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V.A.DJRBASHIAN

PROBABILITIES OF DECAY AND FORMATION PROCESSES

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YEREVAN PHYSICS INSTITUTE

V.A.DJRBASHIAN

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1. Introduction

According to the treatment usually adopted in the literature the decay and formation processes as well as the scattering process can be described by the S-matrix. The latter connects the initial amplitude at $t_0 = -\infty$ with the final one at $t = \infty$. Whereas from the physical meaning it is clear that in the case of decay of a bound system existed at $t_0 = -\infty$ we shall have zero probability for the transition per unit time even at finite t (since infinite time goes from $t_0 = -\infty$). Therefore, unlike the wide-spread treatment the decay and formation processes cannot be described, in principle, by the usual S-matrix theory. The probability amplitudes must satisfy other real initial conditions resulting in certain modification of the S-matrix theory equations [1]. The latter ones are solved and the integral representation of the transition probability amplitude in an arbitrary time interval is found in the present paper, being the generalization and development of works [2]. Following the nature of some physical effects, it is assumed that in states under consideration (non-obligatory stationary states) the energy may have no definite value because of external effect. For the stationary states, which are the particular case of those under consideration, all the results following from the general ones take place. However, owing to the fact that stationary states embrace a narrower range of states, we can formulate a stronger statement concerning the probabilities of transitions between them in the time intervals $[0;t]$ and $[t;t+dt]$. Reasonable probabilities of transitions between two sublevels are obtained for these states. In the general case of non-obligatory stationary states formulae are derived for the probability of transition from a sublevel of the level i, described by the wave function $|m_{oi}>$ at a moment $t=0$, to any of sublevels m_{of} of the level f.

The simplest applications of the obtained formulae for arbitrary time are the descriptions of the decay and formation processes, the first nonvanishing approximation of which are coupling constant first-order effects. In particular, the decay of unstable oriented particles [3] and nuclei in arbitrarily directed magnetic fields as well as the magnetic resonance [4] can be cited as examples.

2. Non-obligatory stationary states

The quantum-mechanical behaviour of the system is described by the amplitude

$$\psi(t) = \sum_{m_o} \phi_{m_o}(t)\, \psi_{m_o}(t), \qquad (2.1)$$

which satisfies the wave equation

$$i\hbar \frac{\partial \psi}{\partial t} = \mathcal{H}\psi, \qquad (2.2)$$

where the complete Hamiltonian

$$\mathcal{H} = \mathcal{H}_o + V, \qquad (2.3)$$

is in general an external field Hamiltonian, V is an operator of the interaction responsible for transitions we are interested in. The wave functions $\psi_{m_o}(t)$ describe the so called non-obligatory stationary states in which the energy may have no definite value. These states have arisen at a moment t, as a result of evolution, from the states $|m_o>$ at $t=0$. The functions $\psi_{m_o}(t)$ are the linear combinations of the functions

$$\psi_{m_o} = \sum_m <m|U|m_o> \psi_m, \qquad (2.4)$$

where

$$\psi_m(t) = e^{-(i/\hbar)E_m t}|m> \qquad (2.5)$$

are the eigenfunctions of $\mathcal{H}_o$. The matrix U, determined for each concrete problem, is unitary. If a state described by ψ_{m_o} is stationary, then $<m/U/m_o> = \delta_{m m_o}$. From eqs. (2.4) and (2.5) it follows that ψ_{m_o} satisfies the wave equation

$$i\hbar \frac{\partial \psi_{m_o}}{\partial t} = \mathcal{H}_o \psi_{m_o} \qquad (2.6)$$

