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VACUUM POLARIZATION BY INVARIANT SOUPCES

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ПОЛЯРИЗАЦИЯ ВАКУУМА ИНВАРИАНТНЫМИ ИСТОЧНИКАМИ

Изучаётся поляризация вакуума инвариантными источниками, генерирующими вакуумное ожидание классического действия S .

Ереванский физический институт
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N.S.ANANIKIAN, G.K.SAVVIDY

VACUUM POLARIZATION BY INVARIANT SOURCES

A vacuum polarization by invariant sources generating the vacuum expectation of classical action S has been studying.

Yerevan Physics Institute

Yerevan 1981

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1. Introduction

Many phenomena in the microworld are apparently due to nontrivial structure of vacuum of gauge fields, which may be, in principle, in different phase states [1].

One may kinematically distinguish these states, when knowing the behaviour of different vacuum averages [2, 3], however, there is not any theoretical regular methods as to dynamical determination of phase states which realize at different values of external parameters, such as, for example, the coupling constant g.

As was shown by Mandelstam [3], in non-abelian gauge theories the finite matrix elements between physical states can be obtained considering only the gauge-invariant operators, such as the local operator

$$\langle 0 | G^a_{\mu\nu}(x)\, G^a_{\mu\nu}(x) | 0 \rangle \tag{1.1}$$

or non-local Wilson operator [1]

$$W(C) = \langle 0 | \mathrm{tr}\, P\exp\left\{ ig \oint_C A_\mu\, dx_\mu \right. | 0 \rangle\,. \tag{1.2}$$

whose two variations at $C \to 0$ turn into (1.1).

As far as the variational approach [4] is the most consistent one to find anomalous solutions corresponding to different phases, we suggest to formulate the variational problem for finding out matrix elements (1.1) and (1.2). For that, it is necessary to build up the effective potential Γ dependent on these matrix elements which can be calculated due to the stationarity condition.

In our previous works [5, 6] we considered potential Γ as dependent on n-point Green functions $\varphi(x)$, $G(x,y)$ .... and quantity $S = \langle 0 | S_{cl} | 0 \rangle$ which in gauge theories is proportional to (1.1). In this work we have been investigating potential $\Gamma(\varphi, S)$ dependent on only $\varphi(x)$ and S which is similar to entropy in statistical physics. We have shown that in one-loop approximation

$$S_{vac} = S(\{\varphi\}_{vac}) \tag{1.3}$$

where $\{\varphi\}_{vac}$ is the set of solutions of the equation

$$\frac{\delta \Gamma}{\delta \varphi} = 0 \tag{1.4}$$

The availability of the non-trivial set of solutions of eq.(1.4) in non-abelian gauge theories was shown in refs. [7, 8, 9, 10].

In sec.2 of this paper the variational problem is formulated and the set of equations determining $\Gamma(\varphi, S)$ is written out. In sec.3 the perturbation theory for Γ is built up, and in sec.4 the questions are discussed being concerned with renormalization of Γ , and the relation (1.3) is obtained.

2. Effective Potential $\Gamma(\varphi, S)$

Let us determine the generating functional $Z(\mathcal{J}, L)$ for the theory with the action $S(\varphi)$ as

$$\exp\left\{\frac{i}{\hbar}Z\right\} = N^{-1}\int \mathcal{D}\varphi_i \exp\frac{i}{\hbar}\left\{S(\varphi) + \mathcal{J}_i\varphi_i + L\cdot S(\varphi)\right\} \tag{2.1}$$

where N is the normalization factor *).

As in [11], we will use the condensed indices i , j , K ... , unifying discrete indices of the Lorentz group and of the internal symmetry group, as well as the space-time continuous coordinates. In (2.1) $\mathcal{J}_i$ is the source of field φ , and the term $\sim L\cdot S(\varphi)$ was introduced in our previous works [5, 6] and is connected with the vacuum polarization by the Lorentz-invariant source**). A diagram technique for calculating $Z(\mathcal{J}, L)$ is given in Appendix A.

