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POSSIBLE CHECK OF THE QCD PARTON MODEL  
IN SEMI-INCLUSIVE  $\mu$  -PRODUCTION OF  $J/\psi$  -MESON

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It is shown that the measurement of azimuthal asymmetry in semi-inclusive production of  $J/\psi$  can serve as a good test for the check of the QC parton model. An additional method of the check of distribution gluon functions in PGF model is suggested

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ВОЗМОЖНАЯ ПРОВЕРКА КХД ПАРТОННОЙ МОДЕЛИ  
В ПОЛУИНКЛУЗИВНОМ  $\mu^-$  - РОЖДЕНИИ  $J/\psi$  - МЕЗОНА

Показано, что измерение азимутальной асимметрии в полуин-  
клузивном рождении  $J/\psi$  может явиться хорошим тестом провер-  
ки КХД партонной модели. Предлагается дополнительный способ  
проверки глюонных функций распределения в  $PGF$  модели.

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In our consideration we shall proceed from the same assumptions which have been used in a number of works on the investigated process [1-7]. Namely the amplitude of the  $J/\psi$  -meson semi-inclusive leptonproduction

$$\mathcal{N}^-(K_1) + N(P_1) \rightarrow \mathcal{N}^-(K_2) + \psi(P_2) + X \quad (1)$$

is factorized into three factors, amplitude of the nucleon transition to partons, amplitude of the incident lepton hard scattering on one of the partons (quarks or gluon), calculated by the perturbation theory, amplitude of the finite parton hadronization. In such a model physical processes at all the distances are described by a Hamiltonian with scale-dependent masses and coupling constants. At the same time the heavy quark masses have a slow scale dependence, and the QCD parton model with a running coupling constant would then be a good approximation in the processes of heavy quark production.

One of the possible tests of checking such an application of the factorization theorem (proved in all the orders of perturbation theory for inclusive processes [8]) is suggested in the work. Namely the measuring of the average  $\cos\psi$  value

defined as

$$\langle \cos\psi \rangle = \int_0^{2\pi} \cos\psi \, d\psi \frac{d\sigma}{d\psi dX_H dy} / \frac{d\sigma}{dX_H dy} \quad (2)$$

where  $\psi$  is the azimuthal angle between the transverse momentum  $\vec{P}_{2\perp}$  of the outgoing  $J/\psi$  meson and the lepton plane (Fig. 1), can give an additional information on the applicability of such assumptions in the processes of the  $J/\psi$  semi-inclusive leptoproduction.

In (2)  $X_H = \frac{Q^2}{2(P_1 q)}$ ,  $y = \frac{P_1 \cdot q}{P_1 \cdot K_1}$ ,  $q$  is the four-momentum of virtual photon,  $Q^2 = -q^2$ .

Note that Politzer [9] was the first to suggest this method of QCD checking applied to the processes of the hadron semi-inclusive production. In the present work this idea spreads on the case of the heavy  $C\bar{C}$  system production.

We also suggest an additional method of checking the gluon distribution function for PGF (photon-gluon fusion) model [1, 3, 7] which is the basis of our consideration.

According to the QCD parton model the differential cross section of the process studied can be written down through a sum of corresponding partonic differential cross sections (see e.g. [4])

$$\begin{aligned} \frac{d\sigma(\mu N \rightarrow \mu + J/\psi + X)}{dX_H dy d\psi} &= \frac{1}{F} \int_{4m_c^2}^{4m_D^2} d\Delta^2 \frac{d\sigma(\mu N \rightarrow \mu + c\bar{c} + X)}{dX_H dy d\psi d\Delta^2} \\ \frac{d\sigma(\mu N \rightarrow \mu + c\bar{c} + X)}{dX_H dy d\psi d\Delta^2} &= \sum_i \int dX dZ d\xi \cdot \delta(X_H - \xi \cdot X) f_i(\xi, \theta^2) \times \\ &\times \frac{d\sigma_i(\mu + P_i \rightarrow \mu + c\bar{c} + X)}{dX dy d\psi d\Delta^2 dZ} \quad (3) \end{aligned}$$