Representing ψ_{m_o} as

$$\psi_{m_o}(t) = e^{-(i/\hbar)\mathcal{H}_o t}|m_o> \qquad (2.7)$$

and making use of eqs. (2.4) and (2.5) we obtain

$$|m_o> = \sum_m |m><m|U|m_o> \qquad (2.8)$$

3. Equations and integral representation for amplitudes

Substituting (2.1) into (2.2) and taking into account (2.3), (2.6) and (2.7) we arrive to a system of equations which determines time variation of physical probability amplitudes $\phi_{m_o}(t)$ for the states described by the functions $\psi_{m_o}(t)$ in the interaction representation

$$i\hbar \frac{\partial \phi_{m_o}(t)}{\partial t} = \sum_{m_o'} < m_o | e^{(i/\hbar)\mathcal{H}_o t} V e^{-(i/\hbar)\mathcal{H}_o t} | m_o' > \tag{3.1}$$

$$\times \phi_{m_o'}(t) + i\hbar \delta_{m_o m_{oi}} \delta(t).$$

The inhomogeneous term in the right-hand side of eq.(3.1) ensures the validity [1] of the equations for both $t > 0$ and $t \leq 0$. This term is necessary since, according to the physical meaning of the problem, the amplitudes $\phi_{m_o}(t)$ must satisfy the conditions

$$\phi_{m_o}(t) = 0 \quad \text{at} \quad t < 0, \tag{3.2a}$$

$$\phi_{m_o}(t) = \delta_{m_o m_{oi}} \quad \text{at} \quad t = +0, \tag{3.2b}$$

i.e. $\phi_{m_{oi}}(t)$ undergoes a jump from zero at $t = -0$ up to unity at $t = +0$. Thus, in the neighbourhood $t = 0$ $\phi_{m_{oi}}(t) = \theta(t)$, $\dot{\phi}_{m_{oi}}(t) = \dot{\theta}(t) = \delta(t)$ [5], just as it follows from eq. (3.1). The latter taken into account, the expression (2.8) can be presented as

$$i\hbar \frac{\partial \phi_{m_o}}{\partial t} = \sum_{m, m_o', m''} < m_o | U^+ | m > e^{(i/\hbar)E_m t} < m | V | m'' >$$

$$\times e^{-(i/\hbar)E_{m''} t} < m'' | U | m_o' > \phi_{m_o'} + i\hbar\delta \tag{3.3}$$

To solve the system (3.3) we introduce the transformations

$$\phi_{m_o}(t) = -\frac{1}{2\pi i} \lim_{\sigma \to 0} \int_{-\infty}^{\infty} dE \sum_{m, m'} < m_o | U^+ | m > \tag{3.4}$$

$$\times e^{(i/\hbar)(E_m - E_{m'} - E - i\sigma)t} < m | G | m' > < m' | U | m_{oi} > ,$$

$$i\hbar \delta_{m_o m_{oi}} \delta(t) = -\frac{1}{2\pi i} \lim_{\sigma \to 0} \int_{-\infty}^{\infty} dE \sum_{m, m'} < m_o | U^+ | m > \tag{3.5}$$

$$\times e^{(i/\hbar)(E_m - E_{m'} - E - i\sigma)t} \delta_{m m'} < m' | U | m_{oi} > ,$$

the last of which holds in virtue of matrix U unitarity. Using this property after substituting (3.4) and (3.5) into (3.3) we obtain

$$-\frac{1}{2\pi i} \lim_{\sigma \to 0} \int_{-\infty}^{\infty} dE \sum_{m, m'} < m_o | U^+ | m > e^{(i/\hbar)(E_m - E_{m'} - E - i\sigma)t} \tag{3.6}$$

$$\times \Big[(E + E_{m'} - E_m + i\sigma) < m | G | m' > - \sum_{m''} < m | V | m'' >$$

$$\times < m'' | G | m' > - \delta_{m m'} \Big] < m' | U | m_{oi} > = 0.$$

whence it follows that the matrix G must satisfy the relation

$$(E + E_{m'} - E_m + i\sigma) < m | G | m' > = \sum_{m''} < m | V | m'' > < m'' | G | m' > + \delta_{m m'} \tag{3.7}$$

Introducing a matrix W we shall look for the solution of this equation in the form

$$< m | G | m' > = < m' | G | m' > \big[\delta_{m m'} + (1 - \delta_{m m'}) < m | W | m' > (E + E_{m'} - E_m + i\sigma)^{-1} \big]. \tag{3.8}$$

