operator have been used recently in Ref. [2] . The analogous method have been applied earlier in Ref. [3].

The plan of this paper is as follows.

The general outline of double dispersion sum rules derivation is presented in Sec. 2 .

Following this outline, the sum rules for the most interesting transitions between C-even $0^{++}, 1^{++}, 2^{++}$, 0^{-+} - levels and vector 1^{--}-levels are obtained in the zeroth order of QCD($\alpha_s = 0$). We discuss the possible influence of gluon corrections and choose the "optimum" moments of the sum rules for which these corrections are expected at the level of $O(\alpha_s) \sim 20\%$. (The calculation of gluon corrections is a rather complicated problem and we postpone it for the future.)

In Sec.3 the obtained sum rules are used to calculate the amplitudes of the electric dipole (E1) transitions ($0^{++}, 1^{++}, 2^{++} \leftrightarrow 1^{--}$) in charmonium. For this purpose the following procedure is used. The set of sum rules is considered as an overdetermined system of linear equations for the unknown amplitudes. Then, the amplitudes are fitted to minimize the sum of squared differences between r.h.s. and l.h.s. of these equations taken over "optimum" moments. This procedure leads to reasonable results, since the fitted values of amplitudes turn out to satisfy all "optimum" moments of the sum rules with the accuracy of 10-20 % i.e. at the level of the QCD zeroth order uncertainty.

The amplitudes obtained in this way determine the widths of E1 transitions in charmonium, which are in a good agreement with available experimental data.

The widths of the lowest E1 transitions in b-quarkonium (the Υ-family) are also estimated in the same way.

In Sec. 4 the magnetic-dipole transitions (M1) are considered between 1^{--} and 0^{-+} levels of quarkonium. The sum rules relating the amplitudes of four M1 transitions between J/ψ, ψ', η_c, η_c'-levels make it possible to estimate the amplitudes of $J/\psi \to \eta_c \gamma$, $\psi' \to \eta_c' \gamma$ diagonal transitions.

If we identify the η_c with the recent SLAC candidate [4] $\eta_c(2977)$, then our estimates do not contradict the preliminary data on $J/\psi \to \eta_c \gamma$ transitions. In a special case our sum rules coinside with simple VDM type relation between $\Gamma(J/\psi \to \eta_c \gamma)$ and $\Gamma(\eta_c \to 2\gamma)$ widths that have been recently obtained and justified from the QCD point of view in Ref. [5].

Finally, in Sec. 5 we present a few comments concerning the comparison of our approach with the traditional nonrelativistic models of radiative transitions.

Note, that some of the results presented here have been published recently [6] in letter form.

2. Dispersion sum rules in the zeroth order of QCD

We start from the general derivation of double dispersion sum rules for the amplitudes of radiative transitions between q-quarkonium levels (q = c,b,...).

Consider a triangle amplitude which is the vacuum average of the T-product of three currents :

$$A_{\mu 12}(k, p_1, p_2) = eQ \int dx\, dy\, e^{-i(kx + p_2 y)} \times$$
$$\times \langle 0 | T\{ j_1(0)\, j^{em}_\mu(x)\, j_2(y) \} | 0 \rangle \equiv$$
$$\equiv I^a_{\mu 12} A_a(s_1, s_2) + I^b_{\mu 12} A_b(s_1, s_2) + \cdots \qquad (1)$$

where Q is the q-quark charge ; $j^{em} = \bar{q}^i \gamma_\mu q^i$ is the electromagnetic current corresponding to the real photon emission with momentum k ; $j_1 = \bar{q}^i\, O_1\, q^i$ and $j_2 = \bar{q}^i\, O_2\, q^i$ are the quark currents with J_1^{PC} and J_2^{PC} quantum numbers determined by the O_1 and O_2 Lorentz structures (sum over color indices) ; p_1 and p_2 are the external momenta at j_1 and j_2 vertices ($p^2_{1,2} = s_{1,2}$) ; $A_{a,b..}$ are the invariant amplitudes; $I^{a,b,..}_{\mu 12}$ are the kinematical structures built out of four-momenta p_1, p_2, k. The number and the form of these structures are defined by $J^{PC}_{1,2}$ quantum numbers. Hereafter, the Lorentz indices correponding to these quantum numbers are omitted.

We shall always choose $I_{\mu 12}$ in the form excluding kinematical singularities on s_1, s_2, so that the analytiwhere $g_{1,2}$ define the "vacuum" matrix elements

$$
\begin{aligned}
& m_1^{c_1} g_1 \tau_1 = \langle 0 | j_1 | R_1 \rangle , \\
& m_2^{c_2} g_2 \tau_2 = \langle 0 | j_2 | R_2 \rangle , \quad m_2'^{c_2} g_2' \tau_2' = \langle 0 | j_2 | R_2' \rangle
\end{aligned}
\tag{4}
$$

that may be extracted from annihilation widths and/or by an independent dispersion method as in Ref. [1]. F and F' define the matrix elements of $R_1 \to R_2\gamma$, $R_2' \to R_1\gamma$ radiative transitions

$$
\begin{aligned}
& (m_1^{d_a} F_a I^a_{\mu 12} + m_1^{d_b} F_b I^b_{\mu 12} + \cdots)\tau_1 \tau_2 = \langle R_1 | j^{em}_\mu | R_2 \rangle \\
& (m_1^{d_a} F_a' I^a_{\mu 12} + m_1^{d_b} F_b' I^b_{\mu 12} + \cdots)\tau_1 \tau_2' = \langle R_1 | j^{em}_\mu | R_2' \rangle
\end{aligned}
\tag{5}
$$

that we are interested in. τ_1, τ_2, τ_2' are the wave functions of R_1, R_2, R_2'; $c_{1,2}$, $d_{a,b..}$ dimensions are chosen so that g_1, g_2 and $F_{a,b}$, $F'_{a,b}$ are dimensionless. (The indices a,b,.. are omitted in Eq.(3) and hereafter).

Finally, $\rho^c(s_1,s_2)$ in Eq. (3) denotes the contribution into the double imaginary part of the amplitude A of the intermediate states sited above the q-flavor threshold. The total set of these states with quantum numbers J_1^{PC} (J_2^{PC}) including both higher quarkonium levels and continuum of states containing q-flavored hadron pairs, we call continuum and denote C_1 (C_2). Defined in this way, the double spectral function $\rho^c(s_1, s_2)$ includes amplitudes

of complicated and mainly unobservable transitions such as $C_2 \to R_1\gamma$, $C_1 \to R_2\gamma$ ($R_2\gamma$), $C_1 \to C_2\gamma$, $C_2 \to C_1\gamma$. The graphical representation of the r.h.s. of Eq. (3) is shown in Fig. 2.

Turn now to the amplitude $A^0(s_1,s_2)$ corresponding to the triangle diagram of Fig. 1.

In general, this amplitude may also be represented in a form of double dispersion integral *):

$$
A^0(s_1,s_2) = \frac{1}{\pi^2} \int \frac{d\tilde{s}_2}{s_2 - \tilde{s}_2} \int \frac{d\tilde{s}_1}{s_1 - \tilde{s}_1} \rho^0(\tilde{s}_1, \tilde{s}_2)
\tag{6}
$$

where the double spectral function $\rho^0(s_1,s_2)$ has the following general form :

$$
\rho^0(s_1,s_2) = 3eQ\,\delta(s_1 - s_2)\, f(s_1)
\tag{7}
$$

Here the function $f(s)$ is defined by $J^{PC}_{1,2}$ quantum numbers and the color factor 3 is explicitly written out.

The origin of representation (6) for the triangle diagram is easily understood if proceeding from the unitarity in the following way.

The imaginary part of the triangle diagram taken on one

*) For simplicity, in Eq.(3) as well as in Eq.(6) we do not take into account the possible subtractions. They may be easily restored in the final expressions (see the comment after Eq.(9)).

of the variables s_1 or s_2 corresponds to the two internal quark lines on the mass shell. Further, we perform the analytical continuation of this imaginary part on the other variable. The only singularity which emerges as a result of this continuation is the pole at $s_1 = s_2$ corresponding to the third internal quark line on the mass shell and to the δ-function in Eq.(7) . The absence of any anomalous threshold for this triangle diagram , as it can be easily verified , is guaranteed by the fact that one of its external masses is equal to zero viz. the photon mass ($k^2 = 0$).

The analytical properties stated above are independent of the $J^{PC}_{1,2}$ quantum numbers of j_1 , j_2 vertices.

