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EXOTIC BARYON RESONANCES WITH ISOSPINS $\geqslant$ 5/2

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А.А.ГРИГОРЯН

СВОЙСТВА ЭКЗОТИЧЕСКИХ БАРИОННЫХ РЕЗОНАНСОВ С ИЗОСПИНАМИ $\geqslant 5/2$

Исследуются свойства экзотических барионных резонансов $a(I,S,\eta)$ с изоспинами $I \geqslant 5/2$, существование которых следует из дисперсионных правил сумм для амплитуд рассеяния редджеонов на частицах. Показано, что спины этих состояний S равны изоспинам $(S = I)$, а внутренние четности η положительны. Найдены выражения для спиральных вершин связи с редджевскими траекториями с $I = 1$ (ρ, A_2, π). Получена формула для ширин распадов $a(I,I,+) \to a(I-1,I-1,+)+\pi$. Обсуждены возможности экспериментального поиска этих состояний.

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EXOTIC BARYON RESONANCES WITH ISOSPINS $\geqslant 5/2$

Properties of exotic baryon resonances $a(I,S,\eta)$ with isospins $I \geqslant 5/2$, whose existence follows from the dispersion sum rules for the reggeon-particle scattering amplitudes, are investigated. It is shown that the spins S of the states $a(I,S,\eta)$ are equal to isospins and intrinsic parities η are positive. The expressions for the helicity couplings of $a(I,S,\eta)$ to the boson reggeons with $I=1$ (ρ, A_2, π) are found and the formula for their decay widths is derived. The experiments to search for the states $a(I,S,\eta)$ are discussed.

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YEREVAN PHYSICS INSTITUTE

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1. Introduction

The dispersion sum rules for the reggeon-particle scattering amplitudes (αa -amplitudes) are successfully applied to the theoretical investigation of strong interactions. This dynamical approach allows one to get a lot of information on the regularities of hadron processes.

The spin and isospin generalization of the sum rules for αa -amplitudes to the case of the scattering of reggeons on the particles has been proposed in the works of Kaidalov and author[1, 2]. The expressions for the resonances contribution to the sum rules have been derived. The general method has been formulated to construct the superconvergent sum rules (SSR). In the subsequent papers[3, 4] these SSR have been applied to investigate the couplings of $I = 1$ boson reggeons with baryons.

Let us consider the SSR for the processes

$$\alpha_i N \to \alpha_k N \qquad (1.1)$$

$$\alpha_i N \to \alpha_k \Delta_{33}$$

$(i, k = \rho, A_2, \pi)$

and saturate the $S(u)$ -channel states with $I_s\,(I_u)$ =1/2 and 3/2 by the contribution of nucleon and Δ_{33}-isobar. Then we obtain a set of equations for the helicity vertices $G^{a\alpha_j b}_{\lambda_a \lambda_b}(a, b = N, \Delta_{33}; j = \rho, A_2, \pi)$. The solution of this set is in good agreement with experimental data.[3]

Reactions $\alpha_i \Delta_{33} \to \alpha_K \Delta_{33}$ are to be considered next. It turns out, however, that the selfconsistency of the sum rules requires the contribution of the states with $I = 5/2$ to these reactions. It has been shown in [4] that one can satisfy all SSR for the processes

$$\begin{aligned} &\alpha_i \Delta_{33} \to \alpha_K \Delta_{33} \\ &\alpha_i N \to \alpha_K E_{55} \\ &\alpha_i \Delta_{33} \to \alpha_K E_{55} \end{aligned} \qquad (1.2)$$

assuming the existence of the exotic baryon resonance with $I = 5/2$ (labeled E_{55}) and saturating the $S(U)$-channel states with $I = 1/2$, 3/2, 5/2 by the nucleon, Δ_{33}-isobar and E_{55}-baryon, respectively. The SSR allow one to determine unambiguously the spin, P-parity and helicity vertices $G^{b\alpha_j E}_{\lambda_b \lambda_E}(b = \Delta_{33}, E_{55})$ of the resonance E_{55} *).

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*) An experimental evidence for the $I = 5/2$ resonance existence has been given recently in JINR experiments on the neutron beam [5]. A narrow peak with a mass 1.44 GeV in $\Delta^{++}\pi^{+}$ and $\Delta^{-}\pi^{-}$-systems has been observed in the reaction $np \to n\pi^{-}\pi^{-}p\pi^{+}\pi^{+}$. The width of this state is close to the theoretical prediction [4, 5] and, as it follows from the angular distribution of the decay products, its spin is greater than 1/2.

Considering now the sum rules for the processes $\alpha_i E_{55} \to \alpha_K E_{55}$ it can be shown that the state with $I = 7/2 - a(7/2)$ must be taken into account, and one can find the relations between the vertices, spin and P -parity of these states from the SSR for the following group of processes

$$\begin{aligned} &\alpha_i E_{55} \to \alpha_K E_{55} \\ &\alpha_i \Delta_{33} \to \alpha_K a(7/2) \qquad (1.3) \\ &\alpha_i E_{55} \to \alpha_K a(7/2) \end{aligned}$$

The successive application of this procedure lead us to the conclusion that there must exist the whole series of baryon resonances with increasing isospins and spins [4]. In accordance with aforesaid, to investigate the properties of these states let us consider the SSR for the processes

$$\begin{aligned} &\alpha_i a(I,I,+) \to \alpha_K a(I,I,+) \\ &\alpha_i a(I-1,I-1,+) \to \alpha_K a(I+1,S_x,\eta_x) \qquad (1.4) \\ &\alpha_i a(I,I,+) \to \alpha_K a(I+1,S_x,\eta_x) \end{aligned}$$

where $a(I,S,\eta)$ denotes the resonance with isospin I, spin S and parity η ; S_x and η_x are unknown spin and parity, and saturate the s - and u -channel states with isospins $I-1$, I and $I+1$ by the contribution of the resonances $a(I-1, I-1,+)$, $a(I,I,+)$ and $a(I+1, S_x, \eta_x)$. As a result of that, a set of equations for the vertices $c\alpha_j a(I+1,S_x,\eta_x)$ $(c = a(I,I,+), a(I+1, S_x, \eta_x))$ arises. The solution of this set for the helicity vertices with $\Delta\lambda = 0$, ±1 ($\Delta\lambda$ is the helicity flip) is given in this paper.

In sec.2 some results of the previous works [1, 2] are presented. The solution of the problem (1.4) is considered in sec.3 . It will be shown that the set of equations for the helicity residues with $\Delta\lambda = 0, \pm 1$ has the selfconsistent solution, and that this solution fixes the spin ($S_x = I + 1$) and parity $(\eta_x = +)$ of the state $a(I+1, S_x, \eta_x)$.

The properties of $a(I, I, +)$ are discussed in sec.4.

The formula which relates the $b \to a + \pi$ decay width (b and a are particles with arbitrary halfinteger spins) with invariant amplitudes will be derived in Appendix.

2. Superconvergent sum rules for $\mathcal{L}a$ -amplitudes

The superconvergent dispersion relations for the amplitudes of $\mathcal{L}a$ - scattering have the usual form

$$\int_0^\infty \mathrm{Im}\, T^{\mathcal{L}_i \mathcal{L}_k (-)}_{a\,b}(\nu, q_i^2, q_k^2, q^2)\, d\nu = 0 \qquad (2.1)$$

where $\nu = (s-u)/4M_N$, $T^{\mathcal{L}_i \mathcal{L}_k (-)}_{a\,b}$ is the crossing-odd amplitude of $\mathcal{L}_i a \to \mathcal{L}_k b$ -scattering

$$T^{\mathcal{L}_i \mathcal{L}_k (-)}_{a\,b} = T^{\mathcal{L}_i \mathcal{L}_k (s)}_{a\,b} - T^{\mathcal{L}_i \mathcal{L}_k (u)}_{a\,b}$$

The formalism of helicity amplitudes in the infinite momentum frame (IMF) has been proposed in [1] in order to take into account the spins of particles. The reggeon-particles vertices and contribution of resonances to the sum rules have a simple form in IMF.

The particle-reggeon-particle helicity vertex in IMF has the form

$$\Gamma^{a\alpha b}_{\lambda_a\lambda_b}(q) = |\vec{q}^{\perp}|^{|\lambda_a-\lambda_b|}\, e^{i\varphi(\lambda_a-\lambda_b)}\, G^{a\alpha b}_{\lambda_a\lambda_b}(\vec{q}^{\perp 2}) \qquad (2.2)$$

where $\vec{q}^{\perp}$ is the transverse component of momentum transfer $q = p_b - p_a$; φ is the angle between $\vec{q}^{\perp}$ and X -axis.