Making use of this expression at $m = m'$ from eq. (3.7) we have

$$< m' | G | m' > = [E + i(\sigma + \hbar\Gamma_{m'}/2)]^{-1}, \tag{3.9}$$

$$\hbar\Gamma_{m'}/2 = i < m' | V | m' > + i \sum_{m'' \neq m'} < m' | V | m'' > \tag{3.10}$$

$$\times < m'' | W | m' > (E + E_{m'} - E_{m''} + i\sigma)^{-1}.$$

At $m \neq m'$ we shall have in a similar way

$$< m | W | m' > = < m | V | m' > + \sum_{m'' \neq m'} < m | V | m'' > \tag{3.11}$$

$$\times < m'' | W | m' > (E + E_{m'} - E_{m''} + i\sigma)^{-1}.$$

Substituting (3.8) and (3.9) into (3.4) and going to the limit wherever this gives rise to no singularities, we obtain the integral representation of the amplitude

$$\phi_{m_0}(t) = -\frac{1}{2\pi i} \lim_{\sigma \to 0} \int_{-\infty}^{\infty} dE \sum_{m, m''} < m_0 | U^{+} | m > [\delta_{mm'} \tag{3.12}$$

$$+ (1 - \delta_{mm'}) < m | W | m' > (E + E_{m'} - E_m + i\sigma)^{-1}]$$

$$\times e^{(i/\hbar)(E_m - E_{m'} - E)t} (E + i\hbar\Gamma_{m'}/2)^{-1} < m' | U | m_{0i} > .$$

Formula (3.2) is the generalization of the well-known expression [1] at $U = 1$, for the case when non-stationary states are also possible.

4. Explicit form of the first approximation amplitude

Consider now the processes the first non-vanishing approximations of which are coupling constant first-order effect. For these processes the matrix element $< m | W | m' >$ in (3.12) in the first approximation of the perturbation theory is determined by the first-hand term of eq. (3.11):

$$< m | W | m' > = < m | V | m' > . \tag{4.1}$$

The imaginary part of $\Gamma_{m'}$ is responsible [1] for the shift of the level m' , i.e. the proper energy indebted for its existence to the virtual field accompanying the particle. As far as we restrict ourselves to the first non-vanishing approximation, we must take $\mathrm{Im}\,\Gamma_{m'} = 0$ and regard $\Gamma_{m'}$ in (3.12) as a real part of $\Gamma_{m'}$ describing the level's width. From (3.10) taking into account (4.1) and using the hermiticity of the operator V we obtain for the real part of $\Gamma_{m'}$:

$$\mathrm{Re}\,\Gamma_{m'}(E) = -\frac{2}{\hbar} \sum_{m'' \neq m'} |< m'' | V | m' >|^2 \,\mathrm{Im} \lim_{\sigma \to 0} (E + E_m - E_{m''} + i\sigma)^{-1}. \tag{4.2}$$

Here, according to (3.12), the value of $\Gamma_{m'}(E)$ is required only at $E = -i\hbar\Gamma_{m'}/2$. As far as we must restrict ourselves to the first non-vanishing approximation in coupling constant, in the E-expansion we shall take into account only the zeroth approximation taking $\Gamma_{m'}(E) = \Gamma_{m'}(0)$ (the next approximation, as is seen from (4.2), will be two orders as high in coupling constant):

$$\mathrm{Re}\,\Gamma_{m'} = \frac{2\pi}{\hbar} \sum_{m'' \neq m'} |< m'' | V | m' >|^2 \delta(E_m - E_{m''}) , \tag{4.3}$$

where

$$\delta(x) = \frac{1}{\pi} \lim_{\sigma \to 0} \frac{\sigma}{x^2+\sigma^2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ixt}\, dt . \tag{4.4}$$

In (4.3) integrating over $E_{m''}$ and replacing $\mathrm{Re}\,\Gamma_{m'}$ by $\Gamma_{m'}$ we obtain

$$\Gamma_{m'} = \frac{2\pi}{\hbar} \sum_{m'' \neq m'} |<m''|V|m'>|^2 \rho_{m''} . \tag{4.5}$$

Here $\rho_{m''}$ is the number of final states m'' per energy interval $dE_{m''}$.

If not final, but initial state belongs to the continuous spectrum, we should integrate over $E_{m'}$ and in this case in (4.5) ρ will be the number of initial states m' per energy interval $dE_{m'}$.