The sum over  $i$  runs over all partons (quarks, antiquarks and gluons).  $\alpha \mathcal{G}_i$  describes the semi-inclusive process

$$\mu(K_1) + i(P_1) \rightarrow \mu(K_2) + C(l_2) + \bar{C}(l_1) + X$$

The parton scaling variables are defined by

$$x = \frac{Q^2}{2P_1 q}, \quad 1-z = \frac{P_1 \Delta}{P_1 q}, \quad \Delta^2 = (l_1 + l_2)^2,$$

where  $\Delta^2$  is the invariant mass of heavy pair.  $m_c$  is the charmed quark mass taken equal to  $m_c = 1.5 \text{ GeV}$  [3];  $m_D$  is the  $D$ -meson mass;  $f_i(\xi, Q^2)$  describes the probability distribution for  $i$  partons with a fraction  $\xi$  of the hadron momentum,  $P_i = \xi P_1 \cdot F$  is the normalization factor equal to the number of resonance states of charmonium in the interval  $2m_c < \sqrt{\Delta^2} < 2m_D$  introduced in agreement with the assumption on semilocal duality [10] (the integration over  $\Delta^2$  is dual to the sum over the charmonium resonances, the colourless state of hadrons is ensured, with a unit probability, by the soft gluon radiation) which has been used when writing down (3).

The gauge invariant set of diagrams giving a contribution of the order  $O(d_s)$  and  $O(d_s^2)$  is shown in Figs 2 and 3 respectively. We have to take into account the diagrams of the order  $O(d_s^2)$  since in the lowest order ( $O(d_s)$ ), according to PGF model, the heavy quarks are produced symmetrically and the nontrivial dependence on  $\varphi$  is absent [1]. All the other contributions of the order  $O(d_s^2)$  are included in the effective gluon distribution function and are considered in the order  $O(d_s)$  differential cross section [2] (see however [5]).

In all our calculations we use  $d_s = [12\pi/(33-2n_f)] \cdot \ln \frac{\Delta^2}{\Lambda^2} \approx 0.375$  for the flavours number  $n_f = 4$  and  $\Lambda = 0.5$  GeV and  $\Delta^2$  in the interesting us region<sup>[3]</sup>. We also introduce a cut-off by  $Q^2$  and  $W \equiv (q + P_1)^2$  -square of the invariant mass of the finite hadronic system. Namely

$$Q^2 \geq Q_0^2 \equiv 3\text{GeV}^2, \quad W^2 \geq W_0^2 \equiv (2m_c + M_N)^2 \quad (4)$$

where  $M_N$  is the nucleon mass. The first restriction is connected with the possibility of applying the perturbation theory, the second one - with the requirement for the conservation of baryon number. To compute the differential cross sections it is necessary to know  $f_i(\xi, Q^2)$ . For the distribution gluon function we adopt two variants later on noted as "conventional" and "broad" [1, 11, 12]

$$f_g(\xi, Q^2) \equiv G^C(\xi) = \frac{3}{\xi} \cdot (1-\xi)^5 \quad (5)$$

$$f_g(\xi, Q^2) \equiv G^B(\xi) = \frac{(1-\xi)^5}{\xi} \cdot (1.07 + 13.50\xi) \quad (6)$$

Both these distributions are normalized with the condition

$$\int_0^1 d\xi \cdot \xi G(\xi) = 0.5$$

which agrees with the fact that the half of the nucleon momentum is taken away by gluons.

From experimental parametrization we have for the quark distribution functions: [2, 13]

$$\sum_i f_i(\xi, Q^2) e_i^2 = \frac{1}{\xi} \cdot F_2^{uN}(\xi, Q^2) =$$

$$= \frac{1}{\xi} (1 - \frac{3}{8}\xi) [3.2(1-\xi)^3 - 4.6(1-\xi)^4 + 1.7(1-\xi)^5] \cdot \left(\frac{Q^2}{3G_2 v^2}\right)^{0.25-\xi} \quad (7)$$