The residues $G^{a\alpha b}_{\lambda_a\lambda_b}(\vec{q}^{\perp 2})$ have no kinematic singularities in $\vec{q}^{\perp 2}$. We will consider in what follows the small $(\vec{q}_{i,k})^2$ region and discuss the quantities $G^{a\alpha b}_{\lambda_a\lambda_b}(0)$.

The reggeon residues $G^{a\alpha b}_{\lambda_a\lambda_b}$ have the following symmetry properties

$$G^{a\alpha b}_{\lambda_a\lambda_b} = \sigma P \eta_a \eta_b (-1)^{S_a - S_b - \lambda_a + \lambda_b} G^{a\alpha b}_{-\lambda_a\,-\lambda_b} \qquad (2.3)$$

$$G^{a\alpha b}_{\lambda_a\lambda_b} = G(-1)^{I} P (-1)^{\lambda_a-\lambda_b} \tilde{G}^{b\alpha a}_{\lambda_b\lambda_a} \qquad (2.4)$$

In (2.3) and (2.4) σ, P, G and I are the signature, P -parity, G -parity and isospin of α (nonstrange boson trajectories are considered); η_i and S_i are the P -parity and spin of particle i .

The coupling $\tilde{G}^{b\alpha a}_{\lambda_b\lambda_a}$ describes the inverse transition $b \to a$. [1, 2]

For reggeons scattering on particles with spins two types of SSR exist

a) Ordinary SSR which take place if $\alpha_\ell - \alpha_i - \alpha_k < -1$, where α_ℓ is the rightmost singularity in the j -plane with the given t -channel quantum numbers. The saturation of these SSR by the resonance contributions lead to the following relations between residues $G^{c\alpha_j f}_{\lambda_c\lambda_f}(0)$.

$$\sum_{I_s} X_{ts} \sum_{d_s} G^{a\mathcal{L}_i d_s}_{\lambda_a \lambda_a+n}(0)\, G^{d_s \mathcal{L}_\kappa b}_{\lambda_a+n\, \lambda_b}(0) - \tag{2.5}$$

$$- \sum_{I_u} X_{tu} \sum_{d_u} G^{a\mathcal{L}_\kappa d_u}_{\lambda_a \lambda_b-n}(0)\, G^{d_u \mathcal{L}_i b}_{\lambda_b-n\, \lambda_b}(0) = 0$$

and

$$\sum_{I_s} X_{ts} \sum_{d_s} \Big[G^{a\mathcal{L}_i d_s}_{\lambda_a \lambda_a-1}(0)\, G^{d_s \mathcal{L}_\kappa b}_{\lambda_a-1\, \lambda_a}(0) - G^{a\mathcal{L}_i d_s}_{\lambda_a \lambda_a+1}(0) \times$$

$$\times G^{d_s \mathcal{L}_\kappa b}_{\lambda_a+1\, \lambda_b}(0) \Big] + \sum_{I_u} X_{tu} \sum_{d_u} \Big[G^{a\mathcal{L}_\kappa d_u}_{\lambda_a \lambda_a-1}(0) \times \tag{2.6}$$

$$\times G^{d_u \mathcal{L}_i b}_{\lambda_a-1\, \lambda_a}(0) - G^{a\mathcal{L}_\kappa d_u}_{\lambda_a \lambda_a+1}(0)\, G^{d_u \mathcal{L}_i b}_{\lambda_a+1\, \lambda_a}(0) \Big] = 0$$

In (2.5) n is integer

$$\min(|\lambda_a|, |\lambda_b|) \leq |\lambda_a+n|\, (|\lambda_b-n|) \leq \max(|\lambda_a|, |\lambda_b|) \tag{2.7}$$

SSR (2.5) and (2.6) correspond to definite t -channel isospin state I_t : $X_{ts} \equiv X(I_t, I_s)$ ($X_{tu} \equiv X(I_t, I_u)$) is the isotopic crossing matrix; d_s and d_u are s and u -channel resonances with given I_s and I_u .

The quantities $G^{c\mathcal{L}_j f}_{\lambda_c \lambda_f}$ are the reduced residues. They are connected with the physical residues in the following way:

$$\left(G^{c\mathcal{L}_j f}_{\lambda_c \lambda_f}\right)^{Phys.} = \varepsilon_c \varepsilon_j \varepsilon_f (-1)^{I_c - I_j + m_f} \begin{pmatrix} I_c & I_j & I_f \\ m_c & m_j & -m_f \end{pmatrix} G^{c\mathcal{L}_j f}_{\lambda_c \lambda_f} \tag{2.8}$$

where $I_i\,(m_i)$ is the isospin (3-rd projection of isospin) of particles; ε_i is the phase connecting the particle state with the basic isospin one; $\begin{pmatrix} I_1 & I_2 & I_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$ is the $3jm$ -symbol.

SSR (2.5) and (2.6) are valid for the amplitudes $T^{\alpha_i \alpha_k}_{\lambda_a \lambda_b}(\sigma_\ell P_\ell)$ with fixed naturality $\sigma_\ell P_\ell$: for (2.5) $(\sigma_\ell P_\ell)(\sigma_i P_i)(\sigma_k P_k) \equiv \tau = +1$, and for (2.6) $\tau = -1$.

Apart from SSR (2.5) and (2.6) for the particles with spins the SSR arise which are valid under weaker restrictions for the position of α_ℓ in j -plane, i.e. $\alpha_\ell - \alpha_i - \alpha_k < 0$ [1]. Saturating these sum rules by the resonance contributions one can get the following equations

$$\Big\{ \sum_{I_s} X_{ts} \sum_{d_s} G^{a\alpha_i d_s}_{\lambda_a\, \lambda_a+n}(0)\, G^{d_s \alpha_k b}_{\lambda_a+n\, \lambda_b}(0) - \sum_{I_u} X_{tu}^{*}$$

$$\times \sum_{d_u} G^{a\alpha_k d_u}_{\lambda_a\, \lambda_b-n}(0)\, G^{d_u \alpha_i b}_{\lambda_b-n\, \lambda_b}(0) \Big\} \Big/ \binom{|n|}{|\lambda_a-\lambda_b|} = \tag{2.9}$$

$$= \Big\{ \sum_{I_s} X_{ts} \sum_{d_s} G^{a\alpha_i d_s}_{\lambda_a\, \lambda_a+n'}(0)\, G^{d_s \alpha_k b}_{\lambda_a+n'\, \lambda_b}(0) - \sum_{I_u} X_{tu}^{*}$$

$$\times \sum_{d_u} G^{a\alpha_k d_u}_{\lambda_a\, \lambda_b-n'}(0)\, G^{d_u \alpha_i b}_{\lambda_b-n'\, \lambda_b}(0) \Big\} \Big/ \binom{|n'|}{|\lambda_a-\lambda_b|}$$

In eq.(2.9) n and n' are integers which satisfy the conditions $n \neq n'$ and (2.7); $\binom{m}{|\lambda_a-\lambda_b|}$ is the binomial coefficient. In order to determine the asymptotic behaviour of given sum rule one must find the position of the rightmost j-plane singularity α_ℓ which can contribute asymptotically to the given crossing-odd amplitude $T_{a\,b}^{\alpha_i \alpha_K (-)}$.

In the case of nonstrange α_i, α_K (with $I_{i,K} = 1$) boson reggeons scattering a simple analysis[2] (which takes into account also the possible contribution of moving cuts) shows that

i) for the amplitudes which lead to the SSR (2.5) and (2.6)

$$\begin{aligned} \alpha_\ell(0) &< 0 && \text{if} \quad I_t = 0 \\ \alpha_\ell(0) &\approx 0 && \text{if} \quad I_t = 1 \\ \alpha_\ell(0) &\approx -0.5 && \text{if} \quad I_t = 2 \end{aligned} \tag{2.10}$$

ii) in the case of sum rules (2.9)

$$\begin{aligned} \alpha_\ell(0) &\approx 0.5 && \text{at} \quad I_t = 0,1 \\ \alpha_\ell(0) &\approx 0 && \text{at} \quad I_t = 2 \end{aligned} \tag{2.11}$$

3. Scattering of reggeons with I = 1 on baryons. General solution of SSR.

In accordance with the scheme developed in Introduction let us consider the SSR (2.5), (2.6) and (2.9) for the following group of processes:

$$\alpha_i \, a \to \alpha_K \, a \tag{3.1}$$

$$\alpha_i \, a\text{-}1 \to \alpha_K \, (a+1)_x \tag{3.2}$$

$$\alpha_i \, a \to \alpha_K \, (a+1)_x \tag{3.3}$$

where α_i and α_K are the reggeons with $I_{i,K} = 1$ (ρ, A_2, π). For simplicity the following notations are used in (3.1)-(3.3):

$a(I,\ S = I,\ \eta = +) \equiv a$ and $a(I+1,\ S_x,\ \eta_x) \equiv (a+1)_x$.