Note, that from the general consideration [6] of the radiation angular distributions it follows that in the absence of external field the level total width (4.5) is independent of its magnetic quantum numbers, i.e. it is the same for different sublevels.

Since $\Gamma_{m'}$ in the approximation under consideration, according to (4.3) is E-independent, it is easy to integrate in the expression (3.12). At $t<0$, closing the integration contour in the upper complex half-plane E, we obtain $\phi_{m_0}(t) = 0$, i.e. the condition (3.2a) holds.

At $t>0$, closing the integration contour in the lower half-plane, according to Cauchy theorem on residues we have

$$\phi_{m_0}(t) = \sum_{m,m'} <m_0|U^+|m>[\delta_{mm'} \exp(-\Gamma_{m'}t/2) + (1-\delta_{mm'}) \tag{4.6}$$

$$\times <m|V|m'> \frac{\exp[i(E_m - E_{m'} + i\hbar\Gamma_{m'}/2)t/\hbar] - 1}{E_{m'} - E_m - i\hbar\Gamma_{m'}/2}] <m'|U|m_{0i}> .$$

It is seen from (4.6) that the initial condition (3.2b) holds, too.

5. Transition probability for stationary states

Consider stationary states of a quantum-mechanical system generally, in external field, that are characterized by a set of quantum numbers m .

From formula (4.6) for a first-order amplitude of perturbation theory we have

$$\phi_m(t) = \delta_{mm_i} \exp(-\Gamma_{m_i}t/2) + (1-\delta_{mm_i}) \tag{5.1}$$

$$\times <m|V|m_i> \frac{\exp[i(E_m - E_{m_i} + i\hbar\Gamma_{m_i}/2)t/\hbar] - 1}{E_{m_i} - E_m - i\hbar\Gamma_{m_i}/2} ,$$

i.e. Wigner-Weisskopf solution [1,7] .

At the moment $t=0$ the system is in the state m_i with a total width Γ_{m_i} . Taking $|\phi_{m_f}(t)|^2$ we calculate the probability of being the system in the state m_f at the moment t ; this probability in virtue of the state stationarity is the probability of the transition $m_i \to m_f$ in the time interval $[0;t]$

For $m = m_i$ from (5.1) one obtains the following probability that the system remains in the initial state

$$W_{m_i \to m_i}(0;t) = \exp(-\Gamma_{m_i} t) , \tag{5.2}$$

which is the radioactive decay law.

For $m_f \neq m_i$ in the case of almost monochromatic beam with $\Delta E \ll \Gamma_{m_i}$ taking into account the fact that the final state belongs to continuous spectrum (emission) we have

$$W_{m_i \to m_f}(0;t) = |\langle m_f|V|m_i\rangle|^2 \tag{5.3}$$

$$\times \left| \frac{\exp[i(E_{m_f} - E_{m_i} + i\hbar\Gamma_{m_i}/2)t/\hbar - 1}{E_{m_f} - E_{m_i} + i\hbar\Gamma_{m_i}/2} \right|^2 \rho \, \Delta E_{m_f} .$$

For the total spectrum the integration over dE_{m_f} is necessary. In order to separate the coupling constant lowest approximation we are interested in, let us write this expression in the form

$$W_{m_i \to m_f}(0;t) = (|\langle m_f|V|m_i\rangle|^2 \rho)_{av} \int_0^\infty A \, dE_{m_f} , \tag{5.4}$$

$$A = \frac{1 + \exp(-\Gamma_{m_i} t) - 2\exp(-\Gamma_{m_i} t/2)\cos(E_{m_f} - E_{m_i})t/\hbar}{(E_{m_f} - E_{m_i})^2 + (\hbar\Gamma_{m_i}/2)^2} \, dE_{m_f}$$

Taking into account that $2E_{m_i}/\hbar\Gamma_{m_i} \gg 1$ and

$$\int_{-\infty}^{\infty} \frac{\cos px}{x^2 + a^2} \, dx = \frac{\pi}{|a|} e^{-|ap|} , \tag{5.5}$$

we obtain

$$W_{m_i \to m_f}(0;t) = \frac{2\pi}{\hbar} (|\langle m_f|V|m_i\rangle|^2 \rho)_{av} \frac{1 - \exp(-\Gamma_{m_i} t)}{\Gamma_{m_i}} \tag{5.6}$$

