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LORENTZ-INVARIANCE AND VACUUM FIELDS

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ЛОРЕНЦ ИНВАРИАНТНОСТЬ И ВАКУУМНЫЕ ПОЛЯ

Показано, что для полного описания полей частиц при $E\to\infty$ необходимо ввести бозе- и ферми-вакуумные поля. Доказано, что в уравнениях Эйнштейна необходимо ввести космологический член пропорциональный квадрату напряженности бозе-вакуумного поля.

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It is shown that for the complete description of the particle fields at $E \to \infty$ it is necessary to introduce bose- and fermi-vacuum fields. It is proved that in the Einstein equations it is necessary to introduce a cosmological term proportional to the square of the bose-vacuum field strength.

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1. Introduction

Ordinary equations for the particle field [1] at the particle energy $E \to \infty$ and, consequently, $m \to 0$ describe only the field with a maximum helicity. It is quite natural to require that equations will describe in the limit $E \to \infty$ all the possible limiting fields [2]. For instance, at $s = 1$ equations must also describe four limiting fields with helicities $j = \pm 1, 0, -1, +1$.

In the present work it is shown that for the complete description of the field $s = 0$, $m^2 > 0$ ($m^2 < 0$) it is necessary to introduce bose- (fermi-) vacuum fields describing the Lorentz group representations of class $P_\mu = 0$. And from the requirement of correct description of limit $E \to \infty$ it follows that in the Einstein equations it is necessary to introduce a cosmological term [3-9] proportional to the square of bose-vacuum field strength.

2. Field with spin $s = 1$

The field of the particles with spin $s = 1$ describes the reducible representation of the Lorentz group

$$D(1,0) \oplus D(0,1) \oplus D(\tfrac{1}{2},\tfrac{1}{2}).$$

In the limit of the particle infinite energy $E \to \infty$ and, consequently, $m \to 0$ the reducible representation falls apart and we have four limiting fields with helicities $j = \pm 1$, 0, -1, +1 describing the representations $D(1,0) \oplus D(0.1)$, $D(½,½)$, $D(1,0)$, $D(0,1)$, respectively. We can say that the Lagrangian describes completely the field $s = 1$ only when it also describes the four limiting fields at $E \to \infty$ and, consequently, it must contain four independent parameters. With this additional requirement the Lagrangian is practically determined unambiguously and has the form

$$\mathcal{L} = -\frac{1}{2}\left(a_1 F^-_{\mu\nu} + a_2 F^+_{\mu\nu}\right)\left(\partial^\mu \overset{*}{A}{}^\nu - \partial^\nu \overset{*}{A}{}^\mu\right) - \frac{1}{2}\left(a_1 \overset{*}{F}{}^-_{\mu\nu} + a_2 \overset{*}{F}{}^+_{\mu\nu}\right)\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) + \frac{1}{2} m_1 \overset{*}{F}_{\mu\nu} F^{\mu\nu} + m_2 \overset{*}{A}_\mu A^\mu , \tag{1}$$

where $F^\pm_{\mu\nu} = \frac{1}{2}\left(F_{\mu\nu} \pm i \tilde{F}_{\mu\nu}\right)$, $\overset{*}{F}{}^\pm_{\mu\nu} = \left(F^\pm_{\mu\nu}\right)^*$

Lagrangian (1) contains four real-valued parameters a_1, a_2, m_1, m_2. It is impossible to exclude any of these parameters by means of some transformations since they take zero values as well. If, for example, parameters $a_1, a_2 \neq 0$ then they can be easily excluded from the Lagrangian by introducing notations $a_1 F^-_{\mu\nu} + a_2 F^+_{\mu\nu} = f_{\mu\nu}$ and carry on a substitution $m_1 \to m_1 a_1 a_2$. If all the parameters are distinct from zero Lagrangian (1) describes the field $s = 1$ with mass square $m^2 = \frac{m_1 m_2}{a_1 a_2}$. If $m_2 = 0$ the (1) describes the field with helicity $j = \pm 1$, and at $m_1 = 0$ describes the field $j = 0$. If $m_2 = 0$, $a_2 = 0$ ($m_2 = 0$, $a_1 = 0$) the Lagrangian describes the field $j = -1$ ($j = +1$). Assuming $m_2 = 0$, $a_2 = 0$ and $m_1 \neq 0$, $a_1 \neq 0$ from (1) we shall obtain the field equation