To proceed, we equate the r.h.s. of dispersion representations (3) and (6) according to the approximation (2) in the whole region $-\infty < s_1 , s_2 \ll 4m^2$. The practical use of the resulting equation becomes possible if one can estimate the total contribution of continuum ($C_1 \oplus C_2$) given by the integral from $\rho^c(s_1,s_2)$ in Eq.(3).

Note that the equality of r.h.s. of Eqs.(3) and(6) leads to the asymptotic coincidence of $\rho^c(s_1,s_2)$ and $\rho^0(s_1,s_2)$ at $s_1,s_2 \to +\infty$ (far from the physical cuts).

Following Ref. [1] , we adopt the simplest hypothesis, based on this coincidence : the total contribution of the continuum can be approximated by the integral from the bare spectral function $\rho^0(s_1,s_2)$ (corresponding to the free quark intermediate states) taken over the $s_0 < s_1,s_2 < \infty$ region (the parton-like approximation of continuum) .

Due to this approximation, we are able to equate the resonance contribution in Eq.(3) with the integral from $\rho^0(s_1,s_2)$ taken over $4m^2 \leqslant s_{1,2} \leqslant s_0$ region. In other words, the resonance contribution is dual to the bare quark states below the q-flavor threshold.

Finally, we obtain :

$$\frac{g_1 g_2 F m_1^{c_1+d} m_2^{c_2}}{(s_1-m_1^2)(s_2-m_2^2)} + \frac{g_1 g_2' F' m_1^{c_1+d} m_2'^{c_2}}{(s_1-m_1^2)(s_2-m_2'^2)} = \frac{3}{\pi^2}\int_{4m^2}^{s_0} \frac{f(\tilde{s})\,d\tilde{s}}{(s_1-\tilde{s})(s_2-\tilde{s}_2)} \qquad (8)$$

The postulated regions of duality between quark and physical intermediate states are shown in Fig. 3 .

Expanding Eq.(8) in powers of s_1,s_2 at the points sited in the asymptotic freedom region $-\infty < s_1,s_2 \ll 4m^2$, we may obtain various sum rules , that express F and F' through the power moments of double spectral function ρ^0 (or f). To obtain a set of such sum rules as complete as possible , we shall expand Eq.(8) not only at $s_1 = 0$, $s_2 = 0$ (following Ref. [1]) but also at $s_1 = -\infty$, $s_2 = -\infty$ (generalizing the method of Ref. [2]).

Then the sum rules take the following final form :

$$F + \frac{g_2'}{g_2} F' \left(\frac{m_2}{m_2'}\right)^{\beta_2} =$$

$$= 3\,\frac{m_1^{\beta_1} m_2^{\beta_2}}{g_1 g_2 \pi^2}\int_{4m^2}^{s_0} \frac{f(\tilde{s})\,d\tilde{s}}{\tilde{s}^{\beta}} , \qquad (9)$$

where $\beta_1 = 2n + 2 - c_1 - d$; $\beta_2 = 2k + 2 - c_2$; $\beta = n + k + 2$. Here $n \geqslant 0$ ($k \geqslant 0$) and $n < 0$ ($k < 0$) correspond to the n-th (k-th) order of Eq.(8) expansion at $s_1 = 0$ ($s_2 = 0$) and $s_1 = -\infty$ ($s_2 = -\infty$), respectively.

Note, that if we had taken into account possible subtractions at $s_1 = 0$ ($s_2 = 0$) to improve the convergence of dispersion integrals, the final result (9) would not have changed. The only difference is as follows. In the case of n_o (k_o) subtractions on s_1 (s_2), the $n \geqslant n_o (k \geqslant k_o)$ and $n < n_o$ ($k < k_o$), moments would correspond to the n-th (k-th) order of the Eq.(8) expansion.

The modification of Eq.(9) for a more complicated spectrum of resonances sited below the treeshold is also obvious.

Suppose that in addition to R_1 , there exists another J_1^{PC} -state R_1' with mass m_1' . Then we should add to the l.h.s. of Eq.(9) two terms corresponding to the two possible transitions $R_1' \to R_2 \gamma$, $R_2' \to R_1' \gamma$ in which R_1' participates (for definiteness the direction of transitions is chosen as if $m_2 < m_1' < m_2'$). The l.h.s. of Eq. (9) becomes

$$F(R_1 \to R_2 \gamma) + \left(\frac{g_2'}{g_2}\right)\left(\frac{m_2}{m_2'}\right)^{\beta_2} F(R_2' \to R_1 \gamma) +$$

$$+\left(\frac{g_1'}{g_1}\right)\left(\frac{m_1}{m_1'}\right)^{\beta_1} F(R_1' \to R_2 \gamma) + \left(\frac{g_1' g_2'}{g_1 g_2}\right)\left(\frac{m_2}{m_2'}\right)^{\beta_2}\left(\frac{m_1}{m_1'}\right)^{\beta_1} F(R_2' \to R_1' \gamma) \qquad (9')$$

where $F(R_i \to R_j \gamma)$ denotes the amplitude (constant) of the

$R_i \to R_j \gamma$ transition. We use this expression in Sec.4 while considering M1 - transitions $\psi \leftrightarrow \eta_c$ in the charmonium.

The completeness of the set of sum rules (9) has the following meaning. Combining positive and negative n and k, we may adjust the relative weights of F and F$'$ in Eq. (9) (or $F(R_i \to R_j \gamma)$ in Eq.(9)) in order to make one of these weights dominant. Thus, in general we have a possibility to extract all the amplitudes of $J_1^{PC} \leftrightarrow J_2^{PC}$ transitions entering the resonance part of the sum rules.As it will become evident below,the situation is not so ideal, since the sum rules are, in fact, valid only at limited n and k . Nevertheless, a lot of information about transition amplitudes still may and will be extracted from Eq.(9).

It is appropriate to note here that the value of s_o in Eq.(9) may not coincide with the physical q-flavor threshold equal to $4M^2$ (where M is the mass of the lightest q-flavored hadron). Really, there are reasons to expect that at s_1 , $s_2 > 4M^2$ the $\rho^c(s_1 , s_2)$ double spectral function is still dominated by resonances that continue the spectra of radial excitations of the lowest levels R_1 and R_2 . Recall, e.g. that the cross section σ ($e^+e^- \to$ charm) that determines the imaginary part of the c-quark polarization operator, in the region $2M_D < \sqrt{s_o} \lesssim 4{,}5$ GeV is saturated by $\psi''(3772)$, $\psi'''(4028)$ and higher resonances. If similar situation takes place for the triangle amplitude A , we may remove the contributions of these above-threshold resonances from the $\rho^c(s_1,s_2)$ and incorporate them into the resonance part of dispersion representation (3) and correspondingly into the l.h.s. of Eq. (9). In this case $\sqrt{s_0}$ is to be larger than the maximum resonance mass. So the choice of "true" continuum region is somewhat uncertain. The natural way to resolve this uncertainty is to compare the sum rules with the experiment (see e.g. the next section). Note, that a sort of "local" duality may occur here, if the resonances in a certain region $s_0 < s_{1,2} < s_0'$ are dual to the quark-parton states.

Turn now to the derivation of Eq. (9) for the most interesting types of radiative transitions between C-even quarkonium levels with $J_1^{PC} = 0^{++}, 1^{++}, 2^{++}, 0^{-+}$ and vector levels with $J_2^{PC} = 1^{--}$. All these transitions are observable in e^+e^- -annihilation.

According to the general outline described above, the problem consists in the calculation of double spectral functions $\rho^0(s_1, s_2)$ of relevant triangle diagrams.