Functional $Z(\mathcal{J}, L)$ depends on two independent sources $\mathcal{J}$ and L . In order to construct potential $\Gamma(\varphi, S)$ as dependent on conjugated variables φ and S

$$\varphi_i \equiv \langle \varphi_i \rangle = Z_{\mathcal{J}i}, \qquad S \equiv \langle S(\varphi) \rangle = Z_L \tag{2.2}$$

let us make the second-order Legendre transformation

$$\Gamma(\varphi, S) = Z - \mathcal{J}_i\varphi_i - L\cdot S \tag{2.3}$$

where it is necessary to express $\mathcal{J}$ and L through φ and S using (2.2). This Legendre transformation can be considered as the coupling·

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*) In this case $N = \int \mathcal{D}\varphi_i \exp\frac{i}{\hbar}\left\{S(\varphi)\right\}$. Normalizations as different from this one will be considered in sec.4.

**) Such a source posesses all symmetries of initial action $S(\varphi)$.

constant transformation[6].

Differentiating (2.3) with respect to φ and S we obtain

$$\Gamma_{\varphi_i} = - J_i \quad , \qquad \Gamma_S = - L \tag{2.4}$$

In the absence of the external sources J and L (in the ground state) eqs.(2.4) $\Gamma_\varphi = 0$ and $\Gamma_S = 0$ determine vacuum expectations φ_{vac} and S_{vac}. Just in this consists the formulation of the variational problem, whose advantage is in the possibility to find anomalous solutions for φ_{vac} and S_{vac} .

It seems that the vacuum expectations are unambiguously determined by the action functional, for example, by

$$S_{vac} = N^{-1} \int \mathcal{D}\varphi_i \, S(\varphi) \, e^{\frac{i}{\hbar} S(\varphi)} = \tag{2.5}$$

$$= \frac{i\hbar}{2} \operatorname{tr} 1 - \frac{\hbar^2}{8}$$  $$+ \cdots ,$$

however in this way only normal solutions can be found. The method's essence consists in constructing the functional Γ , whose stationarity condition brings to the Eulerian equations coinciding with the Schwinger initial equations of motion. The presence of anomalous solutions means non-uniqueness of the stationarity point, and to the spontaneous violation of symmetry corresponds non-invariant stationarity point[4]. From the calculational viewpoint this method's advantage is that Γ can be constructed by the perturbation theory losing no opportunity as to finding anomalous solutions.*).

The quantity of the energy shift between the total ($g \neq 0$) and free

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*) As example, there may serve a polynomial whose stationarity points can depend non-analytically on his parameters.

theories is determined by the formula

$$-\mathcal{E}(g)\int dt = \Gamma\Big|_{\substack{\varphi_{vac}\\ S_{vac}}} - \Gamma_{free}\Big|_{g=0} \tag{2.6}$$

The motion equations for $Z(\mathcal{J}, L)$ can be derived using the measure invariance $\mathcal{D}\varphi$ relative to the translations in (2.1) (Schwinger equation), as well as the coupling between Z_L derivative and $Z_{\mathcal{J}_i\mathcal{J}_j}$, $Z_{\mathcal{J}_i\mathcal{J}_K\mathcal{J}_e}\ldots$ ones, as following from (2.1)[5, 6]. Using (2.2), (2.4) and relations between Z derivatives and Γ derivatives one can obtain the final set of equations for Γ. These equations depend on the considered $S(\varphi)$ theory, therefore we will derive them when considering concrete models.