$$\partial^\nu F^-_{\mu\nu} = 0 , \tag{2}$$

and additional conditions

$$F^-_{\mu\nu} = 0 \tag{3}$$

Thus the general solution of eqs. (2), (3) will be $F^-_{\mu\nu} = 0$. The result obtained means that in the Minkowski space with signature (+ - - -) the field with helicity $j = -1$ describing representation D(1, 0) does not exist in a free state, i.e. it is confined. Quite analogous are the results for the field $j = +1$ describing representation D(0,1). This result does not change at introducing an interaction.

If we formally introduce current into the right-hand side of (2) we shall have

$$\partial^\nu F^-_{\mu\nu} = J_\mu . \tag{4}$$

Assuming $F^-_{\mu\nu} = H_{\mu\nu} - i\tilde{H}_{\mu\nu}$ and $J_\mu = I_\mu - iK_\mu$ where $H_{\mu\nu}$, $I_{\mu\nu}$, K_μ are the real-valued functions, we shall obtain from (4) the Maxwell equations with the Dirac monopoles.

$$\partial^\nu H_{\mu\nu} = I_\mu \quad , \qquad \partial^\nu \tilde{H}_{\mu\nu} = K_\mu . \tag{5}$$

From the fact of absence in free state of particles $j = +1$, -1 it follows that there is no Lagrangian for the system (5) which will satisfy the necessary physical requirements, i.e. the Dirac magnetic monopoles do not exist in free state in the Minkowski space either.

In the Euclidean space the Lagrangian totally describing the field $s = 1$ has the form

$$\mathcal{L} = -\frac{1}{2}\left[a_1(F_{\mu\nu} - \tilde{F}_{\mu\nu}) + a_2(F_{\mu\nu} + \tilde{F}_{\mu\nu})\right](\partial^\mu A^\nu - \partial^\nu A^\mu) + \\ + \frac{1}{2} m_1 F_{\mu\nu} F^{\mu\nu} + m_2 A_\mu A^\mu . \tag{6}$$

Assuming $m_2 = 0$, $a_2 = 0$ for the field $j = -1$ we shall have equations

$$\partial^{\nu} (F_{\mu\nu} - \tilde{F}_{\mu\nu}) = 0 \tag{7}$$

and additional conditions

$$F_{\mu\nu} + \tilde{F}_{\mu\nu} = 0 . \tag{8}$$

Thus in the Euclidean space the particles with helicities $j = +1, -1$ are not confined but exist in free state. If we introduce, on the basis of (6), the Yang-Mills fields and the interaction with an external current I_{μ} and put $m_2 = 0$, $a_1 = 0$ ($m_2 = 0$, $a_2 = 0$) we shall have

$$\mathcal{L} = \frac{1}{2} tr (F^{\mu\nu} \pm \tilde{F}^{\mu\nu}) (\partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} + [A_{\mu}, A_{\nu}]) - \frac{1}{2g^2} tr F^{\mu\nu} F_{\mu\nu} - tr I_{\mu} A^{\mu} . \tag{9}$$

If we put $I_{\mu} = 0$ into (9) then we shall obtain from the Lagrangian the usual duality equations.

3. Vacuum fields

The Lorentz group representations [10] are subdivided into four classes $m^2 > 0$, $m = 0$, $P_{\mu} = 0$, $m^2 < 0$. Respectively physical fields are also divided into four classes. The finite-dimensional representations of the Lorentz group D(A,B) are characterized by two numbers A,B = 0, ½, 1, ... The physical field describing representation D(A,B) is a field of particles with helicity j = B-A. In the virtual state the spin s = A+B should be assigned to the mentioned particles. These expressions are valid at A+B ≠ 0. The ratios suggested by Weinberg [11] are valid not in all cases. The above restriction (A+B ≠ 0) is connected with the fact that the field with scalar strength, describing representation D(0,0), belongs to the class $P_\mu = 0$ and, hence, it is a vacuum field, a field without any particles, so it's no use speaking of the helicity and spin of such a field.