Let us saturate each $s(u)$ -channel state with isospin $I_s(I_u)$ by the contribution of one resonance only (in processes (3.1) $I_s(I_u) = I-1, I, I+1$; in (3.2) $I_s(I_u) = I$ and in (3.3) $I_s(I_u) = I, I+1$). As a result we get a set of relations between the known quantities $G^{c\,\alpha_j\,a}_{\lambda_c\,\lambda_a}$ $(c = a-1, a\ ;\ j = \rho, A_2, \pi)$ *) and unknown residues $G^{f\,\alpha_j\,(a+1)_x}_{\lambda_f\,\lambda_{a+1}}$ $(f = a, (a+1)_x)$. Consider the solution of this set.

Let us show first that the helicity non-flip residues are equal to zero

$$G^{f\,\alpha_j\,(a+1)_x}_{\lambda\ \lambda} = 0 \tag{3.4}$$

For this purpose let us write down the SSR (2.9) for the processes $\alpha_i a-1 \to \alpha_i (a+1)_x$ and $\alpha_i a \to \alpha_i (a+1)_x$ (in $I_t = 2$) at $\lambda_b = \lambda_a + 1$

$$G^{a-1\,\alpha_i\,a}_{\lambda\ \lambda} G^{a\,\alpha_i\,(a+1)_x}_{\lambda\ \lambda+1} - G^{a-1\,\alpha_i\,a}_{\lambda\ \lambda+1} G^{a\,\alpha_i\,(a+1)_x}_{\lambda+1\ \lambda+1} = 0 \tag{3.5}$$

$$G^{a\,\alpha_i\,a}_{\lambda\,\lambda} G^{a\,\alpha_i\,(a+1)_x}_{\lambda\ \lambda+1} - G^{a\,\alpha_i\,a}_{\lambda\ \lambda+1} G^{a\,\alpha_i\,(a+1)_x}_{\lambda+1\ \lambda+1} + Y_x \tag{3.6}$$

$$\times \left[G^{a\,\alpha_i\,(a+1)_x}_{\lambda\ \lambda} G^{(a+1)_x\,\alpha_i\,(a+1)_x}_{\lambda\ \ \lambda+1} - G^{a\,\alpha_i\,(a+1)_x}_{\lambda\ \lambda+1} G^{(a+1)_x\,\alpha_i\,(a+1)_x}_{\lambda+1\ \ \lambda+1} \right] = 0$$

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*). Here and in what follows $G^{c\,\alpha_j\,f}_{\lambda_c\,\lambda_f} \equiv G^{c\,\alpha_j\,f}_{\lambda_c\,\lambda_f}(0)$

In (3.6) $Y = X_{ts}(I \to I+1; I+1, 2)/X_{ts}(I \to I+1; I, 2)$ where $X_{ts}(I_a \to I_b; I_s, I_t)$ is the isotopic crossing matrix of the process $\alpha_i(I=1)\, a(I_a) \to \alpha_K(I=1)\, b(I_b)$ with $s(t)$ -channel isospin $I_s(I_t)$ (for this reaction $X_{tu}(I_a \to I_b; I_s, I_t) = (-1)^{I_t} X_{ts}(I_a \to I_b; I_s, I_t)$.

As it will be shown below, the solution of SSR with the nonzero single-flip residues exists. It follows therefore from equations (3.5) and (3.6) that $G^{a\alpha_i(a+1)_x}_{\lambda\;\lambda} = G^{(a+1)_x\alpha_i(a+1)_x}_{\lambda\;\lambda} = 0$ if $G^{a-1\,\alpha_i a}_{\lambda\;\lambda} = G^{a\alpha_i a}_{\lambda\;\lambda} = 0$.

For the cases $a = N, \Delta_{35}, E_{55}$ it has been shown [3, 4] that the non-flip residues are equal to zero *), hence the statement (3.4) is proved.

Let us now show that the simple relation between the single-flip residues of reggeons α_i and α_K takes place

$$G'^{\,c\alpha_i(a+1)_x}_{\lambda\;\lambda\pm1} = (\mp1)^{\frac{\sigma_i P_i - \sigma_K P_K}{2}}\, G'^{\,c\alpha_K(a+1)_x}_{\lambda\;\lambda\pm1} \tag{3.7}$$

where

$$G'^{\,c\alpha_j(a+1)_x}_{\lambda\;\lambda\pm1} = G^{c\alpha_j(a+1)_x}_{\lambda\;\lambda\pm1} \Big/ G^{N\alpha_j N}_{1/2\,-1/2}$$

Consider the SSR (2.9) (or (2.5) since due to solution (3.4) the SSR (2.9) fall apart into the SSR (2.5)) for the amplitudes with $\lambda_b = \lambda_a \pm 2$ of the reactions $\alpha_i(a-1) \to \alpha_K(a+1)_x$ and $\alpha_i a \to \alpha_K(a+1)_x$ (at $I_t = 2$):

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*) Note that $G^{a\alpha_\pi a}_{\lambda\;\lambda} = 0$ due to (2.4).

$$G^{\prime\, a-1\,\alpha_i\,a}_{\lambda\;\lambda\pm1}\, G^{\prime\, a\,\alpha_k\,(a+1)_x}_{\lambda\pm1\;\lambda\pm2} - G^{\prime\, a-1\,\alpha_k\,a}_{\lambda\;\lambda\pm1}\, G^{\prime\, a\,\alpha_i\,(a+1)_x}_{\lambda\pm1\;\lambda\pm2} = 0 \qquad (3.8)$$

$$\begin{aligned} & G^{\prime\, a\,\alpha_i\,a}_{\lambda\;\lambda\pm1}\, G^{\prime\, a\,\alpha_k\,(a+1)_x}_{\lambda\pm1\;\lambda\pm2} - G^{\prime\, a\,\alpha_k\,a}_{\lambda\;\lambda\pm1}\, G^{\prime\, a\,\alpha_i\,(a+1)_x}_{\lambda\pm1\;\lambda\pm2} + \\ & + Y\Big(G^{\prime\, a\,\alpha_i\,(a+1)_x}_{\lambda\;\lambda\pm1}\, G^{\prime\,(a+1)_x\,\alpha_k\,(a+1)_x}_{\lambda\pm1\;\lambda\pm2} - \\ & - G^{\prime\, a\,\alpha_k\,(a+1)_x}_{\lambda\;\lambda\pm1}\, G^{\prime\,(a+1)_x\,\alpha_i\,(a+1)_x}_{\lambda\pm1\;\lambda\pm2} \Big) = 0 \end{aligned} \qquad (3.9)$$

The relation (3.7) holds for the vertices $G^{\prime\, c\,\alpha_j\,f}_{\lambda\;\lambda\pm1}\ (c, f = N, \Delta_{33}, E_{55})$[3,4]. It follows then from (3.8) and (3.9) that the vertices $G^{\prime\, a\,\alpha_j\,(a+1)_x}_{\lambda\;\lambda\pm1}$ and $G^{\prime\,(a+1)_x\,\alpha_j\,(a+1)_x}_{\lambda\;\lambda\pm1}$ satisfy eq.(3.7) if the vertices $G^{\prime\, a-1\,\alpha_j\,a}_{\lambda\;\lambda\pm1}$ and $G^{\prime\, a\,\alpha_j\,a}_{\lambda\;\lambda\pm1}$ satisfy it, q.e.d.

The relation (3.7) reduces the equations for the single-flip residues to the case of $i = k$ (scattering of the same reggeons). Let $i = k = \pi$. Since $X_{ts}(I_a \to I_b\,;\, I_s, I_t) = (-1)^{I_s} \begin{Bmatrix} 1 & 1 & 1 \\ I_a & I_b & I_s \end{Bmatrix}$ ($\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$ is $6j$ - symbol[2]), taking into account the particular values of the $6j$ -symbols (see table 1) one can get the set of equations for $G^{\prime\, a\,\alpha_\pi\,(a+1)_x}_{\lambda\;\lambda\pm1}$ and $G^{\prime\,(a+1)_x\,\alpha_\pi\,(a+1)_x}_{\lambda\;\lambda\pm1}$ given in table 2. The solutions of this set can be found as the relations (3.4) and (3.7), i.e. by the mathematical induction.