At the same time the connection of Z derivatives with Γ ones is universal for all theories and follows from identities $\delta L/\delta\mathcal{J}_i = 0$ $\delta\mathcal{J}_i/\delta\mathcal{J}_j = 0$ (see [4, 5, 6]). Let us give these relations

$$iG_{ij} \equiv Z_{\mathcal{J}_i\mathcal{J}_j} = Q^{-1}_{ij} \tag{2.7a}$$

$$i^2 H_{ijK} \equiv Z_{\mathcal{J}_i\mathcal{J}_j\mathcal{J}_K} = \left\{Q^{-1}_{ij,\varphi_m} - Q^{-1}_{ij,s}\Gamma^{-1}_{ss}\Gamma_{s\varphi_m}\right\}Q^{-1}_{mK} \tag{2.7b}$$

$$i^3 \mathcal{U}_{ijKe} \equiv Z_{\mathcal{J}_i\mathcal{J}_j\mathcal{J}_K\mathcal{J}_e} = \left\{i^2 H_{ijK,\varphi_m} - i^2 H_{ijK,s}\Gamma^{-1}_{ss}\Gamma_{s\varphi_m}\right\}Q^{-1}_{me} \tag{2.7c}$$

where $$Q_{ij} = \Gamma_{\varphi_i s}\Gamma^{-1}_{ss}\Gamma_{s\varphi_j} - \Gamma_{\varphi_i\varphi_j} . \tag{2.7d}$$

and G_{ij}; H_{ijK}; H_{ijKe} are two-, three- and four-point Green functions in "medium" with vacuum averages φ and S .

Potential Γ , actually, is proportional to the quantity

$$e^{\frac{i}{\hbar}\Gamma(S)} \sim \int \mathcal{D}\varphi_i \; e^{\frac{i}{\hbar}S(\varphi)}\delta(S - S(\varphi)) = \int_{-\infty}^{+\infty} e^{iLS} e^{\frac{i}{\hbar}Z(L)} dL \tag{2.8}$$

being the analog of statistical physics entropy which can be determined by means of the Legendre transformation of statistical sum by the reverse temperature $\beta = \frac{1}{KT}$. Therefore potential Γ can be considered as an entropy in the field theory, being proportional to the logarithm of field transition amplitude with fixed action.

3. Perturbation Theory for $\Gamma(\varphi, S)$

Let us now turn to the concrete models. Consider $g\varphi^3$ -theory with the action

$$S(\varphi) = \frac{i}{2}\varphi_x \mathcal{D}^{-1}_{xy} \varphi_y + g\frac{\varphi_x^3}{3!} \tag{3.1}$$

$$i\mathcal{D}^{-1}_{xy} = -(\Box_x + m^2)\delta(x-y)$$

The equation system for $Z(J, L)$ is to be written in the form:

$$(1+L)\left\{i\mathcal{D}^{-1}_{xy} Z_{J_y} + \frac{g}{2}[Z_{J_x} Z_{J_x} - i\hbar Z_{J_x J_x}]\right\} = -J_x \tag{3.2}$$

$$Z_L = i\mathcal{D}^{-1}_{xy}\frac{\hbar}{2i}[Z_{J_x J_y} + \frac{i}{\hbar}Z_{J_x}Z_{J_y}] +$$

$$+ \frac{g}{3!}\left\{\left(\frac{\hbar}{i}\right)^3 Z_{J_x J_x J_x} + 3\frac{\hbar}{i}Z_{J_x J_x}Z_{J_x} + Z_{J_x}Z_{J_x}Z_{J_x}\right\} \tag{3.3}$$

As it was mentioned above, using (2.2), (2.4) and (2.7) we obtain equations for Γ *)

$$\left(1-\Gamma_S\right)\left\{ i\mathcal{D}^{-1}_{xy}\,\varphi_y + \frac{g}{2}\left[\frac{\hbar}{i}\,Q^{-1}_{xx} + \varphi^2_x\right]\right\} = \Gamma_{\varphi_x} \tag{3.4}$$

$$S - S(\varphi) - \frac{\hbar}{2}\,\Delta^{-1}_{xy}\;Q^{-1}_{yx} = \frac{g\hbar^2}{3!}\,H_{xxx} \tag{3.5}$$

where Δ_{xy} is Green function in external field φ

$$i\Delta^{-1}_{xy} = i\,\mathcal{D}^{-1}_{xy} + g\,\varphi_x\;\delta(x-y) \tag{3.6}$$

We will solve the set (3.4-5) by the perturbation theory method, hence present Γ as

$$\Gamma = \Gamma_0 + g\Gamma_1 + g^2\Gamma_2 + \dots \tag{3.7}$$

After simple calculations we obtain (see Appendix B)