The field of particles with spin $s = 0$ describes the reducible representation $D(½,½) \oplus D(0,0)$ and at the particle energy $E \to \infty$ decays into two independent fields describing representations D(½,½) and D(0,0), i.e. it has two limits - the field with helicity $j = 0$ and the vacuum field.

The Klein-Gordon equation describes the field $s = 0$ not completely since it does not describe the vacuum field in the limit $E \to \infty$. The Lagrangian can completely describe the field $s = 0$ only in the case if it also describes two limiting fields at $E \to \infty$ and, hence, it must contain two arbitrary parameters. This condition is satiafied by the Lagrangian

$$\mathcal{L} = \varphi^\mu \partial_\mu \varphi - \frac{1}{2} m_1 \varphi^\mu \varphi_\mu - \frac{1}{2} m_2 \varphi^2 , \qquad (10)$$

where $m^2 = m_1 m_2$. At $m_2 = 0$ the Lagrangian describes the field with helicity $j = 0$ and at $m_1 = 0$ - the vacuum field with scalar strength which satisfies equation $\partial_\mu \varphi = 0$ with a general solution $\varphi = \text{const}$. The introduced vacuum field is connected with the bose-system $s = 0$, $m^2 > 0$ and is a bose-vacuum field. Thus for a complete description of the limit $E \to \infty$ of the particle field $s = 0$, $m^2 > 0$ it is necessary to introduce a bose-vacuum field.

In the same way one can introduce a fermi-vacuum field describing representation D(0,0) and belonging to class $P_\mu = 0$, though connected with the tachyon fermi-system [12] $\mathfrak{z} = 0$, $m^2 < 0$.

It is convenient to divide the Lorentz group representations into two parts. The first part is the fields connected with the fields $m^2 > 0$. They are subdivided into three classes

$$m^2 > 0, \qquad m' = 0, \qquad \rho'_\mu = 0$$

where $m' = 0$ corresponds to the fields $m^2 > 0$, $m \to 0$, and $\rho'_\mu = 0$ - to the bose-vacuum field. The second part is connected with the fields $m^2 < 0$ and is divided into three classes as well

$$m^2 < 0, \qquad m'' = 0, \qquad \rho''_\mu = 0$$

where $m'' = 0$ corresponds to the fields $m^2 < 0$, $m \to 0$, and $\rho''_\mu = 0$ - to the fermi-vacuum field.

The introduction of bose- and fermi-vacuum fields may be considered as a covariant generalization of bose- and fermi-vacuums respectively.

Thus we come to a conclusion that the set of physical fields belonging to classes $m^2 > 0$ and $m' = 0$ will be closed at $E \to \infty$ only in the case if it is supplemented with a bose-vacuum field of class $\rho'_\mu = 0$.

If the Lorentz invariance is fulfilled globally, as it is the case in the Minkowski space, then the physical theory does not depend on the vacuum field strength. But the picture changes sharply if in the theory the Lorentz invariance is fulfilled locally, as it is the case in the Riemannian space: in this case the theory essentially depends on the strength of the bose-vacuum field.

In the general relativity the action for the field system should be written down in the form

$$S = S_g + S_v + S_m$$

where S_g is the action for gravitation field, S_v - for the bose-vacuum field, S_m - for other fields with $m^2 > 0$ and $m' = 0$. The ordinary Hilbert-Einstein action does not contain a bose-vacuum field and, hence, the set of fields is not closed at $E \to \infty$.

Substituting expressions for actions we shall have

$$S = -\frac{1}{2\varkappa}\int R\sqrt{g}\,d^4x + \int\left(\varphi^{\mu}\partial_{\mu}\varphi - \frac{1}{2}m_2\varphi^2\right)\sqrt{g}\,d^4x + \int \mathcal{L}_m\sqrt{g}\,d^4x. \quad (11)$$

From here follows

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{1}{2}\varkappa m_2 g_{\mu\nu}\varphi^2 = \varkappa T_{\mu\nu}, \quad (12)$$

$$\partial_{\mu}\varphi = 0,$$

where $m_2 = \pm 1$, φ is the bose-vacuum field strength.