The differential cross section of the order  $O(\alpha_s)$  in the framework of the PGF model has been obtained in the works [1, 3, 7] in our variables  $X_H, y, Q^2, \xi$  it has the form

$$\frac{y Q^2 F}{d^2 \alpha_s e_c^2} \cdot \frac{d\sigma}{dx_H dy} = J_1 \cdot y^2 + J_2 \cdot (1-y) \quad (8)$$

where

$$J_1 = \int_{\xi_{\min}}^{\xi_{\max}} \frac{d\xi}{\xi} G(\xi) [\beta_0 \cdot B_1 + \psi \cdot B_2],$$

$$J_2 = 2 \int_{\xi_{\min}}^{\xi_{\max}} \frac{d\xi}{\xi} G(\xi) [\beta_0 \cdot (B_3 + B_4) + \psi (B_2 + B_4)],$$

$$\beta_0 = \left(1 - \frac{\eta X_H}{\xi - X_H}\right)^{1/2}, \quad \eta = \frac{4m_c^2}{Q^2}, \quad \psi = \ln \frac{1 + \beta_0}{1 - \beta_0}, \quad e_c^2 = \frac{4}{9},$$

$$B_1 = -\left(1 - 2 \frac{X_H}{\xi}\right)^2 - \eta \frac{X_H}{\xi} \left(1 - \frac{X_H}{\xi}\right), \quad B_3 = 4 \frac{X_H}{\xi} \left(1 - \frac{X_H}{\xi}\right),$$

$$B_2 = 1 - 2 \frac{X_H}{\xi} \left(1 - \frac{X_H}{\xi}\right) + \eta \frac{X_H}{\xi} \left(1 - \frac{X_H}{\xi}\right) - \frac{1}{2} \left(\eta \frac{X_H}{\xi}\right)^2, \quad B_4 = -2\eta \left(\frac{X_H}{\xi}\right)^2.$$

The interval over  $\xi$  is determined by the kinematic restriction on  $\Delta^2$ ,  $\Delta^2 = (\xi - X_H) \cdot y S_0$ ,  $Q^2 = S_0 y X_H$ ,  $S_0 \approx 2 P_1 K_1$ ,

$$X_H (1 + \eta) \leq \xi \leq X_H (1 + \eta/\gamma), \quad \gamma \equiv m_c^2/m_D^2 = 0.64$$

Substituting in (8) the distributions  $G(\xi)$  from (5) and (6)

and integrating over  $\xi$  (see Table 1) we obtain for the

ratio  $R = \frac{d\sigma^B}{dx_H dy} / \frac{d\sigma^C}{dx_H dy}$  the following values:

$R \simeq 3.5$  at  $X_H = 0.5$ ,  $Q^2 = 27 \text{ GeV}^2$ ;  $R \simeq 2.6$  at  $X_H = 0.3$ ,  
 $Q^2 = 18 \text{ GeV}^2$ ; at small  $X_H$  ( $X_H < 0.2$ )  $R \simeq 1$  (practically at  
any  $0 < y \leq 1$ ).

Note that such an obvious difference at large  $X_H$  of the  
distributions  $G^c$  and  $G^b$  doesn't result in a significant  
difference of total cross sections and single differential  
cross sections (e.g. at  $E_{\mu^-}$  energy of the initial lepton equal  
to 150 GeV which corresponds to  $X_H = 0.3$ ,  $Q^2 = 18 \text{ GeV}^2$ ,  
 $y = 0.2$ ,  $\sigma^b/\sigma^c \simeq 1.25$ ) [1].

For the parton differential cross section of the order  
 $O(d_s^2)$ , keeping only the members  $\sim \cos \psi_g$ , we have the following  
expression (see Appendix, A(2))

$$\frac{Q^2 y}{d^2 \cdot d_s^2} \cdot \frac{d\sigma_i}{dx dy d\psi_g d\Delta^2 dz} = \frac{4 e_i^2}{9\pi^2} \cdot \frac{2m_c^2 + \Delta^2}{\Delta^4} \cdot \beta \cdot A \cos \psi_g, \quad (9)$$

$$A = (1-y)^{1/2} \cdot (2-y) \left\{ x \cdot z [(1-x)(1-z) - \epsilon] \right\}^{1/2} \frac{(1-z)[(1-x)(1-z) + xz] - \epsilon'z}{(1-x)(1-z - \epsilon')^2}$$

Here  $\epsilon' = \Delta^2 / Sy$ . Note that in the limit  $\Delta^2 = 0$  our  
result for A agrees with [9].