As it was shown in [3, 4], the solution of the SSR for the vertices $G^{\prime\, a\,\alpha_\pi\,b}_{\lambda\;\lambda\pm1}\ (a, b = N, \Delta_{33}, E_{55})$ has the form:

$$G^{\prime\, N\,\alpha_\pi\,\Delta}_{\lambda\;\lambda\pm1} = \rho\Big(\mp\sqrt{\tfrac{3}{2}}\, C^{3/2\;\lambda\pm1}_{1\pm1\;1/2\;\lambda}\Big)$$

$$G^{i\Delta\alpha_\pi\Delta}_{\lambda\ \lambda\pm 1} = -3\sqrt{\frac{2}{5}}\, C^{3/2\ \lambda}_{1\ \lambda\pm 1/2\ 1/2\mp 1/2}\, C^{3/2\ \lambda\pm 1}_{1\lambda\pm 1/2\ 1/2\pm 1/2}$$

$$G^{i\Delta\alpha_\pi E}_{\lambda\ \lambda\pm 1} = \gamma\left(\mp\sqrt{3}\, C^{5/2\ \lambda\pm 1}_{1\pm 1\ 3/2\ \lambda}\right) \qquad (3.10)$$

$$G^{iE\alpha_\pi E}_{\lambda\ \lambda\pm 1} = 3\sqrt{\frac{5}{7}}\, C^{5/2\ \lambda}_{2\ \lambda\pm 1/2\ 1/2\mp 1/2}\, C^{5/2\ \lambda\pm 1}_{2\ \lambda\pm 1/2\ 1/2\pm 1/2}$$

where $\varrho = \pm 1$; $\gamma = \pm 1$; $C^{jm}_{j_1 m_1 j_2 m_2}$ is the Clebsch-Gordan coefficients.

One can rewrite formulae (3.10) in the general form

$$G^{ia\alpha_\pi a+1}_{\lambda\ \lambda\pm 1} = \varrho\left(\mp\sqrt{\frac{3}{8(S_a+1)}}\sqrt{(S_a\pm\lambda+2)(S_a\pm\lambda+1)}\right) \qquad (3.11)$$

$$G^{ia\alpha_\pi a}_{\lambda\ \lambda\pm 1} = (-1)^{S_a-1/2}\sqrt{\frac{3(2S_a+1)}{8S_a(S_a+1)}}\sqrt{(S_a\mp\lambda)(S_a\pm\lambda+1)} \qquad (3.12)$$

In (3.11) and (3.12) the evident form of the C-G coefficient is taken into account.

$$C^{S+1\ \lambda\pm 1}_{1\pm 1\ S\lambda} = \sqrt{\frac{(S\pm\lambda+2)(S\pm\lambda+1)}{(2S+2)(2S+1)}}$$

$$C^{S\ \lambda}_{S-1/2\ \lambda\pm 1/2\ 1/2\mp 1/2} = \sqrt{\frac{S\mp\lambda}{2S}} \qquad (3.13)$$

$$C^{S\ \lambda\pm 1}_{S-1/2\ \lambda\pm 1/2\ 1/2\pm 1/2} = \sqrt{\frac{S\pm\lambda+1}{2S}}$$

Substituting the values (3.11) and (3.12) into equations of table 2, one can see that the solution for the residues $G'^{a\mathcal{L}_\pi(a+1)_x}_{\lambda\,\lambda\pm1}$ and $G'^{(a+1)_x\mathcal{L}_\pi(a+1)_x}_{\lambda\,\lambda\pm1}$ has the following form

$$|G'^{a\mathcal{L}_\pi(a+1)_x}_{\lambda\,\lambda\pm1}| = \beta^{a\mathcal{L}_\pi(a+1)_x}_{\lambda\,\lambda\pm1}\, G'^{a\mathcal{L}_\pi(a+1)_x}_{\lambda\,\lambda\pm1} = \sqrt{\frac{3}{8(S_a+1)}}\,\sqrt{(S_a\pm\lambda+2)(S_a\pm\lambda+1)} \tag{3.14}$$

$$|G'^{(a+1)_x\mathcal{L}_\pi(a+1)_x}_{\lambda\,\lambda\pm1}| = \beta^{(a+1)_x\mathcal{L}_\pi(a+1)_x}_{\lambda\,\lambda\pm1} \times G'^{(a+1)_x\mathcal{L}_\pi(a+1)_x}_{\lambda\,\lambda\pm1} = \sqrt{\frac{3(2S_a+3)}{8(S_a+1)(S_a+2)}}\,\sqrt{(S_a\mp\lambda+1)(S_a\pm\lambda+2)} \tag{3.15}$$

where the real phase factors $\beta^{c\mathcal{L}_\pi f}_{\lambda\,\lambda\pm1}$ $((\beta^{c\mathcal{L}_\pi f}_{\lambda\,\lambda\pm1})^2=1)$ satisfy the conditions

$$\beta^{a\mathcal{L}_\pi(a+1)_x}_{\lambda+1\,\lambda}\,\beta^{a\mathcal{L}_\pi(a+1)_x}_{\lambda-1\,\lambda} = -1 \tag{3.16}$$

$$\beta^{(a+1)_x\mathcal{L}_\pi(a+1)_x}_{\lambda\,\lambda-1} = (-1)^{S_a-1/2}\,\beta^{a\mathcal{L}_\pi(a+1)_x}_{\lambda-1\,\lambda}\,\beta^{a\mathcal{L}_\pi(a+1)_x}_{\lambda\,\lambda-1} \tag{3.17}$$

In the work[4], using the analysis of the invariant expression (A.1) for the vertex $a\pi b$ in the IMF, it has been shown that if

$$G^{a\mathcal{L}_\pi b}_{\lambda\,\lambda} = 0 \qquad \text{and} \qquad G^{a\mathcal{L}_\pi b}_{\lambda\,\lambda\pm1} \neq 0 \tag{3.18}$$

then with the necessity

$$\eta_b = \eta_a \; ; \quad S_a \leq S_b \leq S_a + 1 \, , \tag{3.19}$$

$$M_b = M_a \tag{3.19a}$$

and the helicity residues $G^{a\mathcal{L}_\pi b}_{\lambda\,\lambda\pm1}$ are expressed only through the invariant function $G^{a\pi b}_{1}$ (see A.3) and

a) if $S_b = S_a$, then

$$G^{a\mathcal{L}_\pi a}_{\lambda\,\lambda\pm1} = (-1)^{S_a-1/2} G^{a\pi a}_{1} \frac{1}{2S_a}\sqrt{(S_a\mp\lambda)(S_a\pm\lambda+1)} \tag{3.20}$$

b) if $S_b = S_a + 1$, then

$$G^{a\mathcal{L}_\pi a+1}_{\lambda\,\lambda\pm1} = \mp(-1)^{S_a-1/2}\sqrt{2}\, M_a G^{a\pi a+1}_{1}\sqrt{\frac{(S_a\pm\lambda+2)(S_a\pm\lambda+1)}{(2S_a+2)(2S_a+1)}} \tag{3.21}$$

Comparison of the solution (3.4) , (3.14)-(3.17) with (3.18)-(3.21) shows that

i) $$S_x = S_a + 1 \; ; \quad \eta_x = + \tag{3.22}$$

ii) one can write the solution of the SSR for $G^{a\mathcal{L}_\pi a+1}_{\lambda\,\lambda\pm1}$ and $G^{a+1\mathcal{L}_\pi a+1}_{\lambda\,\lambda\pm1}$ in the form (3.11) and (3.12) , and

iii) SSR predict the following values for $G^{a\pi a}_{1}$ and $G^{a\pi a+1}_{1}$

$$G^{a\pi a}_{1} = \sqrt{\frac{3\,S_a(2S_a+1)}{2\,(S_a+1)}} \tag{3.23}$$

$$\left(M_N\, G^{a\pi a+1}_{1}\right)^2 = \frac{3}{8}(2S_a+1) \tag{3.24}$$

The prime in eqs (3.23) and (3.24) has the same meaning as in (3.7).

Note that the solution (3.4), (3.11) and (3.12) for the π -pole residues holds in the limit of the equal masses $(M_{a+1} = M_a = \ldots = M_N)$ The mass equality condition is due to neglecting highlying states contribution to the sum rules[3].

Concluding this section let us dwell on the agreement of SSR solution with the finite energy sum rules (FESR).

Let us consider the crossing-odd amplitude $T^{(\rho)}_{\lambda_a \lambda_a}$ of the processes $\alpha_i a \to \alpha_i a$ $(i = \rho, A_2, \pi)$, in the $I_t = 1$ t -channel isospin state. The asymptotic behaviour of $T^{(\rho)}_{\lambda_a \lambda_a}$ is determined by the ρ -pole contribution in the t -channel.