(3.8)

$$\Gamma = S + \frac{\hbar}{2i}\,\mathrm{tr}\,1\left[\ln\tilde{S} - \frac{1}{\tilde{S}}\,\frac{g\varphi^3}{3!} - \frac{1}{2}\cdot\frac{1}{\tilde{S}^2}\left(\frac{g\varphi^3}{3!}\right)^2 + \dots\right] +$$

$+\ \frac{g\hbar}{2}$  $+\ \frac{ig^2\hbar}{4}$  $+\ \frac{g^2\hbar\tilde{S}}{3!\,\mathrm{tr}\,1}$  $+ \dots$

In (3.8) there are written out the first three terms of expansion (3.7) and introduced the following notations:

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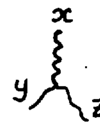
*) Eqs.(3.4-5) can be obtained also from the functional equations for $\Gamma(\varphi, G, S)$ [5] with the use of condition $\Gamma_G = 0$.

$$\text{}$$ $= \varphi_x \;,\quad$  $= \mathcal{D}_{xy} \;,\quad$  $= \delta(x-y)\,\delta(x-z)$ (3.9)

$$\widetilde{S} = S - \frac{i}{2}\,\varphi_x\,\mathcal{D}^{-1}_{xy}\,\varphi_y \;, \qquad \mathrm{tr}\,1 = \int \delta(x-x)\,d^4x \,.$$

In $g\varphi^4$ theory with the action

$$S(\varphi) = \frac{i}{2}\;\varphi_x\,\mathcal{D}^{-1}_{xy}\,\varphi_y + \frac{g\,\varphi_x^4}{4!} \tag{3.10}$$

the equations for $\Gamma(\varphi, S)$ as analogous to (2.4-5) are as follows:

$$(1-\Gamma_S)\Big\{ i\,\mathcal{D}^{-1}_{xy}\,\varphi_y + \frac{g}{3!}\Big[\varphi_x^3 + 3\,\frac{\hbar}{i}\,Q^{-1}_{xx}\,\varphi_x + \hbar^2 H_{xxx}\Big]\Big\} = \Gamma_{\varphi_x} \tag{3.11}$$

$$S - S(\varphi) - \frac{\hbar}{2}\,\Delta^{-1}_{xy}\,Q^{-1}_{yx} - \frac{g\hbar^2}{6}\,H_{xxx}\,\varphi_x + \frac{g\hbar^2}{8}\,Q^{-1}_{xx}\,Q^{-1}_{xx} = \frac{g\hbar^3}{4!}\,\mathcal{M}_{xxxx} \tag{3.12}$$

where

$$i\,\Delta^{-1}_{xy} = i\,\mathcal{D}^{-1}_{xy} + \frac{g}{2}\,\varphi_x^2\,\delta(x-y) \tag{3.13}$$

and expansion (3.7) for Γ has a view (see Appendix B):

$$\Gamma = S + \frac{\hbar}{2i}\,\mathrm{tr}\,1\Big[\ln\widetilde{S} - \frac{1}{\widetilde{S}}\Big(\frac{g\varphi^4}{4!}\Big) - \frac{1}{2}\cdot\frac{1}{\widetilde{S}^2}\Big(\frac{g\varphi^4}{4!}\Big)^2 + \dots\Big] + \tag{3.14}$$

$+ \dfrac{g\hbar}{4}$  $+ \dfrac{ig^2\hbar}{6}$  $+ \dfrac{g^2\hbar}{4}\,\dfrac{\widetilde{S}}{\mathrm{tr}\,1}$  $+ \dfrac{g^2\hbar}{6}\,\dfrac{\widetilde{S}}{\mathrm{tr}\,1}$  $+$

$$+\frac{g\hbar}{4!}\frac{\tilde{S}}{\operatorname{tr} 1}$$  $$-\frac{g\hbar}{4!}\left(\frac{g\varphi^4}{4!}\right)$$  $$+\frac{g^2\hbar}{16i}\frac{\tilde{S}^2}{\operatorname{tr}^2 1}$$  $$^2 +$$

$$+\frac{g^2\hbar\,\tilde{S}^2}{4!\operatorname{tr}^2 1}$$  $$+\frac{g^2\hbar}{2i}\frac{\tilde{S}^2}{\operatorname{tr}^2 1}$$  $$+\cdots$$