The normalization condition is that the sum of transition probabilities and the probability of remaining in the initial state during the time interval $[0;t]$ must be equal to the initial state probability at $t = 0$. With (5.2) kept in mind we have

$$\sum_{m_f \neq m_i} W_{m_i \to m_f}(0;t) + \exp(-\Gamma_{m_i} t) = 1 . \tag{5.7}$$

Hence for the states with $\Gamma_{m_i} = 0$ we obtain $W_{m_i \to m_f} = 0$, $m_f \neq m_i$, i.e. they are stable.

At $\Gamma_{m_i} \neq 0$ from condition (5.7) it follows

$$\lim_{t \to \infty} \sum_{m_f \neq m_i} W_{m_i \to m_f}(0;t) = 1 . \tag{5.8}$$

Considering the expression $(|\langle m_f|V|m_i\rangle|^2 \rho)_{av.}$ proceeding from the condition (5.7), we have to restrict ourselves to the first non-vanishing approximation and let Γ_{m_i} tend to zero since it is proportional to the square of the coupling constant:

$$(|\langle m_f|V|m_i\rangle|^2 \rho)_{av.} = \tag{5.9}$$

$$= \lim_{\Gamma_{m_i} \to 0} \left[\int_0^\infty |\langle m_f|V|m_i\rangle|^2 \rho A \, dE_{m_f} \left(\frac{2\pi}{\hbar} \frac{1 - \exp(-\Gamma_{m_i} t)}{\Gamma_{m_i}} \right)^{-1} \right] .$$

From (5.9) it follows that at $t \gg \hbar/E_{m_i}$

$$(|\langle m_f|V|m_i\rangle|^2 \rho)_{av.} = \int_0^\infty |\langle m_f|V|m_i\rangle|^2 \rho \, \delta(E_{m_f} - E_{m_i}) \, dE_{m_f} . \tag{5.10}$$

Thus, according to (5.6) and (5.10), for the transition probability in the interval $[0;t]$ we have

$$W_{m_i \to m_f}(0;t) = \frac{2\pi}{\hbar} |\langle m_f|V|m_i\rangle|^2 \rho_{m_f} \frac{1 - \exp(-\Gamma_{m_i} t)}{\Gamma_{m_i}} . \tag{5.11}$$

Formula (5.11) gives a generalization of the known expression [1] for the transition probability per time unit in average

$$\frac{W_{m_i \to m_f}(0;t)}{t} = \frac{2\pi}{\hbar} |\langle m_f|V|m_i\rangle|^2 \rho_{m_f} ,$$

which is applicable only at $t \ll 1/\Gamma_{m_i}$. One can understand the time dependence (5.11) (with definition (4.5) for Γ_{m_i}), since if $\Gamma_{m_i} \neq 0$, the system after all must leave the state m_i.

For transition probability in the time interval $[t; t+dt]$, from (5.11), we have

$$W_{m_i \to m_f}(t)\,dt = \frac{2\pi}{\hbar} |\langle m_f|V|m_i\rangle|^2 \rho_{m_f} \exp(-\Gamma_{m_i} t)\,dt. \tag{5.12}$$

Hence it is seen that the transition probability per unit time at an arbitrary moment t $W_{m_i \to m_f}(t)$ differs from the "traditional" one by the factor $\exp(-\Gamma_{m_i} t)$.

At $t \to \infty$ according to formula (5.11) $(2\pi/\hbar)|\langle m_f|V|m_i\rangle|^2 \rho_{m_f}$ is the transition probability divided by the mean lifetime $\tau_{m_i} = 1/\Gamma_{m_i}$ which follows from (5.2). Only in this meaning one should understand the adopted in literature treatment of this expression as a transition probability per unit time at $t \gg \tau_{m_i}$.