However according to (3.4) the non-flip residues of ρ -pole are equal to zero. So the imaginary part of the amplitudes $T^{(\rho)}_{\lambda_a \lambda_a}$ should be small (in mean) and the contributions of different resonances to the sum rules for $T^{(\rho)}_{\lambda_a \lambda_a}$ must cancel.

The contribution of resonances to the sum rules for $T^{(\rho)}_{\lambda_a \lambda_a}$ has the form

$$(I+1)\sum_{(a-1)_j}\Big[\Big(G^{(a-1)_j \alpha_i a}_{\lambda_a-1\,\lambda_a}\Big)^2 + \Big(G^{(a-1)_j \alpha_i a}_{\lambda_a+1\,\lambda_a}\Big)^2\Big] +$$

$$+ \sum_{a_j}\Big[\Big(G^{a \alpha_i a_j}_{\lambda_a\,\lambda_a-1}\Big)^2 + \Big(G^{a \alpha_i a_j}_{\lambda_a\,\lambda_a+1}\Big)^2\Big] - \qquad (3.25)$$

$$- I \sum_{(a+1)_j}\Big[\Big(G^{a \alpha_i (a+1)_j}_{\lambda_a\,\lambda_a-1}\Big)^2 + \Big(G^{a \alpha_i (a+1)_j}_{\lambda_a\,\lambda_a+1}\Big)^2\Big] = 0$$

The summation in (3.25) runs over the resonances with given s -channel isospin. Since all resonances with $I_s = I$ and $I-1$ give a positive contribution to (3.25), the contribution of resonances with $I_s = I+1$ is needed to ensure this equation.

It is not difficult to convince oneself that the solution (3.11) and (3.12) satisfy (3.25) thus confirming the agreement of the SSR solution with FESR.

4. Properties of the states $a(I,I,+)$

The analysis of this work points out the possibility of the existence of the whole series of the exotic baryon resonances with $S = I \geq 5/2$. and positive parity *). In the quark model these resonances are exotic, since they consist of three quarks and n quark-antiquark pairs. In our approach the states $a(I,I,+)$ are the direct analogies of nucleon $N(938)$ and $\Delta_{33}(1232)$ -isobar in the world of states with $I \geq 5/2$. It is worth emphasizing that the existence of the baryon states with increasing isospins and spins has been predicted [7] in the old static bootstrap Chew-Low model which is based on the approach different from above considered.

The most probable decay mode of the states $a(I,I,+)$ is the cascade

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*) As pointed in ref.[4], the SSR show only that the imaginary parts of the corresponding amplitudes must be large enough. However the definiteness of the quantum numbers of $a(I,I,+)$ and their factorized coupling to the external states testify to the resonance interpretation of these states.

proces- $a(I,I,+) \to a(I-1,I-1,+) + \pi \to \dots \to N + n\pi$.

At small decay momenta the simple formula for the width of decay $a(I,I,+) \to a(I-1,I-1,+) + \pi$ follows from exps (A.5), (A.7), (A.26) and (3.24)

$$\Gamma_{a \to a-1\,\pi} = \frac{3\,(G^{p\pi^\circ p})^2}{2(M_N)^2\,4\pi}(p_{a-1})^3\,\frac{2S_a-1}{2S_a+1}\left(1+O(p_{a-1})^2\right) \qquad (4.1)$$

Using (4.1) one can express the width of $a \to a-1\,\pi$ in terms of $\Gamma_{\Delta \to N\pi}$

$$\frac{\Gamma_{a \to a-1\pi}}{\Gamma_{\Delta \to N\pi}} = 2\,\frac{2S_a-1}{2S_a+1}\;\frac{(p_{a-1})^3}{p_N^3} \qquad (4.2)$$

The experimental search for the states $a(I,I,+)$ seems most favourable in the backward scattering processes [4]. For example, in order to observe the resonance $a^{m=I}(I,I,+)$ in $\pi^+ p$ -scattering the fast system, which consists of proton and $(I-1/2)\,\pi^+$ mesons and the system of slow $(I-3/2)\pi^-$ -mesons should be registered in the laboratory frame. In this case the $a(I,I,+)$ resonance is produced via the exchange of Regge trajectory which corresponds to $a(I-1,I-1,+)$. The investigation of the energy and momentum transfer dependence of the backward production cross sections will allow one to determine the position of $a(I,I,+)$ in j-plane.

The diagrams which correspond to the production of $a(I,I,+)$ in the $\pi^\pm p$ -backward scattering are represented in fig. 1a, b.

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APPENDIX: Decay $b \to a + \pi$

In this section we will obtain a formula which expresses the width of $b \to a + \pi$ (b and a are the particles with arbitrary half-integer spins) through the invariant amplitudes.

The invariant expression for the full amplitude of the decay has the form

$$A = \bar{\Psi}^{(S_a)t}_{\mu_1 \cdots \mu_{j_a}}(p_a, \lambda_a)\, \tilde{O}_{ts}\, \Psi^{(S_b)\nu_1 \cdots \nu_{j_b}}_{s}(p_b, \lambda_b)\, A^{\mu_1 \cdots \mu_{j_a}}_{\nu_1 \cdots \nu_{j_b}} \tag{A.1}$$

In (A.1) $\Psi^{(S_i)r}_{\sigma_1 \ldots \sigma_{j_i}}(p_i, \lambda_i)$ is the wave function (w.f.) which describes a particle i with momentum p_i , spin $S_i = j_i + 1/2$ and helicity λ_i (σ_κ (r) is the tensor (bispinor) index) in the Rarita-Schwinger formalism. It is completely symmetric over the tensor indices σ_κ and has the following properties

$$\Psi^{(S_i)r}_{\sigma_1 \ldots \sigma_\kappa \ldots \sigma_{j_i}}(p_i, \lambda_i)\, p_i^{\sigma_\kappa} = 0 \tag{A.2a}$$

$$\Psi^{(S_i)r}_{\sigma_1 \ldots \sigma_\kappa \ldots \sigma_{j_i}}(p_i, \lambda_i)\, \gamma^{\sigma_\kappa}_{rs} = 0 \tag{A.2b}$$

where γ^{σ_κ} is Dirac γ -matrix and is normalized as

$$\begin{aligned} \bar{\Psi}^{(S_i)r}_{\sigma_1 \ldots \sigma_{j_i}}(p_i, \lambda_i)\, \Psi^{(S_i)\sigma_1 \ldots \sigma_{j_i}}_{r}(p_i, \lambda_i') &= \\ = \delta_{\lambda_i \lambda_i'}\, (-1)^{j_i}\, 2M_i & \end{aligned} \tag{A.2c}$$

$A^{\mu_1 \ldots \mu_{j_a}}_{\nu_1 \ldots \nu_{j_b}}$ is the tensor which is constructed from the metric tensor $g_{\mu\nu}$ and momenta p_b^{μ} , $p_{a\nu}$

$$A^{\mu_1\cdots\mu_{J_a}}_{\nu_1\cdots\nu_{J_b}} = A^{\mu_1\cdots\mu_J}_{\nu_1\cdots\nu_J}\, \varepsilon^{S_a}_{S_b}\,(p_{a\nu}, p_b^{\mu})$$

$$\begin{aligned} A^{\mu_1\cdots\mu_J}_{\nu_1\cdots\nu_J} &= G_1^{a\pi b}(q^2)\, g_{\mu_1\nu_1}\cdots g_{\mu_J\nu_J} + \\ &+ G_2^{a\pi b}(q^2)\, g_{\mu_1\nu_1}\cdots g_{\mu_{J-1}\nu_{J-1}}\, p_{a\nu_J} p_b^{\mu_J} + \cdots + \\ &+ G_{J+1}^{a\pi b}(q^2)\, p_{a\nu_1}\cdots p_{a\nu_J}\, p_b^{\mu_1}\cdots p_b^{\mu_J} \end{aligned} \qquad \text{(A.3)}$$

$$(\, q^2 = (p_a - p_b)^2 \,;\ J = \min(J_a, J_b))$$

$$\varepsilon^{S_a}_{S_b}(p_{a\nu}, p_b^{\mu}) = \begin{cases} p_{a\nu_{J+1}}\cdots p_{a\nu_{J_b}} & \text{if} \quad S_b > S_a \\ p_b^{\mu_{J+1}}\cdots p_b^{\mu_{J_a}} & \text{if} \quad S_b < S_a \\ 1 & \text{if} \quad S_b = S_a \end{cases}$$

In (A.1) $\tilde{O} = I\ (\gamma_5)$ if the product $\eta_a(-1)^{S_a-1/2}\,\eta_b(-1)^{S_b-1/2} =$ $= -1\ (+1)$.