At various J_1^{PC} quantum numbers (various j_1 currents) the amplitude (1) in terms of invariant amplitudes has the following form :

a) $J_1^{PC} = 0^{++}$; $j_1 = \bar{q}q$,

$$A_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{k_\nu p_{2\mu}}{(kp_2)}\right) A(s_1, s_2) ; \qquad (10a)$$

b) $J_1^{PC} = 1^{++}$; $j_1 = \eta_{\lambda\rho} \bar{q} \gamma_\rho \gamma_5 q$,

$$A_{\mu\nu\lambda} = \frac{k_\alpha \varepsilon^{\alpha\mu\nu\rho}}{(kp_2)} A_a(s_1, s_2) +$$
$$+ \left(p_{2\alpha} \varepsilon^{\alpha\mu\nu\rho} - \frac{p_{2\mu} p_{2\alpha} k_\beta \varepsilon^{\alpha\beta\nu\rho}}{(kp_2)}\right) A_b(s_1, s_2) ; \qquad (10b)$$

c) $J_1^{PC} = 2^{++}$, $j_1 = i\bar{q}(\gamma_\lambda \overleftrightarrow{\partial}_\rho + \gamma_\rho \overleftrightarrow{\partial}_\lambda - \frac{2}{3} \eta_{\lambda\rho} \overleftrightarrow{\hat{\partial}})q$,

$$e_\mu A_{\mu\nu\lambda\rho} = \frac{2\delta_{\nu\lambda} F_{\rho\alpha} p_{2\alpha}}{(kp_2)} A_a(s_1, s_2) +$$
$$+ \frac{2F_{\nu\lambda} p_{2\rho}}{(kp_2)^2} A_b(s_1, s_2) +$$
$$+ \frac{2F_{\nu\alpha} p_{2\alpha} k_\lambda p_{2\rho}}{(kp_2)^3} A_c(s_1, s_2) ; \qquad (10c)$$

d) $J_1^{PC} = 0^{-+}$; $j_1 = i\bar{q}\gamma_5 q$,

$$A_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta} k_\alpha p_{1\beta} A(s_1, s_2) \qquad (10d)$$

where $\eta_{\lambda\rho} = \delta_{\lambda\rho} - (p_{1\lambda} p_{1\rho}/s_1)$; $F_{\rho\alpha} = e_\rho k_\alpha - e_\alpha k_\rho$ - is the e.m.field tensor (e_μ is the photon polarization vector) According to their definition (5), the matrix elements of $J_1^{PC} \leftrightarrow 1^{--}$ transitions have the same form (10) where instead of $A_{a,b...}$ invariant amplitudes there are corresponding $F_{a,b...}$ invariant constants and the kinematical structures are multiplied by the wave functions of photon and 1^{--}, J_1^{PC} -resonances with μ and $\nu, \rho, \lambda, ...$ Lorentz indices,respectively.Since we are interested in these matrix elements only , we have left in Eq. (10) only those independent kinematical structures which survive after being multiplied by the transverse wave functions. In Eq,(10c) we have also taken into account the symmetry on $\lambda\rho$ (reflected by coefficient 2) and the irreducibility (the vanishing trace) of the 2^{++}-resonance wave function .

The results of calculation of double spectral functions (7) corresponding to the invariant amplitudes (10) are presented in Table 1 as well as the dimensions d and c_1 of matrix elements (4) and (5). Together with the dimension of "vacuum" matrix element (4) of the vector (1^{--}) current $c_2=2$, the contents of the Table 1 entirely determine the sum rules (9) for all types of transitions under consideration.

Before turning to the physical applications, it is necessary to determine the intervals of n,k variation

$$-n_2 \leq n \leq +n_1 \quad ; \quad -k_2 \leq k \leq +k_1 \tag{11}$$

for which our sum rules are expected to hold.

If the sum rules (9) were valid at arbitrary large n, $k > 0$, it would mean that the physical and the bare quark amplitudes coincide with each other in the whole region s_1, $s_2 > 0$. Such coincidence is in fact impossible since at $s_1, s_2 \to 4m^2$ the asymptotic freedom is violated and the zeroth order of QCD is inapplicable (for details see Ref. [1]). Hence, the sum rules are reliable up to certain maximum moments n_1, k_1.

On the other hand, the parton approximation of continuum used to obtain Eq.(9) is actually based on the assumption that

$$\rho^c(s_1,s_2)-\rho^o(s_1,s_2) \sim O(s_1^{-n_2}\, s_2^{-k_2})$$

at $s_1, s_2 \to +\infty$, where n_2, k_2 are some finite numbers. Therefore, the sum rules would be violated also at $n < -n_2$, $k < -k_2$.

To choose proper upper boundaries in Eq.(11) we refer to the well-studied sum rules obtained in Ref. [1] for the c-quark polarization operator by expanding one-dimensional dispersion relation at s=0. The comparison with experiment shows that these sum rules are valid up to the 4-th moment. For higher moments the agreement is restored if one takes into account, as in Ref. [7] , the so called power gluon corrections sharply growing with increasing the number of moment. These corrections are not reproduced by perturbation theory in α_s and reflect the essential violation of asymptotic freedom at $s \to 4m^2$ (connected with the confinement). For lower moments the power corrections in the charmonium [7] as well as in the b-quarkonium [8] are negligible. With this in mind, we choose in Eq.(11) n_2, $k_2 = 3$. Moreover, such a limitation allows to neglect the Coulomb type gluon corrections for $b\bar{b}$ and heavier quarkonia [8] .

The "negative" moments of sum rules for the c-quark polarization operator have been derived in Ref. [2] by means of expansion at $s = -\infty$. It have been found out that these "reversed" sum rules fulfil fairly well up to the 20-th moment, if J/ψ, ψ', and $\psi''(3770)$ contributions are taken into account. This agreement indicates an almost ideal duality between the continuum and the c-quark states in the 1^{--}-channel of charmonium. We make use of this fact as a motivation to apply the sum rules (9) at $n,k < 0$ choosing in Eq.(11) n_1, $k_1 = -5$ with some reserve. (Note, however, that, strictly speaking, the thoretical status of negative moments is less clear so far than that of positive ones.)

Thus, we have chosen a certain set of "optimum" moments of sum rules. We expect that the accuracy of these moments is mainly determined by the usual $O(\alpha_s)$ perturbation theory corrections given by gluon exchanges at small distances in the triangle diagram. The calculation of these corrections is a rather complicated problem. Nevertheless, we expect that they do not exceed $\sim 20\%$ which is the order of $O(\alpha_s)$ correction to the polarization operator (see Ref. [1]). The numerical estimates of transition amplitudes given below are expected to be at the same level of accuracy.

3. The electric dipole transitions

Here and in the next section we shall try to realize whether the obtained sum rules give us an actual possibility to calculate the widths of radiative transitions and whether the results of these calculations agree with available experimental data.

Up to now the most complete experimental information has been accumulated concerning

$$\chi_J \to J/\psi\,\gamma \quad , \quad \psi' \to \chi_J \gamma \qquad (12)$$

radiative transitions in the charmonium.

The χ_J-levels (J=0,1,2) are $\chi_0(3415)$, $\chi_1(3510)$, $\chi_2(3550)$ with $J^{PC}=0^{++},1^{++},2^{++}$. They are intermediate between $J/\psi(3095)$ and $\psi'(3685)$. Following the nonrelativistic terminology, we call the transitions (12) electric dipole (E1) ones.

Evidently, the E1 transitions in the b-quarkonium

$$\Upsilon' \to \chi_J^b \gamma \;,\; \chi_J^b \to \Upsilon\gamma \;,\; \Upsilon'' \to \chi_J^b \gamma \qquad \text{etc.}$$

sooner or later will also be discovered. (χ_J^b are the b-quark analogs of χ_J-levels).

Returning to the charmonium, we note that the arrangement of χ and ψ levels coincide with the one considered above in general case ($R_1=\chi_J$, $R_2=J/\psi$, $R_2'=\psi'$).

Substituting into Eq.(9) for each value of J the corresponding functions $f(s)$ and dimensions d, c_1 from Table 1 (recall that $c_2=2$) , we obtain the sum rules for transition constants $F^{(J)}$ and $F'^{(J)}$. Recall that the a,b,... indices count the invariant amplitudes whereas J=0,1,2 denotes the spin of χ_J in the transitions (12). When these details are inessential, the indices are omitted.

What are the numerical values of parameters entering Eq.(9) ?

The masses $m_1=m_{\chi_J}$, $m_2=m_\psi$, $m_2'=m_{\psi'}$ are written down above (in MeV) at the symbols of resonances. As for the c-quark mass value , we shall use $m=m_c=1{,}25 \div 1{,}30$ GeV evaluated in Ref [1] . Recall that this value has the meaning of the current quark mass or the mass at small distances. According to the discussion presented above, the value of $\sqrt{s_0}$ is to be chosen in the following interval

$$m_{\psi'} = 3{,}685 \text{ GeV} < \sqrt{s_0} < m_{\psi''} = 3{,}770 \text{ GeV}.$$

Let us fix an initial value of $\sqrt{s_0} = 3{,}70$ GeV and then try to trace the variation of results with increasing this parameter. The values of $g_2 = g_\psi$ and $g_2 = g_{\psi'}$ defining the matrix elements (4) have been calculated in Ref. [1] by means of sum rules for the c-quark polarization operator. These values turn out to agree fairly well with the experimental ones

$$|g_\psi| = 0{,}125 \quad , \quad |g_{\psi'}| = 0{,}0755$$

that may be extracted from J/ψ and ψ' leptonic widths $\Gamma^{ee}_{\psi} =$ 4,8 KeV, $\Gamma^{ee}_{\psi'} = 2{,}1$ keV (we take central exp. values)[9] according to

$$\Gamma^{ee}_{\psi} = \frac{4\pi\alpha^2 Q_c^2}{3} g_\psi^2 m_\psi , \qquad (13)$$

$$(Q_c = 2/3)$$

Recall that, just the succesful calculation of g_ψ has stimulated the development of dispersion approach in various directions.