Similarly, one can obtain the expansion for Γ in the other theories as well; we will not present them here, note only that, for example, in quantum electrodynamics as in $g\varphi^3$ and $g\varphi^4$ theories there will be vacuum diagrams  along with the diagrams with external lines. All the terms of expansion Γ may be divided into three groups of addenda: dependent on only external lines φ, on only $\tilde{S}$, and mixed ones, dependent on both φ and $\tilde{S}$. Addenda dependent on only $\tilde{S}$ are multiplied by vacuum diagrams which diverge as the fourth power of momentum, and to extract the finite part from them is impossible. Therefore we, unfortunately, cannot determine phase states in which S is nonzero and $\varphi = 0$. *)

The appearance of vacuum diagrams is connected with the determination of the generating functional (2.1) and matrix elements (2.2). A detailed discussion of analogous matrix elements on the example of kinetic energy in quantum mechanics can be found in sec.7 of the book by Feynman and Hibbs [12].

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*) Though, if the theory is superconvergent as, for example, $g\varphi^3$, then in expansion $\Gamma(0,S) = S + \frac{\hbar}{2i}\operatorname{tr} 1 \cdot \ln S + \frac{g^2\hbar}{3!}\frac{S}{\operatorname{tr} 1}$  $+\cdots$ the terms with high powers of S are finite (beginning with S^3) and therefore one can obtain a finite expression for Γ.

In the next section we have considered the means of eliminating these divergences.

4. Renormalization

In order to obtain finite matrix elements let us define the generating functional of the theory so that to exclude from it all the vacuum diagrams. For this purpose, it is sufficient to normalize Z as follows:

$$e^{\frac{i}{\hbar} Z^r(J,L)} = \frac{\int \mathcal{D}\varphi_i \, e^{\frac{i}{\hbar}[S(\varphi) + J_i \varphi_i + L \cdot s(\varphi)]}}{\int \mathcal{D}\varphi_i \, e^{\frac{i}{\hbar}[S(\varphi) + L \cdot s(\varphi)]}} \tag{4.1}$$

whence it is seen that

$$Z^r(J,L) = Z(J,L) - Z(0,L)$$

where $Z(O,L)$ is equal to (see Appendix A)

$$Z(0,L) = -\frac{\hbar}{2i} \, tr\, 1 \cdot \ln(1+L) + \frac{\hbar^2}{8} \frac{}{1+L} + \ldots$$

and is the set of vacuum diagrams at "included" source L.

Determine new variables which are now finite after the usual renormalization, with the help of (4.1)

$$S^r = Z^r_L(J,L) = Z_L(J,L) - Z_L(O,L) \tag{4.2}$$

$$\varphi_i^r = Z^r_{J_i}(J,L) = Z_{J_i}(J,L) \tag{4.3}$$

From (2.2) it follows that they are expressed by the former variables φ and S through the relations:

$$S^{P} = S - \mathcal{Z}_L\,(0, L) \qquad (4$$

$$\varphi_i^{r} = \varphi_i \qquad (.$$

Expression (4.4) points out that from S all vacuum diagrams are s tracted, therefore the new variable S^{r} is finite, after the usual r normalization.