The explicit form of w.f. $\Psi^{(S_i)r}_{\sigma_1\cdots\sigma_{J_i}}(p_i, \lambda_i)$ is[8]

$$\Psi^{(S_i)r}_{\sigma_1\cdots\sigma_{J_i}}(p_i, \lambda_i) = \left[\frac{2^{J_i}(J_i+m_i+1)!\,(J_i-m_i)!}{(2J_i+1)!}\right]^{1/2} \times$$

$$\times \sum_{\beta_1\cdots\beta_J} \Big\{ \left[(1+\beta_1)!\,(1-\beta_1)!\cdots(1+\beta_{J_i})!\,(1-\beta_{J_i})!\right]^{-1/2} \times \qquad \text{(A.4)}$$

$$\times\ \xi_{\sigma_1}(p_i,\beta_1)\ldots\xi_{\sigma_{j_i}}(p_i,\beta_{j_i})\, u^r\Big(p_i,\lambda_i-\sum_{k=1}^{j_i}\beta_k\Big)\Big\}$$

where $m_i=\lambda_i-1/2$; $\xi(p_i,\beta_k)$ is the w.f. of the particle with unity spin and helicity β_k ; $u^r(p_i,\gamma)$ is the bispinor (γ is the helicity).

The width of the decay $b\to a+\pi$ has the form

$$\Gamma_{b\to a\pi}=\frac{|\vec{p}_a|}{8\pi(2S_b+1)M_b^2}\sum_{\lambda_a,\lambda_b}|\langle\lambda_a|A|\lambda_b\rangle|^2 \qquad (A.5)$$

In order to calculate the $\sum_{\lambda_a,\lambda_b}|\langle\lambda_a|A|\lambda_b\rangle|^2$, i.e. express $\Gamma_{b\to a\pi}$ through the invariant functions $G_i^{a\pi b}(q^2)$ it is easier to consider first the helicity amplitudes of the decay. The $b\to a+\pi$ decay helicity amplitude has the form[9]

$$\langle\lambda_a|A|\lambda_b\rangle=e^{i\varphi(\lambda_b-\lambda_a)}d^{S_b}_{\lambda_b\lambda_a}(\theta)\,a_{S_b}(\lambda_a) \qquad (A.6)$$

ince

$$\sum_{\lambda_b=-S_b}^{S_b}d^{S_b}_{\lambda_b\lambda_a}(\theta)\,d^{S_b}_{\lambda_b\lambda_a'}(\theta)=\delta_{\lambda_a\lambda_a'}$$

then

$$\sum_{\lambda_a,\lambda_b}|\langle\lambda_a|A|\lambda_b\rangle|^2=\sum_{\lambda_a=-S_a}^{S_a}|a_{S_b}(\lambda_a)|^2= \qquad (A.7)$$

$$=2\sum_{\lambda_a=+1/2}^{S_a}|a_{S_b}(\lambda_a)|^2$$

The latter equality in (A.7) is the consequence of the symmetry property of $a_{s_b}(\lambda_a)$ [9]

$$|a_{s_b}(\lambda_a)| = |a_{s_b}(-\lambda_a)| \tag{A.8}$$

In order to find the relations between $a_{s_b}(\lambda_a)$ and $G_i^{a\pi b}(q^2)$, project the invariant form (A.1) on the helicity states. Let us first write down the projections of some invariant expressions

$$\bar{u}(p_a, 1/2)\, u(0, 1/2) = N \cos\theta/2 \tag{A.9a}$$

$$\bar{u}(p_a, -1/2)\, u(0, 1/2) = -N \sin\theta/2\, e^{i\varphi} \tag{A.9b}$$

$$\bar{u}(p_a, 1/2)\, \gamma_5\, u(0, 1/2) = N' \cos\theta/2 \tag{A.10a}$$

$$\bar{u}(p_a, -1/2)\, \gamma_5\, u(0, 1/2) = N' \sin\theta/2\, e^{i\varphi} \tag{A.10b}$$

In exps (A.9)-(A.10) $N = \sqrt{2M_b(E_a+M_a)}$; $N' = \sqrt{2M_b(E_a-M_a)}$

$$\xi^*(p_a, 0)\, \xi(0, 1) = \frac{E_a}{\sqrt{2}\, M_a} \sin\theta\, e^{i\varphi} \tag{A.11a}$$

$$\xi^*(p_a, +1)\, \xi(0, 1) = -\cos^2\theta/2 \tag{A.11b}$$

$$\xi^*(p_a, -1)\, \xi(0, 1) = -\sin^2\theta/2\, e^{2i\varphi} \tag{A.11c}$$

$$\xi^*(p_a, \lambda)\, p_b = \delta_{0\lambda} \frac{|\vec{p}_a|\, M_b}{M_a} \tag{A.12a}$$

$$p_a \xi(0,1) = \frac{1}{\sqrt{2}} |\vec{p}_a| \sin\theta \, e^{i\varphi} \qquad \text{(A.12b)}$$

Let us introduce the tensors $D^{L,\ell}_{\mu_1 \dots \mu_L}$ and $F^{\mu_1 \dots \mu_L}$

$$D^{L,\ell}_{\mu_1 \dots \mu_L} = \sum_{\substack{\beta_1 \dots \beta_L \\ \Sigma \beta_i = \ell}} \left[(1+\beta_1)!(1-\beta_1)! \dots (1+\beta_L)!(1-\beta_L)! \right]^{-1/2} \times$$
$$\times \xi^*_{\mu_1}(p_a, \beta_1) \dots \xi^*_{\mu_L}(p_a, \beta_L) \qquad \text{(A.13)}$$

$$F^{\mu_1 \dots \mu_L} = \sum_{i=1}^{L+1} G_i^{a\pi b}(q^2) \, \xi^{\mu_1}(0,1) \dots \xi^{\mu_{L-i+1}}(0,1) \times$$
$$\times p_b^{\mu_{L-i+2}} \dots p_b^{\mu_L} \left(p_a \xi(0,1) \right)^{i-1} \qquad \text{(A.14)}$$

Then

$$\overline{\Psi}^{(S_a)}_{\mu_1 \dots \mu_{J_a}}(p_a, \lambda_a) = C(J_a, m_a) \Big\{ D^{J_a, m_a}_{\mu_1 \dots \mu_{J_a}} \overline{u}(p_a, 1/2) +$$
$$+ D^{J_a, m_a+1}_{\mu_1 \dots \mu_{J_a}} \overline{u}(p_a, -1/2) \Big\} \qquad \text{(A.15)}$$

where

$$C(J_a, m_a) = \left[\frac{2^{J_a} (J_a + m_a + 1)! \, (J_a - m_a)!}{(2J_a + 1)!} \right]^{1/2}$$

As (see (A.4))

$$\psi^{(S_b)\nu_1 \dots \nu_{J_b}}(0, \lambda_b = S_b) = \xi^{\nu_1}(0,1) \dots \xi^{\nu_{J_b}}(0,1) \, u(0, 1/2) \qquad \text{(A.16)}$$

then it follows from eqs (A.1), (A.4), (A.12a), (A.14)-(A.16) that

$$\langle \lambda_a | A | \lambda_b \rangle = \chi^n C(J_a, m_a) \Big\{ D^{J,m_a}_{\mu_1 \dots \mu_J} \bar{u}(p_a, 1/2) + \qquad \text{(A.17)}$$

$$+ D^{J,m_a+1}_{\mu_1 \dots \mu_J} \bar{u}(p_a, -1/2) \Big\} F^{\mu_1 \dots \mu_J} \,\tilde{O}\, u(0, 1/2)$$

where

$$\chi = \begin{cases} p_a\, \xi(0,1) & \text{if} \quad S_b > S_a \\ p_b\, \xi^{*}(p_a, 0) & \text{if} \quad S_b < S_a \end{cases} \qquad \text{(A.18)}$$

and $n = | S_b - S_a |$

Let us consider the contraction

$$D^{L,\ell}_{\mu_1 \dots \mu_L} \xi^{\mu_1}(0,1) \dots \xi^{\mu_L}(0,1) \equiv H^{L,\ell} \qquad \text{(A.19)}$$

One can show (taking into account the exps (A.11)-(A.13) that

$$H^{L,\ell} = 2^{-L/2} (-1)^{\ell} L! (\cos\theta/2)^{L+\ell} (\sin\theta/2)^{L-\ell} \times$$

$$\times\, e^{i\varphi(L-\ell)} \sum_{\gamma=0}^{d} \frac{x^{L-\ell-2\gamma}}{(\ell+\gamma)!\,(L-\ell-2\gamma)!\,\gamma!} \qquad \text{(A.20)}$$

where

$$d = \begin{cases} (L-\ell)/2 & \text{if} \quad L-\ell \quad \text{is even} \\ (L-\ell-1)/2 & \text{if} \quad L-\ell \quad \text{is odd} \end{cases}$$

$x = 2E_a/M_a$.