Integrating over  $\psi$  and  $z$  (see (3) and A(3)) we obtain

$$\int \cos \psi d\psi \left( \frac{Q^2 y F}{d^2 d_s^2} \cdot \frac{d\sigma}{dx_H dy d\psi} \right) = \frac{4}{9\pi} (2-y) \cdot (1-y)^{1/2} \cdot J_3 \quad (10)$$

$$J_3 = \int_{4m_c^2}^{4m_s^2} d\Delta^2 \cdot \frac{\beta(2m_c^2 + \Delta^2)}{\Delta^4} \cdot \int_{X_H}^{1/(1+\frac{\Delta^2}{Q^2})} dx \frac{Z(x, \Delta^2)}{[x(1-x)]^{1/2}} \left( \sum_i f_i \left( \frac{X_H}{x}, Q^2 \right) \cdot e_i^2 \right)$$

where

$$Z(x, \Delta^2) = A_\Delta \cdot \Delta^4 + B_\Delta \cdot \Delta^2 + C_\Delta + \left[ \frac{\Delta^2 \cdot \alpha}{x \cdot (1 - (1-x) \Delta^2 \alpha)} \right]^{1/2} (\bar{A}_\Delta \cdot \Delta^4 + \bar{B}_\Delta \cdot \Delta^2 + \bar{C}_\Delta)$$

$$x = \frac{x}{Q^2(1-x)}$$

$$A_{\Delta} = x^2 \left[ -\frac{27}{8} + \frac{49}{4}x - 15x^2 + 6x^3 \right], \quad \bar{A}_{\Delta} = -x^2(1-x)^2 [1 - 6x + 6x^2],$$

$$B_{\Delta} = x \cdot \left[ \frac{9}{4} - \frac{11}{2}x + 4x^2 \right], \quad \bar{B}_{\Delta} = x(1-x) \cdot \left[ \frac{3}{2} - \frac{13}{2}x + 7x^2 \right],$$

$$C_{\Delta} = \frac{1}{8}(1+2x), \quad \bar{C}_{\Delta} = -\frac{1}{2} + x - \frac{3}{2}x^2$$

It is convenient to change the order of integration in (10). Integrating over  $\Delta^2$  and numerically over  $X$  with consideration of (7) we obtain for the value  $\langle \cos \psi \rangle$

$$\langle \cos \psi \rangle = \frac{\alpha_s}{\pi} \cdot \frac{(2-y)(1-y)^{1/2} J_3}{J_1 \cdot y^2 + J_2 \cdot (1-y)} \quad (11)$$

For  $J_1, J_2, J_3$  at fixed  $X_H$  and  $Q^2$  we have the values presented in Table 1:

Table 1

|       | $X_H=0.05$           |                      | $X_H=0.1$            |                      | $X_H=0.3; Q^2=18\text{GeV}^2$ |                      | $X_H=0.5; Q^2=27\text{GeV}^2$ |                      |
|-------|----------------------|----------------------|----------------------|----------------------|-------------------------------|----------------------|-------------------------------|----------------------|
|       | $Q^2=30\text{GeV}^2$ | $Q^2=18\text{GeV}^2$ | $Q^2=30\text{GeV}^2$ | $Q^2=18\text{GeV}^2$ | $G^C$                         | $G^B$                | $G^C$                         | $G^B$                |
| $J_1$ | 1.65                 | 1.95                 | 0.74                 | 0.66                 | $1.55 \cdot 10^{-2}$          | $3.99 \cdot 10^{-2}$ | $4.94 \cdot 10^{-4}$          | $1.75 \cdot 10^{-3}$ |
| $J_2$ | 3.68                 | 4.44                 | 1.68                 | 1.50                 | $3.5 \cdot 10^{-2}$           | $8.98 \cdot 10^{-2}$ | $10.94 \cdot 10^{-4}$         | $3.96 \cdot 10^{-3}$ |
| $J_3$ | 0.15                 | 0.09                 | 0.05                 | 0.03                 | $4.29 \cdot 10^{-4}$          |                      | $8.78 \cdot 10^{-6}$          |                      |