Thus it is necessary to introduce a cosmological term $\Lambda \sim \varphi^2$ in the Einstein equations.

The Einstein equations with a cosmological term are equations which describe in the Riemannian space fields of three classes $m^2 > 0$, $m' = 0$, $P'_{\mu} = 0$.

In conclusion the author expresses his deep gratitude to Prof. S.G.Matinyan for useful discussions.

Appendix

The Lagrangian describing the field $s = ½$ completely has the following form

$$\mathcal{L} = \frac{1}{2} i \left[\bar\psi \gamma^\mu (a_1 \eta_1 + a_2 \eta_2) \partial_\mu \psi - \partial_\mu \bar\psi \gamma^\mu (a_1 \eta_1 + a_2 \eta_2) \psi \right] - m_1 \bar\psi \psi,$$

where projection matrices $\eta_{1,2} = \frac{1}{2}(1 \pm \gamma_5)$, $m^2 = \frac{m_1^2}{a_1 a_2}$

The field $s = 2$ may be described by the Lagrangian

$$\mathcal{L} = -\frac{1}{2}\left[a_1 ({}_\mu \overset{*}{H}_{\lambda\nu} + i\, {}_\mu \overset{*}{H}_{\tilde\lambda\tilde\nu}) + a_2 ({}_\mu \overset{*}{H}_{\lambda\nu} - i\, {}_\mu \overset{*}{H}_{\tilde\lambda\tilde\nu})\right] \partial^\lambda h^{\mu\nu} -$$

$$-\frac{1}{2}\left[a_1 ({}_\mu H_{\lambda\nu} - i\, {}_\mu H_{\tilde\lambda\tilde\nu}) + a_2 ({}_\mu H_{\lambda\nu} + i\, {}_\mu H_{\tilde\lambda\tilde\nu})\right] \partial^\lambda \overset{*}{h}{}^{\mu\nu} +$$

$$+\frac{1}{2} m_1 \left({}_\mu \overset{*}{H}_{\lambda\nu}\, {}^\mu H^{\lambda\nu} - \overset{*}{H}_\lambda H^\lambda\right) - m_2 \left(\overset{*}{h}_{\mu\nu} h^{\mu\nu} - \overset{*}{h} h\right),$$

where ${}_\mu H_{\lambda\nu} = -{}_\mu H_{\nu\lambda}$, ${}^\mu H_{\mu\lambda} = H_\lambda$, $h_{\mu\nu} = h_{\nu\mu}$, $h^\mu_\mu = h$ and tilda means a dual conjugation by corresponding indices. This Lagrangian contains four parameters a_1, a_2, m_1, m_2 and describes four limits at $E \to \infty$. If all the parameters are distinct from zero the Lagrangian describes the field $s = 2$ with $m^2 = \frac{m_1 m_2}{a_1 a_2}$. At $m_2 = 0$ ($m_1 = 0$) it describes the field with helicity $j = \pm 2$ ($j = 0$), and at $m_2 = 0$, $a_2 = 0$ ($m_2 = 0$, $a_1 = 0$) describes the field $j = -2$ ($j = +2$). And the particles with helicities $j = -2$ and $j = +2$ are confined, i.e. they do not exist in free state in the Minkowski space. In the Euclidean space these particles are not confined. The result does not change when introducing an interaction. The suggested Lagrangian does not describe the field $s = 2$ completely. Neither it describes the limiting fields $j = \pm 1, -1, +1$.

Considering fields with the highest spin we came to a conviction that it is impossible to construct a Lagrangian which would satisfy the necessary physical requirements and describe completely fields with spins $s \geq \frac{3}{2}$, i.e. describe also all kinds of limiting fields at $E \to \infty$ and, consequently, at $m \to 0$ as well.

In the Minkowski space particles with helicities $j = -1, +1$ at $s = 1$ are confined, but the mentioned particles are the carriers of the Dirac monopole interaction and for this reason the Dirac magnetic monopoles are confined in the Minkowski space as well. Analogously, particles with helicities $j = +2, -2$ at $s = 2$ are confined too, but these particles are the carriers of the tachyon interaction [7], hence, the tachyons are confined in the Minkowski space as well.

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