Let us consider now the expression

$$\left\{ D^{L,\ell}_{\mu_1\ldots\mu_L}\bar{u}(p_a,1/2) + D^{L,\ell+1}_{\mu_1\ldots\mu_L}\bar{u}(p_a,-1/2)\right\}\times$$

$$\times\,\xi^{\mu_1}(0,1)\ldots\xi^{\mu_L}(0,1)\begin{pmatrix}I\\ \gamma_5\end{pmatrix}u(0,1/2) = H^{L,\ell}\,\bar{u}(p_a,1/2)\begin{pmatrix}I\\ \gamma_5\end{pmatrix}\times \qquad \text{(A.21)}$$

$$\times\, u(0,1/2) + H^{L,\ell+1}\,\bar{u}(p_a,-1/2)\begin{pmatrix}I\\ \gamma_5\end{pmatrix}u(0,1/2) \equiv K^{L,\ell}\begin{pmatrix}I\\ \gamma_5\end{pmatrix}$$

Taking into account the formula for $H^{L,\ell}$ (A.20) and (A.9), (A.10) it is easy to show that

$$K^{L,\ell}\begin{pmatrix}I\\ \gamma_5\end{pmatrix} = M^{L,\ell}\begin{pmatrix}I\\ \gamma_5\end{pmatrix}\Phi^{L,\ell}\begin{pmatrix}I\\ \gamma_5\end{pmatrix} \qquad \text{(A.22)}$$

with

$$M^{L,\ell}\begin{pmatrix}I\\ \gamma_5\end{pmatrix} = 2^{-L/2}(-1)^{\ell}(\cos\theta/2)^{L+\ell+1}\times \qquad \text{(A.23)}$$

$$\times\,(\sin\theta/2)^{L-\ell}\,e^{i\varphi(L-\ell)}\,L!\begin{pmatrix}N\\ N'\end{pmatrix}$$

and

$$\Phi^{L,\ell}\begin{pmatrix}I\\ \gamma_5\end{pmatrix} = \frac{1}{\left(\frac{L+\ell}{2}\right)!\left(\frac{L-\ell}{2}\right)!} + \sum_{\gamma=0}^{\frac{L-\ell-2}{2}}\frac{1}{(L-\ell-2\gamma-1)!}\times \qquad \text{(A.24)}$$

$$\times\,\frac{x^{L-\ell-2\gamma-1}}{(\ell+\gamma)!\,\gamma!}\left(\frac{x}{L-\ell-2\gamma} \pm \frac{1}{\ell+\gamma+1}\right) \equiv \Phi^{L,\ell}_{\text{even}}\begin{pmatrix}I\\ \gamma_5\end{pmatrix}$$

if $L-\ell$ is even, and

$$\Phi^{L,\ell}\binom{I}{\gamma_5} = \sum_{\gamma=0}^{\frac{L-\ell-1}{2}} \frac{X^{L-\ell-2\gamma-1}}{(\ell+\gamma)!\,(L-\ell-2\gamma-1)!\;\gamma!} \times \tag{A.25}$$

$$\times\left(\frac{X}{L-\ell-2\gamma} \pm \frac{1}{\ell+\gamma+1}\right) \equiv \Phi^{L,\ell}_{\text{odd}}\binom{I}{\gamma_5}$$

at odd $L-\ell$.

The plus (minus) sign in (A.24) and (A.25) and the choice of $N\,(N')$ in (A.23) correspond to $\hat{O} = I\;(\gamma_5)$

Since

$$d^{S_b}_{S_b\lambda_a}(\theta) = (-1)^{J_b-m_a}\left[\frac{(2J_b+1)!}{(J_b-m_a)!\,(J_b+m_a+1)!}\right]^{1/2}\times$$

$$\times\,(\cos\theta/2)^{J_b+m_a+1}\,(\sin\theta/2)^{J_b-m_a}$$

then, finally

$$a_{S_b}(\lambda_a)\binom{I}{\gamma_5} = 2^{n/2}(-1)^{J_b}\,|\vec{p}_a|^n\,\mathcal{C}\binom{N}{N'}\times$$

$$\times\left[\frac{(J_a+m_a+1)!\,(J_a-m_a)!\,(J_b+m_a+1)!\,(J_b-m_a)!}{(2J_a+1)!\,(2J_b+1)!}\right]^{1/2}\times \tag{A.26}$$

$$\times\left\{\sum_{i=1}^{J+1} G_i^{a\pi b}(q^2)\,(J-i+1)!\left(\frac{2p_a^2M_b}{M_b}\right)^{i-1}\Phi^{J-i+1,m_a}\binom{I}{\gamma_5}\right\}$$

In the formula (A.26) $n = |S_b - S_a|$

$$\zeta = \begin{cases} 1 & \text{if} \quad S_b \geq S_a \\ (M_b/M_a)^n & \text{if} \quad S_b < S_a \end{cases}$$

Let us remind that $\gamma = \min(\gamma_b, \gamma_a)$

Let us write down some values of $\Phi^{L,\ell}\left(\frac{I}{\gamma_s}\right)$

$$\Phi^{L,\ell=L}\left(\frac{I}{\gamma_5}\right) = \frac{1}{L!} \; ; \; \Phi^{1,0}\left(\frac{I}{\gamma_5}\right) = \pm 1 + X \; ;$$

$$\Phi^{2,0}\left(\frac{I}{\gamma_5}\right) = 1 \pm X + \frac{1}{2}X^2 \; ; \; \Phi^{2,1}\left(\frac{I}{\gamma_5}\right) = \pm\frac{1}{2} + X \; ;$$

$$\Phi^{3,0}\left(\frac{I}{\gamma_5}\right) = \pm\frac{1}{2} + X \pm \frac{1}{2}X^2 + \frac{1}{6}X^3 \; ;$$

$$\Phi^{3,1}\left(\frac{I}{\gamma_5}\right) = \frac{1}{2}(1 \pm X + X^2) \; ; \; \Phi^{3,2}\left(\frac{I}{\gamma_5}\right) = \frac{1}{2}\left(\pm\frac{1}{3} + X\right) ;$$

$$\Phi^{4,0}\left(\frac{I}{\gamma_5}\right) = \frac{1}{4} \pm \frac{1}{2}X + \frac{1}{2}X^2 \pm \frac{1}{6}X^3 + \frac{1}{24}X^4 \; ;$$

$$\Phi^{4,1}\left(\frac{I}{\gamma_5}\right) = \pm\frac{1}{6} + \frac{1}{2}X \pm \frac{1}{4}X^2 + \frac{1}{6}X^3 \; ;$$

$$\Phi^{4,2}\left(\frac{I}{\gamma_5}\right) = \frac{1}{6} \pm \frac{1}{6}X + \frac{1}{4}X^2 \; ; \; \Phi^{4,3}\left(\frac{I}{\gamma_5}\right) = \frac{1}{6}\left(\pm\frac{1}{4} + X\right)$$

Using these values one can write down the expressions for the decay widths for several particular cases

$a(3/2,3/2,+) \to a(1/2,1/2,+) + \pi \; (\Delta_{33} \to N\pi)$

$$\Gamma_{\Delta \to N\pi} = \frac{p_N^3 (E_N + M_N)}{12\pi M_\Delta} \left(G^{N\pi\Delta}\right)^2 \qquad \text{(A.27)}$$

$a(5/2,5/2,+) \to a(3/2,3/2,+) + \pi \; (E_{55} \to \Delta_{33}\pi)$

$$\Gamma_{E_{55} \to \Delta\pi} = \frac{p_\Delta^3 (E_\Delta + M_\Delta)}{180\pi M_{E_{55}}} \left\{ 6 \left(G_1^{\Delta\pi E_{55}}\right)^2 + \right. \qquad \text{(A.28)}$$

$$\left. + \left[\left(1 + \frac{2E_\Delta}{M_\Delta}\right) G_1^{\Delta\pi E_{55}} + 2 \frac{M_{E_{55}} p_\Delta^2}{M_\Delta} G_2^{\Delta\pi E_{55}} \right]^2 \right\}$$

$a(7/2,7/2,+) \to a(5/2,5/2,+) + \pi \; (E_{77} \to E_{55}\pi)$

$$\Gamma_{E_{77} \to E_{55}\pi} = \frac{(p_{E_{55}})^3 (E_{E_{55}} + M_{E_{55}})}{8400\pi M_{E_{77}}} \left\{ 150 \times \right.$$

$$\times \left(G_1^{E_{55}\pi E_{77}}\right)^2 + 10 \left[G_1^{E_{55}\pi E_{77}} (1 + 2x) + \right.$$

$$\left. + y G_2^{E_{55}\pi E_{77}} \right]^2 + 3 \left[G_1^{E_{55}\pi E_{77}} (2 + 2x + x^2) + \right. \qquad \text{(A.29)}$$

$$\left. \left. + y G_2^{E_{55}\pi E_{77}} (1 + x) + y^2 G_3^{E_{55}\pi E_{77}} \right]^2 \right\}$$

In (A.29) $x = 2E_{E_{55}}/M_{E_{55}}$; $y = 2M_{E_{77}} p_{E_{55}}/M_{E_{55}}$.