The analogous calculation of $g_J = g\,\chi_J$ gives [1] :

$$|g_{\chi_0}| = 0{,}105, \quad |g_{\chi_1}| = 0{,}095 \ , \ |g_{\chi_2}| = 0{,}13 \qquad (14)$$

The g_χ constants have no direct physical sense. However in nonrelativistic approach the g_{χ_0} and g_{χ_2} may be extracted from $\chi_{0,2} \to 2\gamma$ widths. After substitution of numerical values of parameters, corresponding to the given spin J , each moment (n,k) of sum rules turns into an independent linear equation for dimensionless unknowns F^J and F'^J.

To avoid the possible influence of the J^{++} states cited above the χ_J , we confine ourselves to the positive n's : $1 \leq n \leq 3$. At the same time, according to the option made above, we take $-5 \leq k \leq 3$. The negative moments ($k < 0$) give us information about the F' constant, while at $k > 0$

the F contribution is suppressed by $(m_\psi / m_{\psi'})^{2k}$ weight.

Finally, we obtain a highly overdetermined system of 24 linear equations (9) for the two unknowns . Since there are no reasons to prefer some of these equations to others, we have to find an approximate solution for F and F' equally satisfying all the equations. We suggest the following procedure. Let us determine the unknowns F and F' from the minimization condition of the sum of squared differences between l.h.s. and r.h.s. of Eq.(9) denoted by $L_k(F', F)$ and $R_{n,k}(m, s_0)$ respectively) taken over all the selected moments:

$$\sum_{\substack{1 \leq n \leq 3 \\ -5 \leq k \leq 3}} \left(\frac{L_k - R_{n,k}}{R_{n,k}} \right)^2 = \min \qquad (15)$$

In this way we obtain definite values of F and F' satisfying Eq.(15). At $\sqrt{s_0} = 3{,}70$ GeV and $m_c = 1{,}25$ GeV (1,30 GeV) these values are:

$$\begin{aligned}
F^{(0)} &= 0{,}48\ (0{,}33), & F'^{(0)} &= 0{,}17(0{,}18), \\
F_a^{(1)} &= 0{,}59\ (0{,}41), & F_a'^{(1)} &= 0{,}35(0{,}31), \\
F_a^{(2)} &= 0{,}39\ (0{,}27), & F_a'^{(2)} &= 0{,}20(0{,}19), \\
F_b^{(2)} &= 0{,}006(0{,}003), & F_b'^{(2)} &= 0{,}010(0{,}005).
\end{aligned} \qquad (16)$$

Besides this,

$$F_b^{(1)} = F_b'^{(1)} = 0 \ ; \ F_c^{(2)} = -(3/2)F_b^{(2)} ; F_c'^{(2)} = -(3/2)F_b'^{(2)} \quad (16a)$$

Eqs.(16a) follow immediately from Eq.(9) with the corresponding $f(s)$ from Table 1 in r.h.s.

Note that the relative sign of F and F' turns out to be positive everywhere*). Our method of approximate solution actually leads to reasonable results. Indeed, if we substitute the values (16) back to Eq.(9), all "optimum" moments of sum rules are then satisfied with the expected accuracy of the QCD zeroth order. Besides that, the numerical analysis shows that the minimization procedure is stable against small variation of parameters entering Eq.(9).

Having the values of $F^{(J)}$ and $F'^{(J)}$, it is easy to calculate the widths of radiative transitions. To obtain the expressions for these widths we must square the matrix elements (5) (substituting the corresponding kinematical structures $I_{\mu\lambda 2}$ from Eqs.(10)) and take into account the phase space volume.

In this way, we obtain :

$$\Gamma(\chi_J \to J/\psi\gamma) = \frac{\alpha Q_c^2 m_{\chi_J}}{2(2J+1)} \left| M_J(m_{\chi_J}, m_\psi, F^{(J)}_{a,b,..}) \right|^2 \left(1 - \frac{m_\psi^2}{m_{\chi_J}^2}\right),$$

$$\Gamma(\psi' \to \chi_J\gamma) = \frac{\alpha Q_c^2}{6} \frac{m_{\chi_J}^2}{m_{\psi'}} \left| M_J(m_{\chi_J}, m_{\psi'}, F'^{(J)}_{a,b,..}) \right|^2 \left(1 - \frac{m_{\chi_J}^2}{m_{\psi'}^2}\right), \qquad (17)$$

where $|M_0(m_{\chi_0}, m_\psi, F^{(0)})|^2 = |F^{(0)}|^2,$

$$|M_1(m_{\chi_1}, m_\psi, F_a^{(1)})|^2 = \left(1 + \frac{m_{\chi_1}^2}{m_\psi^2}\right) |F_a^{(1)}|^2,$$

*) Note that since we substitute the absolute values of constants g, this sign is in fact the sign of g_2F and $g_2'F'$ products. (This will be important for us in Sec. 5.)

$$|M_2(m_{\chi_2}, m_\psi, F^{(2)}_{a,b})|^2 = 2\left\{ \left[\frac{(m_{\chi_2}^2 - m_\psi^2)^2}{4 m_{\chi_2} m_\psi} + \frac{10}{3} \right] |F_a^{(2)}|^2 + \left[\frac{1}{3} + \frac{m_{\chi_2}^2}{m_\psi^2} \right] \left(|F_b^{(2)}|^2 - F_a^{(2)} F_b^{(2)} \right) \right\}.$$

where we have taken into account Eq.(16a).

The widths calculated by means of Eqs.(15) and (17) are given in Table 2 where they are compared with available experimental data [4,9,10] .Strictly speaking, the only measured widths are $\Gamma(\psi' \to \chi_J\gamma) = BR(\psi' \to \chi_J\gamma)\,\Gamma^{\psi'}_{tot}$ whereas $\Gamma(\chi_J \to J/\psi\gamma)$ presented in Table 2 are not free of ambiguities :

$$\Gamma(\chi_J \to J/\psi\gamma) = BR(\psi' \to \chi_J\gamma \to \gamma\gamma J/\psi) \frac{\Gamma^{\chi_J}_{tot}}{BR(\psi' \to \chi_J\gamma)};$$

where for $\Gamma_{tot}(\chi_J)$ we have taken the whole interval of theoretical predictions from the review paper [1] .

The agreement is rather good within the theoretical uncertainty of the QCD zeroth order ($\sim 20\ \%$ in amplitudes i.e. $\sim 40\%$ in widths) and within the experimental errors. It is worth noting that the accuracy of experiment is not high so far. E.g., even the $\Gamma_{tot}(\psi')$ has an error of $\sim 25\%$.

We postpone a more detailed discussion of comparison with experiment until more accurate data appear and the $O(\alpha_s)$ corrections are calculated. In any case, the qualitative agreement is evident indicating that we are in the right way.

It is worth discussing, however, the sensitivity of our predictions to the choice of parameters of Eq.(9).

Note, first, the dependence of $\chi_J \to J/\psi\gamma$ constants $F^{(J)}$ on the c-quark mass. The best agreement with the experiment (with the provisos made above) seems to be at $m_c = 1{,}30$ GeV. May be this last circumstance reflects a somewhat effective renormalization (due to gluon corrections) of the original value m_c = 1,25 GeV for which the best agreement is achieved for the one-dimensional sum rules [1] (with the gluon corrections) .

The values of F and F$'$ are also influenced by g_1, g_2, g_2 variations. Recall that we have used in our sum rules (9) those g values obtained by the dispersion method [1] . The agreement of (9) with experiment means , to our mind, a nontrivial accordance between various types of dispersion sum rules for both annihilation and radiative transitions.