The Legendre transformation (2.3) looks now as

$$\Gamma^{P}(\varphi^{P}; S^{r}) = \mathcal{Z}^{r}(\mathcal{J}, L) - \mathcal{J}_i\,\varphi_i^{r} - L\cdot S^{r} = \qquad (4$$

$$= \Gamma(\varphi, S) - \Gamma(\varphi, S)\Big/_{\mathcal{J}=0}$$

and (2.4) turns into

$$\Gamma^{r}_{\varphi_i^{P}} = -\mathcal{J}_i\ , \qquad \Gamma^{P}_{S^{r}} = -L \qquad ($$

As far as we will use in what follows only new variables, symbol r be omitted. The first terms of Γ expansion in coupling constant ha view:

$$\Gamma = S + \frac{\hbar}{4}$$  $$+ \dots$$

Here variables φ and S are coupled in the ven order by the re

$$S = S(\varphi)$$

The next terms of the loop expansion can be obtained in the follow We unify in (4.1) addenda with $S(\varphi)$ and make non-total Legendre t mation (only with respect to $\mathcal{J}$). As a result we will obtain the usual expansion for the effective potential with re-determined action $(1+L)S(\varphi)$ and propagator in the external field $\Delta(\varphi)/(1+L)$

$$\Gamma^{L\cdot}_{(\varphi)} = \mathcal{Z}(\mathcal{J}, L) - \mathcal{J}_i\, \varphi_i = \tag{4.10}$$

$$= (1+L)S(\varphi) + \hbar\,\Gamma_1(\varphi) + \frac{\hbar^2}{(1+L)^2}\,\Gamma_2(\varphi) + \cdots$$

where Γ_n is the loop correction.

Having made now the Legendre transformation also with respect to L , which in the sum is equivalent to (4.6) , we will obtain one-loop approximation

$$\Gamma = S + \hbar\,\Gamma_1(\varphi) \tag{4.11}$$

having , as in the first-order coupling constant one , a relation which couples φ and S :

$$S = S(\varphi) \tag{4.12}$$

Using (4.11-12) one can define $\{\varphi_{vac}\}$ from equation

$$\frac{\delta\Gamma}{\delta\varphi_i} = 0 \tag{4.13}$$

and then find S_{vac} from (4.12)

$$S_{vac} = S(\{\varphi\}_{vac}) \tag{4.14}$$

These relations point out that in renormalized theories the nonzero vacuum average S may arise only in the case when there is the non-trivial solution of equation (4.13).

In principle, another means as to determine S_{vac} is possible. For that, one has to find φ from (4.12) as a function of S (this solution is many-valued) and then substitute it into (4.11)

$$\widetilde{\Gamma}(S) \equiv S + \hbar \Gamma_1(\varphi(S)) + \cdots \tag{4.15}$$

then from $\widetilde{\Gamma}_S = 0$ find S_{vac} .

Consider from this viewpoint quantum electrodynamics and chromodynamics. In the first one eq.(4.13) does not contain non-trivial solutions [13] and $S_{vac}^{QED} = 0$, in the second one [14] there are non-trivial solutions [7, 8, 9, 10] and hence $S_{vac}^{QCD} \neq 0$

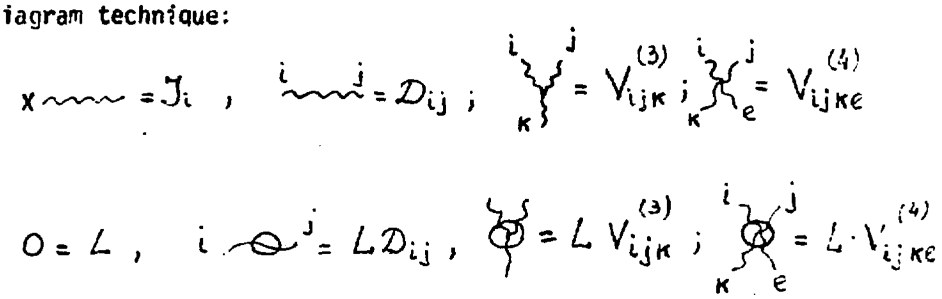
In conclusion the authors should like to express their gratitude to S.G.Matinian, G.M.Asatrian, A.N.Vasil'ev and A.G.Sedrakian for their interest in the work and valuable discussions.