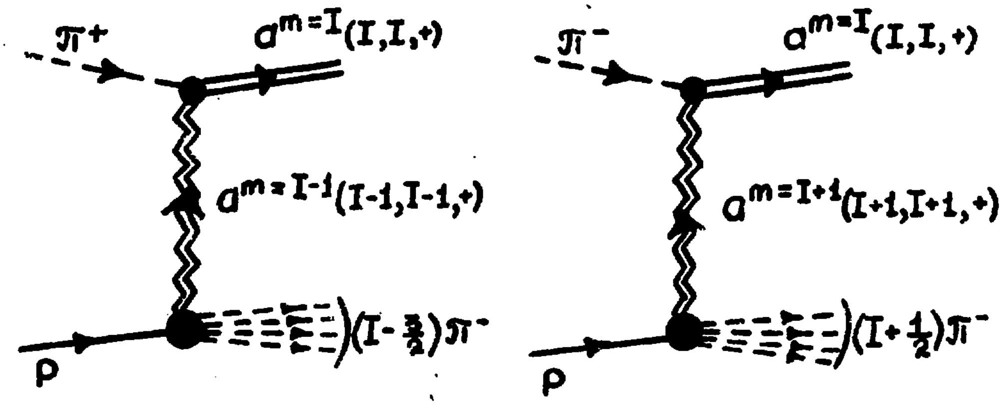
Table 1

Values of $6j$ -symbols[6]

$\begin{Bmatrix}1 & 1 & 2\\ I_a & I_b & I_s\end{Bmatrix}$	$\begin{Bmatrix}1 & 1 & 1\\ I_a & I_b & I_s\end{Bmatrix}$	$\begin{Bmatrix}1 & 1 & 0\\ I_a & I_b & I_s\end{Bmatrix}$
$\begin{Bmatrix}1 & 1 & 2\\ I & I & I\end{Bmatrix} = \sqrt{\frac{(2I+3)(2I-1)}{5}}\,K$	$\begin{Bmatrix}1 & 1 & 1\\ I & I & I-1\end{Bmatrix} = -(I+1)K$	$-\begin{Bmatrix}1 & 1 & 0\\ I & I & I\end{Bmatrix} =$
$\begin{Bmatrix}1 & 1 & 2\\ I & I & I-1\end{Bmatrix} = (I+1)\sqrt{\frac{2I+3}{5(2I-1)}}\,K$	$\begin{Bmatrix}1 & 1 & 1\\ I & I & I+1\end{Bmatrix} = I\,K$	$= \begin{Bmatrix}1 & 1 & 0\\ I & I & I+1\end{Bmatrix} =$
$\begin{Bmatrix}1 & 1 & 2\\ I & I & I+1\end{Bmatrix} = I\sqrt{\frac{2I-1}{5(2I+3)}}\,K$		$= \begin{Bmatrix}1 & 1 & 0\\ I & I & I-1\end{Bmatrix} =$
$\begin{Bmatrix}1 & 1 & 2\\ I & I+1 & I\end{Bmatrix} = -\sqrt{\frac{3I(I+2)}{5}}\,K$	$\begin{Bmatrix}1 & 1 & 1\\ I & I+1 & I+1\end{Bmatrix} = -\sqrt{\frac{I(2I+1)(I+2)}{3+2I}}\,K$	$= \sqrt{2I(I+1)}\,K$
$\begin{Bmatrix}1 & 1 & 2\\ I & I+1 & I+1\end{Bmatrix} = -I\sqrt{\frac{3(2I+1)}{5(3+2I)}}\,K$	$K = \begin{Bmatrix}1 & 1 & 1\\ I & I & I\end{Bmatrix}$	

Equations for π -meson helicity residues with $\Delta\lambda = \pm 1$ Table 2

Amplitudes	EQUATIONS	I_t
	$\alpha_\pi a \rightarrow \alpha_\pi a$ -scattering	
$\lambda \rightarrow \lambda$	$\left(G^{a\alpha_\pi a-1}_{\lambda\,\lambda-1}\right)^2-\left(G^{a\alpha_\pi a-1}_{\lambda\,\lambda+1}\right)^2-\frac{2I-1}{I+1}\left[\left(G^{a\alpha_\pi a}_{\lambda\,\lambda-1}\right)^2-\left(G^{a\alpha_\pi a}_{\lambda\,\lambda+1}\right)^2\right]+\frac{I(2I-1)}{(I+1)(2I+3)}\left[\left(G^{a\alpha_\pi(a+1)_x}_{\lambda\,\lambda-1}\right)^2-\left(G^{a\alpha_\pi(a+1)_x}_{\lambda\,\lambda+1}\right)^2\right]=0$	2
	$\left(G^{a\alpha_\pi a-1}_{\lambda\,\lambda-1}\right)^2-\left(G^{a\alpha_\pi a-1}_{\lambda\,\lambda+1}\right)^2+\left(G^{a\alpha_\pi a}_{\lambda\,\lambda-1}\right)^2-\left(G^{a\alpha_\pi a}_{\lambda\,\lambda+1}\right)^2+\left(G^{a\alpha_\pi(a+1)_x}_{\lambda\,\lambda-1}\right)^2-\left(G^{a\alpha_\pi(a+1)_x}_{\lambda\,\lambda+1}\right)^2=0$	0
$\lambda-1 \rightarrow$ $\rightarrow \lambda+1$	$G^{a\alpha_\pi a-1}_{\lambda-1\,\lambda}G^{a-1\,\alpha_\pi a}_{\lambda\,\lambda+1}+\frac{1}{I+1}G^{a\alpha_\pi a}_{\lambda-1\,\lambda}G^{a\alpha_\pi a}_{\lambda\,\lambda+1}-\frac{I}{I+1}G^{a\alpha_\pi(a+1)_x}_{\lambda-1\,\lambda}G^{(a+1)_x\alpha_\pi a}_{\lambda\,\lambda+1}=0$	1
	$\alpha_\pi a-1 \rightarrow \alpha_\pi (a+1)_x$ -scattering	
$\lambda \rightarrow \lambda$	$G^{a-1\,\alpha_\pi a}_{\lambda\,\lambda-1}G^{a\alpha_\pi(a+1)_x}_{\lambda-1\,\lambda}-G^{a-1\,\alpha_\pi a}_{\lambda\,\lambda+1}G^{a\alpha_\pi(a+1)_x}_{\lambda+1\,\lambda}=0$	2
	$\alpha_\pi a \rightarrow \alpha_\pi (a+1)_x$ -scattering	
$\lambda\pm1 \rightarrow$ $\rightarrow \lambda\mp1$	$G^{a\alpha_\pi a}_{\lambda\pm1\,\lambda}G^{a\alpha_\pi(a+1)_x}_{\lambda\,\lambda\mp1}+\sqrt{\frac{(2I+1)(I+2)}{(3+2I)}}\,G^{a\alpha_\pi(a+1)_x}_{\lambda\pm1\,\lambda}G^{(a+1)_x\alpha_\pi(a+1)_x}_{\lambda\,\lambda\mp1}=0$	1
$\lambda \rightarrow \lambda$	$G^{a\alpha_\pi a}_{\lambda\,\lambda-1}G^{a\alpha_\pi(a+1)_x}_{\lambda-1\,\lambda}-G^{a\alpha_\pi a}_{\lambda\,\lambda+1}G^{a\alpha_\pi(a+1)_x}_{\lambda+1\,\lambda}-\frac{(2I+1)I}{(3+2I)(I+2)}\Big[G^{a\alpha_\pi(a+1)_x}_{\lambda\,\lambda-1}\times G^{(a+1)_x\alpha_\pi(a+1)_x}_{\lambda-1\,\lambda}-G^{a\alpha_\pi(a+1)_x}_{\lambda\,\lambda+1}G^{(a+1)_x\alpha_\pi(a+1)_x}_{\lambda+1\,\lambda}\Big]=0$	2



a) b)

Fig.1. Production of the states $a^{m=I}(I,I,+)$: a) in π^+p - and b) in π^-p-backward scattering.

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(на английском языке)

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