6. Transition probability for non-obligatory stationary states

In general case of non-obligatory stationary states, including non-stationary ones, using (4.6), we can answer only the following correctly formulated question. What is the probability of transition from a sublevel, described by the function $|m_{oi}\rangle$ at $t = 0$, to any of sublevels m_{of} of the level in the time interval $[0; t]$? Taking $|\phi_{m_{of}}(t)|^2$ we calculate the probability of finding the system in the state described by

$$\psi_{m_{of}}(t) = \sum_{m_f, m'_{of}} |m'_{of}\rangle\langle m'_{of}|U^+|m_f\rangle \exp\left(-\frac{i}{\hbar} E_{m_f} t\right)\langle m_f|U|m_{of}\rangle$$

and arising at the moment t as a result of evolution from the state described by $|m_{of}\rangle$ at $t = 0$. However, this expression is not generally the probability of the transition to the state m_{of} in the interval $[0; t]$, since in the case of non-stationary states (due to the operator U) at various moments t_1, $0 < t_1 \leq t$, $\psi_{m_{of}}$ is various combination of the wave functions $|m'_{of}\rangle$ of the sublevels of the level f. Summing the obtained modulus square over m_{of}, we obtain the probability of transition from the sublevel, described by $|m_{oi}\rangle$ at $t = 0$, to the level f :

$$\sum_{m_{of}} |\phi_{m_{of}}(t)|^2 = \sum_{m_{of}} \Big| \sum_{m_f, m_i} \langle m_{of}|U^+|m_f\rangle\langle m_f|V|m_i\rangle \tag{6.1}$$

$$\times \frac{\exp[i(E_{m_f} - E_{m_i} + i\hbar\Gamma_{m_i}/2)t/\hbar - 1}{E_{m_i} - E_{m_f} - i\hbar\Gamma_{m_i}/2} \langle m_i|U|m_{oi}\rangle\Big|^2,$$

since for f differing from i $\delta_{m_f m_i} = 0$.

The expression obtained from the right-hand side of (6.1) after summation over m_{of} by using the unitarity of the matrix U needs integration over E_{m_f} taking into account that the final state belongs to continuous spectrum. For this purpose we represent the probability we are interested in as

$$W_{m_{oi} \to f}(0; t) = \int dE_{m_f} \sum_{m_{of}} |\phi_{m_{of}}(t)|^2 \rho\, d\Omega \tag{6.2}$$

$$= \sum_{m_f, m_i, m'_i} \int dE_{m_f} \langle m_f|V|m_i\rangle^* \langle m_f|V|m'_i\rangle \langle m_i|U|m_{oi}\rangle^*$$

$$\times \langle m'_i|U|m_{oi}\rangle \frac{i}{\hbar} \int_0^t \exp[-i(E_{m_f} - E_{m_i} - i\hbar\Gamma_{m_i}/2)t'/\hbar]\,dt'$$

$$\times \left(-\frac{i}{\hbar}\right) \int_0^t \exp[i(E_{m_f} - E_{m_i} + i\hbar\Gamma_{m'_i}/2)t''/\hbar]\,dt''\, \rho\, d\Omega.$$

Here, unlike the case of the stationary states (5.3), the solid angle is separated from ρ .

Taking into account the fact that according to (6.1) by the order of magnitude the difference between the energies of the final and initial states is no more than the level width, let us integrate (6.2) over E_{m_f} in the interval $]-\infty;\infty[$

Since

$$\int_{-\infty}^{\infty} dE_{m_f} \exp[i(t''-t')E_{m_f}/\hbar] = 2\pi\hbar\,\delta(t''-t'), \tag{6.3}$$

after the integration over E_{m_f}, it is easy to integrate over t''. Thus, for (6.2) we obtain

$$W_{m_{oi}\to f}(0;t) = \int_0^t W_{m_{oi}\to f}(t')\,dt', \tag{6.4}$$

where

$$W_{m_{oi}\to f}(t)\,dt = \frac{2\pi}{\hbar}\sum_{m_f,m_i} \Big| < m_f|V|m_i > \tag{6.5}$$

$$\times \exp[-i(E_{m_i} - i\hbar\Gamma_{m_i}/2)t/\hbar] < m_i|U|m_{oi}> \Big|^2 \rho_{m_f}\, d\Omega\, dt$$

is the probability of transition from sublevel of i described by $|m_{oi}>$ at $t = 0$, to an arbitrary sublevel of the level f in the time interval $[t, t + dt]$.