APPENDIX A

Let us present the action $S(\varphi)$ in the form of

$$S(\varphi) = \frac{i}{2}\varphi_i \mathcal{D}^{-1}_{ij}\varphi_j + \frac{1}{3!}V^{(3)}_{ijk}\varphi_i\varphi_j\varphi_k + \frac{1}{4!}V^{(4)}_{ijk\ell}\varphi_i\varphi_j\varphi_k\varphi_\ell \tag{A.1}$$

where $\mathcal{D}^{-1}_{ij}$ is the Green function, and $V^{(3)}_{ijk}$ and $V^{(4)}_{ijk\ell}$ are vertices. In tne case when the source L is a sma'l quantity, tne exnansion in powers of L, $V^{(3)}$ and $V^{(4)}$ may be obtained using the following diagram technique:



(A.2)

The invariant source L is labelled by a circle $\bigcirc$, and the source $\mathcal{J}_i$ by a cross $\times$. In case only vertices V are considered small quantities, summarizing in exponential (2.1) the mean-squared over field φ_i addenda, for $g\varphi^4$ theory we obtain:

(A.3)

$\mathcal{Z}_{tot}(\mathcal{J},L) = -\frac{\hbar}{2i}\,\mathrm{tr}\,1\ln(1+L) + \frac{i}{2}$  $/\,(1+L)\; +$

$+\,g\Big\{\frac{1}{4}$  $/\,(1+L)^3 - \frac{\hbar}{2}$  $/\,(1+L)^2 + \frac{\hbar^2}{8}$  $/\,(1+L)\Big\} + \dots$

In order to get $Z(\mathcal{J},L)$ (2.1) it is necessary to subtract $Z_{tot}(0,0)$ from $\mathcal{Z}_{tot}(\mathcal{J},L)$, i.e.

$$Z(J, L) = Z_{tot}(J, L) - Z_{tot}(0, 0)$$

However, after this subtraction in $Z(J, L)$ remain vacuum diagrams which diverge as momentum fourth power. All vacuum diagrams can be excluded by means of the following subtraction:

$$Z^r(J, L) = Z_{tot}(J, L)\, Z_{tot}(0, L)$$

This subtraction is equivalent to the following choice of the normalization factor

$$N(L) = \int \mathcal{D}\varphi_i \exp \frac{i}{\hbar}\left[S(\varphi) + L S_1(\varphi)\right]$$

having been considered in sec.4.

APPENDIX B

Equation (3.4) in the zero in g order has a view:

$$\Gamma_{0\varphi_x} = (1 - \Gamma_{0s})\, i \mathcal{D}_{xy}^{-1}\, \varphi_y \tag{B.1}$$

whence it follows that

$$\Gamma_0 = S + F_0 (\tilde{S}) \tag{B.2}$$

where $F(S)$ is a function of $\tilde{S} = S - \frac{i}{2} \varphi_x \mathcal{D}_{xy}^{-1} \varphi_y$ only.
Operator Q_{xy} in the zero in g order will be expressed through F_0 as

$$Q_{xy}^{(0)} = i \mathcal{D}_{xy}^{-1}\, F_0' \tag{B.3}$$

where F' means differentiation of F with respect to $\tilde{S}$.

Taking into account (B.2) and (B.3) eq.(3.5) in the zero in g order will be written as

$$\tilde{S} - \frac{\hbar}{2i}\, \frac{tr\, 1}{F_0'} = 0 \tag{B.4}$$

Having solved (B.4), we find F_0 and substitute it into (B.2):

$$\Gamma_0 = S + \frac{\hbar}{2i}\, tr\, 1\, \ln \tilde{S} \tag{B.5}$$

Write down eq.(3.4) in the first in g order taking into account (B.3):

$$\Gamma_{1\varphi_x} = - \Gamma_{1S}\, i \mathcal{D}_{xy}^{-1}\, \varphi_y - \frac{\hbar}{4i}\, \frac{tr\, 1}{\tilde{S}}\, \varphi_x^2 + \frac{\hbar}{2}\, \mathcal{D}_{xx} \tag{B.6}$$