Finally, we should like to draw attention to the substantial sensitivity of the $\psi' \to \chi_J \gamma$ constants $F'^{(J)}$ to the value of threshold energy $\sqrt{s_0}$. If we increase $\sqrt{s_0}$ up to 3,77 GeV and "forget" to take into account the ψ'' contribution in the l.h.s. of Eq.(9) , then the fitted $F'^{(J)}$ values for all J's become approximately twice the values presented in Eq.(16). This already results in sharp contradiction of predicted widths $\Gamma(\psi' \to \chi_J \gamma)$ with experiment. At the same time the $F^{(J)}$ values are not influenced by $\sqrt{s_0}$ variation since this variation manifests itself only in the $k < 0$ moments where $F'^{(J)}$ dominate . If we take into account the $\psi''(3770)$ resonance i.e. add to the l.h.s. of Eq.(9) the third term $g''_2/g_2 \left(m''_\psi/m_\psi\right)^{\beta_2} F''$ (where g_2 = 0,027 is determined from $\Gamma^{ee}_{\psi''} \cong 0,3$ keV according to Eq.(13) and $F''^{(J)}$ is the $\psi'' \to \chi_J \gamma$ transition constant), then at $m_{\psi''} < \sqrt{s_0} \lesssim m_{\psi'''}$ = 4,03 GeV $F'^{(J)}$ values are restored close to those of Eq.(16) and simultaneously $F''^{(J)} \sim F'^{(J)}$. Therefore we conclude that the contributions of 1^{--}-states heavier than ψ' are essential in the sum rules (9) at $k < 0$ (c.f. Ref. [2]

where the ψ'' contribution was important). Moreover, we do not exclude the existence of local duality between above-threshold ψ -resonances and quark states in comparatively narrow intervals of s_2.

Turn now to E1 transitions in the b-quarkonium. Here the spectrum of 1^{--}-states below threshold consists of three levels Υ(9460), Υ'(10015), and Υ''(10400) (see , e.g. Ref. [11]). Two groups of intermediate χ^b_J-levels are also expected : $m_\Upsilon < m_{\chi^b} < m_{\Upsilon'} < m_{\chi^{b'}} < m_{\Upsilon''}$ (J = 0,1,2). A crude estimate gives $m_{\chi^b} - m_\Upsilon \cong m_{\chi^{b'}} - m_{\Upsilon'} \cong m_\chi - m_\psi \cong 0,3$ GeV whereas more elaborated predictions of nonrelativistic models are : m_{χ_b} = 9,9 GeV, $m_{\chi'_b}$=10,3 GeV (see , e.g. Refs. [12,13]). Recall that the spin-orbit splitting in the b-quarkonium is expected to be negligible :

$$m_{\chi^b_{J+1}} - m_{\chi^b_J} \sim \left(\frac{m_c}{m_b}\right)^2 \left(m_{\chi_{J+1}} - m_{\chi_J}\right) \cong 10 \text{ MeV } (J=0,1)$$

In such system of levels , for each J there are six $\Upsilon \leftrightarrow \chi^b$ E1 transitions. Consequently, the l.h.s. of sum rules (9) contain now six terms instead of two, i.e. there are six transition constants F,F$'$,... . In this case the minimization procedure suggested above leads to rather uncertain values of these constants since (within our limited intervals of n,k) their weights in the l.h.s. of Eq.(9) are too correlated (the variations of the ratios of these weights with n,k are insufficient) to extract one or another F distinctly. We can only estimate the order of value of the lowest $\chi^b_J \to \Upsilon\gamma$ transition constants $F^{(J)}$. To do this we take the n,k = 2,3 moments of sum rules (9) and leave only F in the l.h.s. of Eq.(9) using the fact that the weights of other constants are small due to $g'/g \ll 1$ factors). Then we substitute into Eq.(9) $m_2 = m_\Upsilon = 9{,}46$ GeV, $m = m_b = 4{,}65$ GeV (the b-quark mass at small distances obtained in Ref. [8] by dispersion method) taking $\sqrt{s_0} = 11{,}0$ GeV (b-flavor threshold) and $g_2 = g_\Upsilon = 0{,}07$;*) $g_1 = g_{\chi^b} \cong g_\Upsilon$. The resulting $F^{(J)}$ values are:

$$F^{(0)} \cong F_a^{(1)} \cong F_a^{(2)} \cong 0{,}05 \div 0{,}1\ ; \quad F_b^{(2)} \ll F_a^{(2)} \qquad (18)$$

By means of Eq,(17) we obtain ($Q_b = -1/3$) :

$$\Gamma(\chi_0^b \to \Upsilon\gamma) = 1{,}0 \div 3{,}5 \text{ keV}; \quad \Gamma(\chi_1^b \to \Upsilon\gamma) = 0{,}5 \div 2{,}0 \text{ kev};$$
$$\Gamma(\chi_2^b \to \Upsilon\gamma) = 1{,}0 \div 4{,}0 \text{ keV}.$$

4. Magnetic dipole transitions.

Since the discovery of the J/ψ resonance, the problem of pseudoscalar (0^{-+}) $c\bar{c}$ - levels η_c and η_c' have been the "stumbling-block" of the charmonium spectroscopy.

In e^+e^--annihilation, the promising sources of η_c-resonances are $\psi \leftrightarrow \eta_c$ magnetic dipole (M1) transitions that we are going to consider here.

Just recently, after unsuccessful "finds" such as X(2,83), the situation around M1 transitions have been considerably clarified. A real candidate for η_c with mass $m_{\eta_c} = 2{,}98 \pm 0{,}02$ GeV have been discovered [4] at SLAC. There are also some preliminary results on the M1 transitions :

$$BR\,(\psi' \to \eta_c\gamma) = 0{,}2 \div 0{,}5\ \% \qquad (19a)$$

*) from $\Gamma_\Upsilon^{ee} = 1{,}28$ keV [11]

and still an upper bound [14]:

$$BR\,(J/\psi \to \eta_c\gamma\ ) < 3\cdot 10^{-5} \qquad (19b)$$
$$\qquad \hookrightarrow 2\gamma$$

Recall that the experimental value of m_{η_c} fit well not only the nonrelativistic model of "hyperfine" $J/\psi - \eta_c$ splitting but also the much stringent limits [15]

$$2{,}98 \text{ GeV} < m_{\eta_c} < 3{,}02 \text{ GeV}$$

that follows from dispersion sum rules for the $\langle 0|\, j_5\, j_5 \,|0\rangle$ vacuum average (including the main gluon corrections) where $j_5 = i\bar{c}\gamma_5 c$ is the c-quark 0^- current . As a byproduct, these sum rules [15,5] give also the value of "vacuum" constant

$$|g_1| = |g_{\eta_c}| = 0{,}12 \qquad (20)$$

defining the matrix element (4) for $j_1 = j_5$ current.

The second 0^{-+}-charmonium level η_c' is expected to be near its vector analog ψ'. Thus, for example, the nonrelativistic calculations give (see , e.g. Ref. [16])

$$m_{\psi'} - m_{\eta_c'} \simeq \frac{m_{\psi'}^2}{m_\psi^2}\left(\Gamma_{\psi'}^{ee}/\Gamma_\psi^{ee}\right)(m_\psi - m_{\eta_c}) = 73 \pm 19 \text{ MeV}, \qquad (21a)$$

and $\quad g_{\eta_c}'/g_{\eta_c} = g_{\psi'}/g_\psi = 0{,}604 \qquad (21b)$

(Recall that in terms of nonrelativistic approach $g \sim \psi(0)$)

These estimates satisfy the lowest moments of sum rules[15] in which the η_c' contribution is not suppressed. Therefore, we shall use (21) along with (20) below.

Two levels $R_1 \equiv \eta_c$ and $R_1' \equiv \eta_c'$ (with $J_1^{PC} = 0^{-+}$) are related to $R_2 \equiv J/\psi$, $R_2' \equiv \psi'$ (with $J_2^{PC} = 1^{--}$) by four M1 transitions. Their matrix elements are defined by four invariant constants $F\,(R_i \to R_j\gamma)$. The sum rules relating these constants have the general form (9) with the l.h.s. given by Eq. (9') and

and $f^{(0^-)}$ from Table 1 in the r.h.s. The final answer is :

$$F(J/\psi\to\eta_c\gamma)+\left(\frac{g_{\eta_c}}{g'_{\eta_c}}\right)\left(\frac{m_{\eta_c}}{m_{\eta'_c}}\right)^{2n+1}F(\eta'_c\to J/\psi\gamma)+$$
$$+\left(\frac{g_{\psi'}}{g_\psi}\right)\left(\frac{m_\psi}{m_{\psi'}}\right)^{2K}\left\{F(\psi'\to\eta_c\gamma)+\left(\frac{g'_{\eta_c}}{g_{\eta_c}}\right)\left(\frac{m_{\eta_c}}{m_{\eta'_c}}\right)^{2n+1}F(\psi'\to\eta'_c\gamma)\right\}=$$
$$=\frac{3}{2\pi^2 g_\psi g_{\eta_c}}\left(\frac{m_\psi}{2m_c}\right)^{2K}\left(\frac{m_{\eta_c}}{2m_c}\right)^{2n+1}\int_0^{V_0} v\,dv\,(1-v^2)^{n+K}\ln\frac{1+v}{1-v} \qquad (22)$$
$$\left(V_0=(1-4m_c^2/s_0)^{1/2}\right)$$

As it was explained above, we expect that Eq.(22) holds with $\sim 20\ \%$ accuracy in the interval $-5\lesssim n,k,\lesssim 3$.