At small  $X_H < 0.2$  the "conventional" and "broad" distributions of gluons make approximately equal contributions to  $J_1$  and  $J_2$ . Note that the chosen values of  $X_H$  are determin-

ed by the restriction  $X_H \leq \left(1 + \frac{4m_c(m_c + M_N)}{Q^2}\right)^{-1}$ , following from (4). The dependence of  $\langle \cos \psi \rangle$  on  $y$  is presented in Fig. 4. The chosen values of  $X_H, Q^2$  correspond to the maximum values of  $\langle \cos \psi \rangle$  at available energies  $E_M$  (at  $y = 0.6, X_H = 0.1, Q^2 = 30 \text{ GeV}^2, E_M \approx 260 \text{ GeV}$  ( $\langle \cos \psi \rangle \approx 0.5\%$ ). Note that the maximum values for  $\langle \cos \psi \rangle \approx 1\%$  are obtained at  $X_H = 0.05, Q^2 = 30 \text{ GeV}^2$  and asymptotically large energies.

In all our preceding calculations we have ignored the intrinsic transverse component  $K_{\perp}$  of initial partons (according to the approach of the works [1,9]). The consideration of intrinsic transverse motion essentially changes the results obtained. Already the diagram of the lowest order  $O(d_S)$  (Fig. 2) gives a non-zero contribution to  $\langle \cos \psi \rangle$ . Generalizing the results of the work [1] for this case (practically substituting  $2(P_1 K_1) \rightarrow S_0 \xi - 2K_{\perp} \cdot K_{\perp} \cos \psi$ ) one may obtain for  $\langle \cos \psi \rangle$ , keeping the linear members by  $K_{\perp}$ , the following expression

$$\langle \cos \psi \rangle \approx - \frac{2X_H \langle K_{\perp} \rangle}{\sqrt{Q^2}} (1-y)^{1/2} \frac{J_2'/y - J_1'}{J_1 + \frac{(1-y)}{y^2} J_2} \quad (12)$$

Note that the result obtained  $\langle \cos \psi \rangle \sim \frac{\langle K_{\perp} \rangle}{\sqrt{Q^2}}$  coincides with the one of the work [4] where an analogous value has been considered in a parton model. Here  $\langle K_{\perp} \rangle$  is the mean transverse momentum of the initial gluon taken equal to  $\approx 0.6 \text{ GeV}$  [5].

$J_1' = J_1, J_2' = J_2/2$  with substitution  $G(\xi) \rightarrow G(\xi)/\xi$  in the subintegral expression for  $J_1, J_2$  of the formula (8).

The obtained for  $J_1'$  and  $J_2'$  results are presented in Table 2.

Table 2

|        | $X_H = 0.05$             |                          | $X_H = 0.1$              |                          | $X_H = 0.3$              | $X_H = 0.5$              |
|--------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|        | $Q^2 = 30 \text{ GeV}^2$ | $Q^2 = 18 \text{ GeV}^2$ | $Q^2 = 30 \text{ GeV}^2$ | $Q^2 = 18 \text{ GeV}^2$ | $Q^2 = 18 \text{ GeV}^2$ | $Q^2 = 27 \text{ GeV}^2$ |
| $J'_1$ | 23.8                     | 23.5                     | 5.27                     | 3.96                     | $3.17 \cdot 10^{-2}$     | $6.97 \cdot 10^{-2}$     |
| $J'_2$ | 26.5                     | 26.7                     | 5.97                     | 4.51                     | $3.55 \cdot 10^{-2}$     | $7.70 \cdot 10^{-4}$     |

The dependence of  $\langle \cos \Delta \varphi \rangle$  on  $y$  is presented in Fig. 5. Note that the consideration of  $K_1$  has increased the value of asymmetry (e.g. 4 times at  $X_H = 0.1$ ,  $Q^2 = 30 \text{ GeV}^2$ ,  $y = 0.6$ ) and which is most interesting has led to the change of the sign of  $\langle \cos \Delta \varphi \rangle$ . Thus the measurement of the azimuthal asymmetry sign can give additional information on the role of intrinsic transverse motion of partons (in nucleon) in the processes of the  $J/\psi$  semi-inclusive production.