It follows from (B.6) that

$$\Gamma_1 = \frac{\hbar}{2} \mathcal{D}_{xx} \varphi_x - \frac{\hbar}{2i} \frac{\mathrm{tr}\, 1}{\tilde{S}} \frac{\varphi_x^3}{3!} + F_1(\tilde{S}) \tag{B.7}$$

Substitute (B.7) into eq.(3.5). To find $F_1(\tilde{S})$ one must know Q^{-}_{xy} and H_{xyz} in the zero order and Q^{-1}_{xy} in the first order. The zero-order Q_{xy} is determined from (B.3), H_{xyz} is zero, and Q^{-1}_{xy} in the first order will be some function of $F_1(S)$. Having substituted all these expressions into (3.5), we obtain eq. $F_1' = 0$, i.e. F_1 is $\tilde{S}$ independent. Similarly, one can obtain F_2, F_2 etc. both in $g\varphi^3$ and $g\varphi^4$ theories.

Let us give expressions for operators Q^{-1}_{xy}, H_{xyz}, $\mathcal{M}_{xyzt}$ used when constructing $\Gamma(\varphi, S)$ by perturbation theory:

a) $g\varphi^3$ theory

$$Q_{oxy} = \frac{2\tilde{S}\, \mathcal{D}_{xy}}{\hbar\, \mathrm{tr}\, 1} \tag{B.8a}$$

$$H_{oxyz} = 0 \tag{B.8b}$$

$$Q^{-1}_{1xy} = \frac{2i\, \tilde{S}}{\hbar\, \mathrm{tr}\, 1} \left[\mathcal{D}_{xz} \varphi_z \mathcal{D}_{zy} + \frac{i}{3!} \frac{\varphi^3}{\tilde{S}} \mathcal{D}_{xy} \right] \tag{B.8c}$$

$$H_{1xyz} = -i \left(\frac{2\tilde{S}}{\hbar\, \mathrm{tr}\, 1} \right)^2 \mathcal{D}_{xt}\, \mathcal{D}_{yt}\, \mathcal{D}_{zt} \tag{B.8d}$$

b) $g\varphi^4$ theory

$$Q^{-1}_{oxy} = \frac{2\tilde{S}\, \mathcal{D}_{xy}}{\hbar\, \mathrm{tr}\, 1} \tag{B.9a}$$

$$H_{0\,xyz} = 0 \tag{B.9b}$$

$$\mathcal{M}_{0\,xyzt} = 0 \tag{B.9c}$$

$$Q^{-1}_{1\,xy} = -\frac{2}{\hbar}\left(\frac{\tilde{S}}{t\tau 1}\right)^2 \Big[\mathcal{D}_{xy}\,\frac{t\tau 1}{\tilde{S}^2}\quad \frac{\varphi^4}{4!} + \tag{B.9d}$$

$$\frac{t\tau 1}{2i\,\tilde{S}}\mathcal{D}_{xz}\,\varphi_z^2\,\mathcal{D}_{zy} - \mathcal{D}_{xz}\,\mathcal{D}_{zz}\,\mathcal{D}_{zy} + \frac{\mathcal{D}_{xy}}{2t\tau 1}\,\mathcal{D}^2_{zz}$$

$$H_{1\,xyz} = -i\left(\frac{2\tilde{S}}{\hbar t\tau 1}\right)^2 \mathcal{D}_x + \mathcal{D}_{yt}\,\mathcal{D}_{zt}\,\varphi_t \tag{B.9e}$$

$$\mathcal{M}_{1\,xyzt} = -\left(\frac{2\tilde{S}}{\hbar t\tau 1}\right)^3 \mathcal{P}_x \tag{B.9f}$$

In formulae (B.8-9) $Q^{-1}{}_n$, H_n , $\mathcal{M}_n$ means n -term in expansion in g of the given operator (i.e. $A = \sum_n g^n A_n$).

Using (B.8-9) one can readily restore the expansions (3.8) and (3.14). Note that (3.8) and (3.14) can be obtained immediately from (A.3) as well , however this means does not allow one to investigate general properties of Γ as well as construct any other approximations of Γ .

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