The numerical fit of the four constants F over the resulting set of linear equations (22) gives the following values for the constants of "diagonal" transitions $J/\psi\to\eta_c\gamma$ and $\psi'\to\eta'_c\gamma$

$$F(J/\psi\to\eta_c\gamma)=3{,}6\ (2{,}5)$$
$$F(\psi'\to\eta'_c\gamma)=2{,}3\ (1{,}9) \qquad (23a)$$

at m_c= 1,25 GeV (1,30 GeV) , $\sqrt{s_0}$ = 3,70 GeV .

As for the "nondiagonal" transitions $\psi'\to\eta_c\gamma$ and $\eta'_c\to J/\psi\gamma$ the fitting procedure leads only to the definite upper limit

$$|F(\psi'\to\eta_c\gamma)|\sim|F(\eta'_c\to J/\psi\gamma)|\lesssim 0{,}5 \qquad (23b)$$

The actual values of nondiagonal constants are not fitted since even their maximal weights in Eq.(22) (at $n=-5$, $k=3$ or at $n=3$, $k=-5$) are not large enough to extract these constants distinctly. So the total balance between l.h.s. and r.h.s. of Eq. (22) is mainly determined by the values of diagonal constants.

According to the definition of transition matrix element (5) with corresponding kinematical structure (10d) the diagonal

M 1 widths are :

$$\Gamma(V\to P\gamma)=\frac{\alpha Q_c^2}{24}\left|F(V\to P\gamma)\right|^2\frac{m_V^3}{m_P}\left(1-\frac{m_P^2}{m_V^2}\right)^3 \qquad (24)$$

where $V(P)$ is J/ψ or ψ' (η_c or η'_c).

Substituting the values (23) into Eq.(24) , we have

$$\Gamma(J/\psi\to\eta_c\gamma)=2{,}45\,\text{keV}\,(1{,}2\ \text{keV})\times\left(\frac{m_\psi-m_{\eta_c}}{118\ \text{MeV}}\right)^3$$
$$\Gamma(\psi'\to\eta'_c\gamma)=0{,}2\ \text{keV}\,(0{,}15\ \text{keV})\times\left(\frac{m_{\psi'}-m_{\eta'_c}}{70\ \text{MeV}}\right)^3 \qquad (25)$$

If we adopt $BR(\eta_c\to2\gamma)$ =1/845 (the familiar QCD estimate [17]) then $\Gamma^{exp}(J/\psi\to\eta_c\gamma)<1{,}7$ keV. The obtained prediction for $\Gamma(J/\psi\to\eta_c\gamma)$ is compatible with this bound (within $\sim 40\%$ uncertainty of our result and within $O(\alpha_s)$ uncertainty of $BR(\eta_c\to2\gamma)$)

The nondiagonal transition widths due to our result (23b) are limited :

$$\Gamma(\eta'_c\to J/\psi\gamma)=\frac{\alpha Q_c^2}{8}m_{\eta'_c}\left(1-\frac{m_\psi^2}{m_{\eta_c}'^2}\right)^3\left|F(\eta'_c\to J/\psi\gamma)\right|^2<7\ \text{keV}$$
$$\Gamma(\psi'\to\eta_c\gamma)=\frac{\alpha Q_c^2}{24}m_{\psi'}\left(\frac{m_{\psi'}}{m_{\eta_c}}\right)\left(1-\frac{m_{\eta_c}^2}{m_{\psi'}^2}\right)^3\left|F(\psi'\to\eta_c\gamma)\right|^2<8\ \text{keV}$$

The last limit is too high as compared with the experimental value (19a). Therefore, we conclude that our sum rules are not very informative for the nondiagonal transitions.

The $\Gamma(J/\psi\to\eta_c\gamma)$ value close to our result (25) have been obtained recently in Ref. [5] by means of simple VDM type relation:

$$\Gamma(J/\psi\to\eta_c\gamma)=\frac{2}{9}\alpha\frac{m_\psi^4}{m_{\eta_c}^3}\frac{\Gamma(\eta_c\to2\gamma)}{\Gamma(J/\psi\to ee)}\left(1-\frac{m_{\eta_c}^2}{m_\psi^2}\right)^3\left(1+O(\alpha_s)\right). \qquad (26)$$

There is nothing surprising in such coincidence since (26), in fact, may be derived from our sum rules (22) in a particular case. Indeed, at k=0 the r.h.s. of Eq.(22) coincides with the one-dimensional dispersion integral with n subtractions

$$\frac{3eQ_c}{2\pi^2(2m)^{2n+1}}\int_0^{v_0} v\,dv(1-v^2)^n \ln\frac{1+v}{1-v}=\frac{1}{\pi}\int_{4m_c^2}^{s_0}\frac{Im_{s_1}\overset{o}{A}(s_1,0)}{s_1^{n+1}}ds_1 \qquad (27)$$

corresponding to the bare triangle diagram with two photon vertices ($k^2 = 0$, $s_2 = 0$) and 0^{-+} vertex. This integral at $s_1 = 0$ can be expressed through the resonance contributions (in zeroth order in α_s and in the quark - continuum duality limit at $s_1 > s_0$). As a result , the sum rules with various n emerge for $\eta_c \to 2\gamma$, $\eta_c' \to 2\gamma$ amplitudes. These sum rules, among others have been obtained in Ref. [1] (see,e.g. Eq.(5.34) from the review paper [1]). Substituting these resonance terms into the r.h.s. of Eq.(22) instead of (27) at k = 0 , we obtain

$$g_{\eta_c} g_\psi F(J/\psi\to\eta_c\gamma) + g_{\eta_c'} g_\psi \left(\frac{m_{\eta_c}}{m_{\eta_c'}}\right)^{2n+1} F(\eta_c'\to J/\psi\gamma)$$
$$+ g_{\psi'} g_{\eta_c} F(\psi'\to\eta_c\gamma) + g_{\eta_c'} g_{\psi'} \left(\frac{m_{\eta_c}}{m_{\eta_c'}}\right)^{2n+1} F(\psi'\to\eta_c'\gamma) =$$
$$= g_{\eta_c} A(\eta_c\to 2\gamma) + \left(\frac{m_{\eta_c}}{m_{\eta_c'}}\right)^{2n+1} g_{\eta_c'} A(\eta_c'\to 2\gamma) \qquad (28)$$

where $A(\eta_c(\eta_c')\to 2\gamma)$ defines the matrix element

$$M(\eta_c(\eta_c')\to 2\gamma) = e^2 Q_c^2\, \varepsilon_{\mu\nu\alpha\beta}\, e_\mu^1 e_\nu^2 k_\alpha^1 k_\beta^2 A(\eta_c(\eta_c')\to 2\gamma)\, m^{-1}_{\eta_c(\eta_c')}$$

It is naturally to split Eq.(28) into η_c and η_c' parts .Then, neglecting $F(\psi'\to\eta_c\gamma)$ as in Ref. [5] we have

$$A(\eta_c\to 2\gamma) = g_\psi F(J/\psi\to\eta_c\gamma)$$

This relation ,being squared and written in terms of widths, immediately transforms into Eq.(26). The $O(\alpha_s)$ corrections in Eq. (26) correspond to the analogous corrections to the r,h.s. of Eq.(22).

Finally, the sum rules (22) written for the b-quarkonium allow to estimate the amplitude of the lowest $\Upsilon\to\eta^b\gamma$ M1 transition (η^b is the b-quark analog of the η_c). Leaving in the r.h.s. of Eq.(22) only $F(\Upsilon\to\eta^b\gamma)$ we obtain at $n,k = 2,3$

$$F(\Upsilon\to\eta_b\gamma) \sim 1.2\div 2.0$$
$$\Gamma(\Upsilon\to\eta_b\gamma) \sim (5\div 10)\cdot 10^2\left(1-m_{\eta_b}^2/m_\Upsilon^2\right)^3 \text{keV}$$

at $m_{\eta^b} = m_\Upsilon$, $g_{\eta^b} = g_\Upsilon$, m_b = 4,65 GeV, $\sqrt{s_0}$ = 11,0 GeV.