In conclusion let us note that, by varying the limits of the integration over  $\Delta^2$ , the consideration carried out in our work can be easily continued in the case of production of systems with a larger mass ( $D\bar{D}, \gamma$ ).

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## Appendix

Here is the detailed calculation of differential cross section, order  $O(d_s^2)$  determined by the Fig. 3 diagrams. The parton differential cross section has the following form:

$$d\hat{\sigma}_i = \frac{1}{4(2\pi)^8 (K_1 P_1)} \cdot \overline{|M|^2} \cdot \frac{d^3 \vec{K}_2}{2E_2} \cdot \frac{d^3 \vec{P}_2}{2E_2} \cdot \frac{d^3 \vec{\ell}_1}{2\ell_{10}} \cdot \frac{d^3 \vec{\ell}_2}{2\ell_{20}} \times \delta(K_1 + P_1 - K_2 - P_2 - \ell_1 - \ell_2) \quad A(1)$$

Here  $(E_2, \vec{K}_2)$ ,  $(E_2, \vec{P}_2)$ ,  $(\ell_{20}, \vec{\ell}_2)$ ,  $(\ell_{10}, \vec{\ell}_1)$  are the energy and momentum of finite lepton, quark, heavy quarks and antiquark respectively.

$$\overline{|M|^2} = \frac{e^4 g^4}{\Delta^4 q^4} \left( \frac{1}{4} \cdot \frac{2}{3} \right) \ell_i^2 \cdot J_{\mu\nu}(K_1, K_2) \cdot J_{\rho\sigma}(\ell_1, \ell_2) \cdot K_{\mu\nu\rho\sigma}(P_1, P_2, q, \Delta)$$

The factor  $2/3$  ( $1/4$ ) is connected with averaging by colour (spin);  $\ell_i$  is the charge of light quarks in the electron charge units;  $J_{\mu\nu}(K_1, K_2)$  is the lepton spur.

$J_{\rho\sigma}(\ell_1, \ell_2)$ ,  $K_{\mu\nu\rho\sigma}(P_1, P_2, q, \Delta)$  are the spurs of heavy and light quarks respectively.

A(1) can be rewritten in the following form:

$$d\hat{\sigma}_i = \frac{e^4 g^4 \cdot \ell_i^2}{24(2\pi)^8 (K_1 P_1) \cdot \Delta^4 q^4} \cdot J_{\mu\nu}(K_1, K_2) \cdot K_{\mu\nu\rho\sigma}(P_1, P_2, q, \Delta) \times \\ \times f_{\rho\sigma}(\Delta) d\Delta^2 \frac{d^3 \vec{K}_2}{2E_2} \cdot \frac{d^3 \vec{P}_2}{2E_2} \cdot \frac{d^3 \vec{\Delta}}{2\Delta^2} \cdot \delta(K_1 + P_1 - K_2 - P_2 - \Delta)$$

$$f_{\rho\sigma}(\Delta) = J_{\rho\sigma}(\ell_2, \ell_1) \frac{d^3 \vec{\ell}_1}{2\ell_{10}} \cdot \frac{d^3 \vec{\ell}_2}{2\ell_{20}} \delta(\Delta - \ell_1 - \ell_2) = \\ = -\frac{2\pi}{3} (2m_c^2 + \Delta^2) \cdot \left(1 - \frac{4m_c^2}{\Delta^2}\right)^{1/2} \cdot \left(g_{\rho\sigma} - \frac{\Delta_\rho \Delta_\sigma}{\Delta^2}\right)$$