The value between E_{m_i} and $E_{m_i'}$ must be substituted as E_{m_f} to the right-hand side of (6.5).

The solution of the problem to find the transition probability in an arbitrary time interval can be readily derived from formula (6.5) by integrating over t .

With the help of the expression (6.5) writing down the integral (6.4) in the form

$$W_{m_{oi}\to f}(0;t) = 2\pi\sum_{m_f,m_i,m_i'} < m_f|V|m_i >^* < m_f|V|m_i' > \tag{6.6}$$

$$\times \frac{1 - \exp\{i[E_{m_i} - E_{m_i'} + i\hbar(\Gamma_{m_i} + \Gamma_{m_i'})/2]t/\hbar\}}{i(E_{m_i'} - E_{m_i}) + \hbar(\Gamma_{m_i} + \Gamma_{m_i'})/2} < m_i|U|m_{oi}>^*$$

$$\times < m_i'|U|m_{oi}> \rho_{m_f}\, d\Omega$$

and taking into account that for magnetic quantum numbers we have [6]

$$\frac{2\pi}{\hbar}\int \sum_{m_f} < m_f|V|m_i >^* < m_f|V|m_i' > \rho_{m_f}\, d\Omega = \delta_{m_i m_i'} \sum_{f\neq i} \gamma_{m_{if}} \tag{6.7}$$

where

$$\gamma_{m_{if}} = \frac{2\pi}{\hbar}\sum_{m_f}\int |< m_f|V|m_i >|^2 \rho_{m_f}\, d\Omega \tag{6.8}$$

is the partial width *), we obtain

$$\sum_{f\neq i} W_{m_{oi}\to f}(0;t) = \sum_{m_i}[1 - \exp(-\Gamma_{m_i}t)]\,|< m_i|U|m_{oi}>|^2 \tag{6.9}$$

$$\times \Big(\sum_{f\neq i}\gamma_{m_{if}}\Big)/\Gamma_{m_i}$$

for the integrated over radiation angles total probability of transition from sublevel m_{oi} to any different from i level in the time interval $[0; t]$.

Let us now consider the probability of remaining in the i-state, with the possibility of radiation taken into account. As far as the interference terms in the square modulus of (4.6) after integrating over radiation angles vanish due to the relation [6]

$$\int < m_i|V|m_i' > d\Omega = \delta_{m_i m_i'}\int < m_i|V|m_i > d\Omega, \tag{6.10}$$

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*) In the right-hand side of (6.8) $\sum_{m_f}$ means summation over the sublevels of the given level f, whereas in the left-hand side of (6.7) the summation has been carried out over sublevels of all the levels f $(f\neq i)$.

the total probability turns out equal to

$$W_{m_{0i}\to i}(0;t)=\sum_{m_i}\left\{\exp(-\Gamma_{m_i}t)+\left[1-\exp(-\Gamma_{m_i}t)\right]\gamma_{m_i i}/\Gamma_{m_i}\right\} \times\left|\langle m_i|U|m_{0i}\rangle\right|^2, \tag{6.11}$$

where the partial width $\gamma_{m_i i}$ is given by the right-hand side of (6.8) in which summation over m_f is replaced by summation over m_i' $(m_i' \neq m_i)$.

Taking into account the formulae (6.9) and (6.11) we obtain

$$W_{m_{0i}\to i}(0;t)+\sum_{f\neq i} W_{m_{0i}\to f}(0;t)=1, \tag{6.12}$$

since, according to (4.5) and (6.8), the total width is equal to the sum of partial widths:

$$\Gamma_{m_i}=\gamma_{m_i i}+\sum_{f\neq i}\gamma_{m_i f}, \tag{6.13}$$

i.e. the probability amplitudes (4.6) are normalized in such a way that for an arbitrary time interval $[0;t]$ the sum of the probabilities to remain at level i, without radiating (precessing), as well as to go into the continuous spectrum, is equal to unity.

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