5. The comparison with nonrelativistic models

The dispersion approach developed above represents a model-independent and purely relativistic treatment of radiative transitions.

On the other hand, there exists a traditional way of describing these processes based on the nonrelativistic models of quarkonium quantum mechanics. In these models the radiative transitions are viewed as simple dipole transitions in a certain (model-dependent) interquark potential with confinement. Here we present a few brief comments concerning the comparison of our approach with the nonrelativistic one.

First, let us establish relations between invariant matrix elements of radiative transitions and the corresponding nonrelativistic amplitudes . For definiteness, we consider $\chi_J \to J/\psi\,\gamma$ and $J/\psi \to \eta_c\gamma$ transitions. The results for other transitions are similar.

In the rest system of χ_J (or η_c) the expressions (5) written for the $\chi_J \to J/\psi\,\gamma$ (or $J/\psi\to\eta_c\gamma$) with kinematical structures taken from Eq.(10) take the following form in the nonrelativistic limit $\frac{\omega}{m_\psi} \to 0$ (where $\omega \cong m_{\chi_J} - m_\psi$ (or $m_{\psi} - m_{\eta_c}$) is the photon energy)

$$e_\mu \langle \chi_0 | j_\mu^{em} | J/\psi \rangle = \vec{\psi}\cdot\vec{e}\,\chi_0 F^{(0)} m_\psi , \tag{29a}$$

$$e_\mu \langle \chi_1 | j_\mu^{em} | J/\psi \rangle = \left\{ \vec{e}\cdot(\vec{\psi}\times\vec{\chi}) - \frac{\omega}{m_\psi}(\vec{n}\cdot\vec{\psi})\,\vec{n}\cdot(\vec{e}\times\vec{\chi}) \right\} F_a^{(1)} m_\psi$$
$$+ \left\{ \vec{e}\cdot(\vec{\psi}\times\vec{\chi}) + \frac{\omega^2}{m_\psi^2}(\vec{n}\cdot\vec{\psi})\,\vec{n}(\vec{e}\times\vec{\chi}) \right\} F_b^{(1)} m_\psi , \tag{29b}$$

$$e_\mu \langle \chi_2 | j_\mu^{em} | J/\psi \rangle = \psi_i e_k \chi_{ik} F_a^{(2)} m_\psi - \left\{ (\vec{\psi}\vec{e}) n_i n_k \chi_{ik} + \right.$$
$$\left. + (\vec{\psi}\vec{n}) e_i n_k \chi_{ik} \right\} F_b^{(2)} m_\psi - (\vec{\psi}\vec{e}) n_i n_k \chi_{ik} F_c^{(2)} m_\psi , \tag{29c}$$

$$e_\mu \langle \eta_c | j_\mu^{em} | J/\psi \rangle = \vec{\psi}\cdot(\vec{e}\times\vec{n})\,\omega F(J/\psi \to \eta_c\gamma) . \tag{29d}$$

where $\vec{n} = \frac{\vec{K}}{\omega}$ and $\vec{e}$, η_c , $\vec{\psi}$, χ_0 , $\vec{\chi}$, χ_{ik} are the three-dimensional wave functions of photon , η_c , J/ψ , χ_0 , χ_1 , χ_2 respectively. Eqs.(29a) and (29d) correspond to trivial E1 and M1 nonrelativistic amplitudes. A nontrivial fact is the E1-dominance in the two other structures (29b),and (29c) since the terms of these structures corresponding to M2 and M2,E3 amplitudes of $\chi_1 \to J/\psi\,\gamma$ and $\chi_2 \to J/\psi\,\gamma$ transitions are suppressed due to small numerical factors ω/m_ψ and $F_b^{(2)}/F_a^{(2)}$, $F_c^{(2)}/F_a^{(2)}$ respectively(see Eq.(16)). The E1 amplitudes F ($F^{(0)}$, $F_a^{(1)}$, $F_a^{(2)}$) and F' ($F'^{(0)}$, $F_a'^{(1)}$, $F_a'^{(2)}$)

are related with the nonrelativistic overlap integrals as follows :

$$F^{(J)} = -\omega^{(J)} I^{(J)} , \quad F'^{(J)} = \omega'^{(J)} I'^{(J)} \tag{30}$$

(with some inessential normalization factors) where $\omega^{(J)} = m_{\chi_J} - m_\psi$; $\omega'^{(J)} = m_{\psi'} - m_{\chi_J}$,

$$I^{(J)} = \int R_\psi(r) R_{\chi_J}(r)\, r^3 dr$$
$$I'^{(J)} = \int R_{\psi'}(r) R_{\chi_J}(r)\, r^3 dr$$

R_ψ , $R_{\psi'}$ and R_{χ_J} are the radial wave functions of $J/\psi\,(^3S_1)$ $\psi'(2^3S_1)$ and χ_J (^3P_J) charmonium levels. The minus sign in the first equation (30) is caused by the fact that the sequence of states in the matrix element $\langle \chi \,| j_\mu^{em} | J/\psi \rangle$ is reversed as compared with the "natural" sequence $\langle f | i \rangle \equiv \langle J/\psi | \chi \rangle$ On the other hand , the g_ψ , $g_{\psi'}$, g_{χ_J} constants in nonrelativistic language are proportional to $R_\psi(0), R_{\psi'}(0), \frac{d}{dr}R_\chi(0)$ respectively. Recently a rather general theorem has been proved[18] concerning the sign of 2S $\to$ 1P dipole matrix element. From this theorem it follows that the relative sign of $R_\psi(0) I^{(J)}$ and $R_{\psi'}(0) I'^{(J)}$ products is negative. Just the same sign follows from our sum rules if we take into account Eq.(30) and the definitions of g's and F's (see the footnote after Eq.(16)).

We turn now to the comparison of numerical results for the E1 transitions. For this purpose, in the fourth column of Table 2 we present the E1 widths calculated in one of the most elaborated nonrelativistic models [12] . The general agreement with our results is obvious. The $\Gamma(\chi_J \to J/\psi\,\gamma)$ predictions also agree with the results of nonrelativistic sum rules[19] given in the fifth column of Table 2 . Note however that the $\Gamma(\psi' \to \chi_0\gamma)$ width given by the dispersion sum rules is smaller (and closer to the experimental value) than the nonrelativistic model prediction. It is important to remind the accordance between radiative and leptonic (annihilation) widths in our approach whereas in the nonrelativistic charmonium there exists an unambigous discrepancy between $\psi(0)$ and the overlap integrals, as it was noted in Ref. [12] .

As for the b-quarkonium, our predictions for $\Gamma(\Upsilon' \to \Upsilon\gamma)$ obtained above are much smaller than those of nonrelativistic model [12] .

Our final comment refer to the nonrelativistic limit of M1 amplitude F($J/\psi \to \eta_c\gamma$). that is easily obtained comparing Eq.(24) in the limit $m_\psi - m_{\eta_c}/m_\psi \to 0$ with the familiar nonrelativistic expression

$$\Gamma(J/\psi \to \eta_c\gamma) = \frac{16}{3}\mu^2\omega^3 I^2$$

where μ is the c-quark magnetic moment and $I = \int R_\psi(r) R_{\eta_c}(r) r^2 dr$. Then $F(J/\psi \to \eta_c\gamma) = 4\left(\frac{\mu}{\mu_0}\right) \cdot I$ where $\mu_0 = \frac{eQ_c}{m_\psi}$. If one neglects the effects of hyperfine splitting , then $R_\psi = R_{\eta_c}$, $I = 1$, $\mu \simeq \mu_0$ and F $(J/\psi \to \eta_c\gamma)$ = 4 . Similarly, F($\psi' \to \eta_c'\gamma$) = F($J/\psi \to \eta_c\gamma$) = 4, F($\psi' \to \eta_c\gamma$)= F($\eta_c' \to J/\psi\,\gamma$) = 0 and also F($\Upsilon \to \eta^b\gamma$) =4. These values are of order of our results obtained above from the numerical analysis of dispersion sum rules (22).

Thus, we may conclude that there is an obvious qualitative agreement between dispersion sum rules and nonrelativistic models which are based on the potentials with confinement. This conclusion supports the general viewpont inferred in Ref. [1] from the analysis of one-dimensional sum rules.

6. Conclusion

We should like to emphasize two main results of this paper.

The first result (which is of practical interest) is a new model-independent method of calculation of radiative transition widths. This method may successfully compete with the traditional nonrelativistic models.

The second result (which is of theoretical interest) is that QCD is able to predict not only the properties of separate $\bar{q}q$ - resonances , as it has been found out in Ref. [1] but also the amplitudes of the simplest processes where two such resonances participate. The basic elements of the one-dimensional dispersion approach such as the validity of dispersion relations and the duality between quarks and continuum seem to work well in the two-dimensional case.

We hope to continue the investigation of these interesting subjects in the nearest future.

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Table 1 . The double spectral functions of triangle diagrams and the dimensions c_1,d of matrix elements (4),(5) for various transitions $J^{PC}_{1} \leftrightarrow 1^{--}$.

J^{PC}_1	$\rho^{(J)}(s_1,s_2) = 3eQ\,\delta(s_1-s_2)\,f^{(J)}(s_1)$	c_1	d
0^{++}	$f^{(0)}(s) = \frac{ms}{4}\left[2v-(1-v^2)\ln\frac{1+v}{1-v}\right]$ *)	2	1
1^{++}	$f^{(1)}_a(s) = \frac{s^2}{8}\left[2v-(1-v^2)\ln\frac{1+v}{1-v}\right]$	2	2
	$f^{(1)}_b(s) = 0$		
2^{++}	$f^{(2)}_a(s) = \frac{s^2}{16}\left\{(1-v^2)(3-v^2)\ln\frac{1+v}{1-v} - 6v + \frac{10}{3}v^3\right\}$	3	1
			3
	$f^{(2)}_b(s) = \frac{s^3}{16}\left\{(1-v^2)(5-v^2)\ln\frac{1+v}{1-v} - 10v + \frac{26}{3}v^3\right\}$		3
	$f^{(2)}_c(s) = -\frac{3}{2}f^{(2)}_b(s)$		3
0^{-+}	$f^{(0^-)}(s) = \frac{m}{2}\ln\frac{1+v}{1-v}$	2	-1

*) $v \equiv \left(1-\frac{4m^2}{s}\right)^{1/2}$

Table 2 . The widths of E1 transitions in charmonium. The exp.values are obtained from measured branching ratios and relevant total widths: $\Gamma^{\psi'}_{tot} = 228 \pm 56$ keV [9] ; $\Gamma^{\chi_0}_{tot} = 3 \div 5$ MeV; $\Gamma^{\chi_1}_{tot} = 0,1 \div 0,4$ MeV; $\Gamma^{\chi_2}_{tot} = 1 \div 2$ MeV [1]

Transition	Predictions of sum rules at m_c=1,25 (1,30) GeV (keV)	Experiment (keV)	Predictions of nonrelativistic models (keV) [12]	[13]
$\psi' \to \chi_0\gamma$	7 (8)	16 ± 8 a)	50	
$\psi' \to \chi_1\gamma$	40 (31)		45	
$\psi' \to \chi_2\gamma$	34 (31)		29	
$\chi_0 \to J/\psi\,\gamma$	228 (108)	100±30 ÷ 165±50 a) 30±30 ÷ 55±55 b) 20 ÷ 35 c)	141	100 ÷ 200
$\chi_1 \to J/\psi\,\gamma$	330 (160)	24±8 ÷ 96±30 a) 36±5 ÷ 144±20 b) 30±10 ÷ 120±40 c)	289	200 ÷ 400
$\chi_2 \to J/\psi\,\gamma$	280 (136)	160±30 ÷ 320±60 a) 170±40 ÷ 340±80 b) 160±80 ÷ 320±160 c)	398	300 ÷ 500

a)Ref. [9] ;b)Ref. [10] ;c)Ref. [4] .

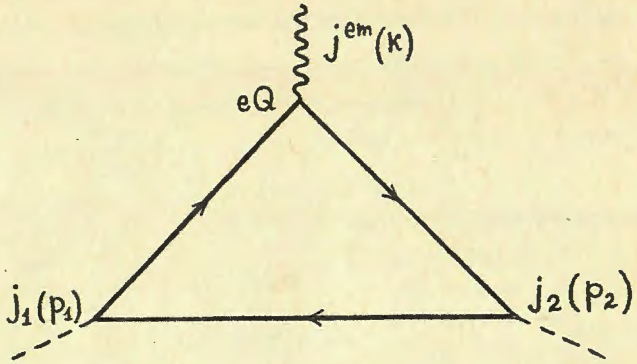

Fig.1

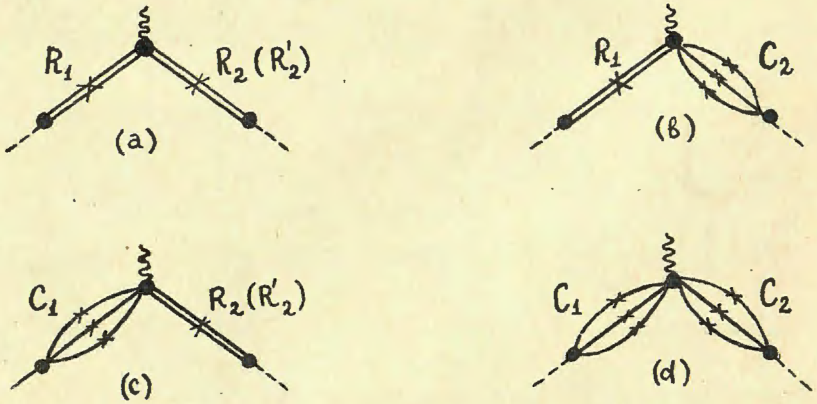

Fig.2

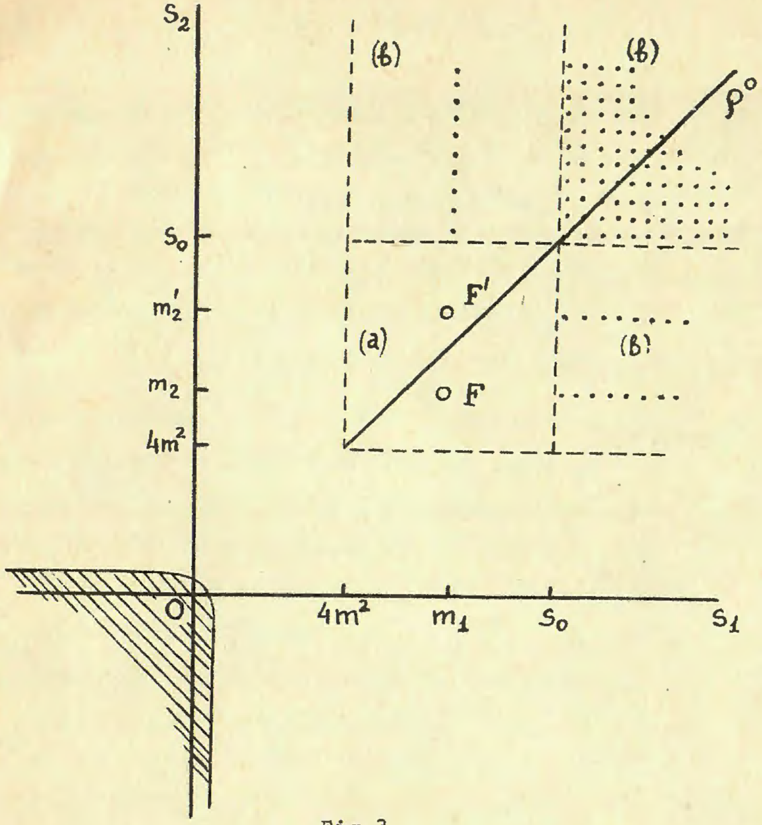

Fig.3

FIGURE CAPTIONS

Fig.1 The Feynman diagram corresponding to the zeroth order in α_s of the amplitude (1) (plus the diagram with j^{em} and j_2 interchanged)

Fig.2 The graphical representation of the r.h.s. of Eq. (3): the diagram (a) ((b) - (d)) corresponds to the resonance (continuum) contribution.

Fig.3 The postulated regions of duality between quark and physical intermediate states in the s_1, s_2 plane. The quark contribution nonvanishing on the straight line ρ^o is dual to the resonance contribution (F and F' points denoted by circles) in the region (a) ($4m^2 \leq s_{1,2} \leq s_0$) and is dual to the continuum contribution (dotted lines and region) in the region (b) ($s_{1,2} > s_0$). The asymptotic freedom region $s_{1,2} \ll 4m^2$ is shaded.

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А.Ю.ХОДЖАМИРЯН

РАДИАЦИОННЫЕ ПЕРЕХОДЫ В КВАРКОНИИ
И КВАНТОВАЯ ХРОМОДИНАМИКА

(на английском языке)

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