Using the gauge invariance, after standard calculations (ignoring masses of leptons and light quarks  $K_1^2 = K_2^2 = P_1^2 = P_2^2 = 0$ ) we obtain

$$d\sigma_i = \frac{4e_i^2}{9(2\pi)^3} \cdot \frac{d^2 \cdot d_s^2}{\Delta^4 \cdot Q^4 \cdot S} (2m_c^2 + \Delta^2) \beta \cdot Q(K_1, K_2, P_1, P_2) d\Delta^2 d\Gamma$$

where

$$\beta = \left(1 - \frac{4m_c^2}{\Delta^2}\right)^{1/2}; \quad S = (K_1 + P_1)^2 = 2(K_1, P_1) \quad A(2)$$

$$d\Gamma = \frac{d^3 \Delta}{2\Delta_0} \cdot \frac{d^3 K_2}{2E_2} \cdot \frac{d^3 P_2}{2E_2} \cdot \delta(K_1 + P_1 - K_2 - P_2 - \Delta)$$

$$\begin{aligned} Q(K_1, K_2, P_1, P_2) &= g_{\rho\sigma} \cdot J_{\mu\nu}(K_1, K_2) K_{\mu\nu\rho\sigma}(P_1, P_2, q, \Delta) = \\ &= -16 \left\{ Q^2 \left( \frac{\mathcal{X}_1}{\mathcal{X}_2} + \frac{\mathcal{X}_2}{\mathcal{X}_1} \right) + 2 \frac{\Delta^2}{\mathcal{X}_1^2} S(S - \mathcal{X}_1 - Q^2) + \Delta^2 Q^4 \left( \frac{1}{\mathcal{X}_1^2} + \frac{1}{\mathcal{X}_2^2} \right) + \right. \\ &+ 8 \frac{\Delta^2}{\mathcal{X}_2^2} (K_1, P_2) \cdot \left( (K_1, P_2) + \frac{\mathcal{X}_2 + Q^2}{2} \right) - \frac{4}{\mathcal{X}_1 \mathcal{X}_2} \left[ -\frac{Q^2 S}{2} (S - \mathcal{X}_1 - Q^2) - \right. \\ &- Q^2 (K_1, P_2) \cdot \left( (K_1, P_2) \cdot 2 + \mathcal{X}_2 + Q^2 \right) - Q^4 (P_1, P_2) + \Delta^2 Q^2 (P_1, P_2) - \\ &\left. \left. - \Delta^2 (K_1, P_2) \cdot (2S - \mathcal{X}_1 - Q^2) - \frac{\Delta^2}{2} S(\mathcal{X}_2 + Q^2) \right] \right\} \end{aligned}$$

Here  $\mathcal{X}_1 = (P_1 + q)^2 = Sy(1-x)$ ;  $\mathcal{X}_2 = (P_2 - q)^2 = -Sy(1-z) + \Delta^2$

The phase space in the chosen invariant variable has the form

$$d\Gamma = \frac{\pi S y}{8} dx dy dz d\psi_g$$

Kinematic restrictions are as follows

$$0 \leq \varphi_g \leq 2\pi,$$

A(3)

$$0 \leq z \leq 1 - \frac{\Delta^2}{S_4(1-x)}, \quad X_H \leq X \leq 1 - \frac{\Delta^2}{S_4}$$

where  $\varphi_g$  is the azimuthal angle of the outgoing gluon with respect to the lepton plane (according to the assumption of semilocal duality  $\varphi_g = \varphi$ ). It is easy to make sure that in the photon and quark "c" system  $\vec{q} + \vec{P}_1 = \vec{P}_2 + \vec{\Delta} = 0$ , the dependence on  $\varphi$  in A(2) remains only in the members  $\sim (K_1 P_2)$ . Singling out this dependence in obvious form we obtain the formula (9) given in the text.

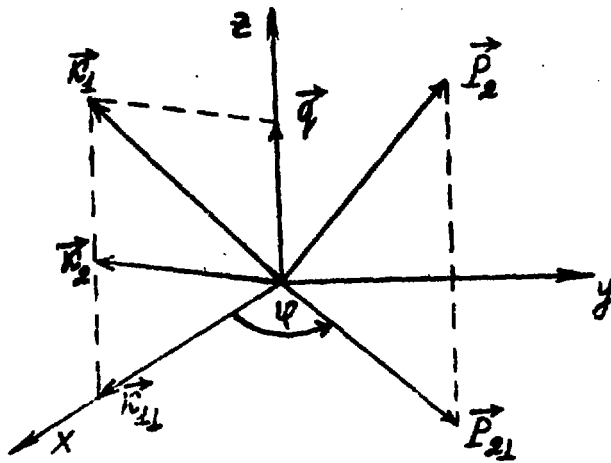


FIG. 1

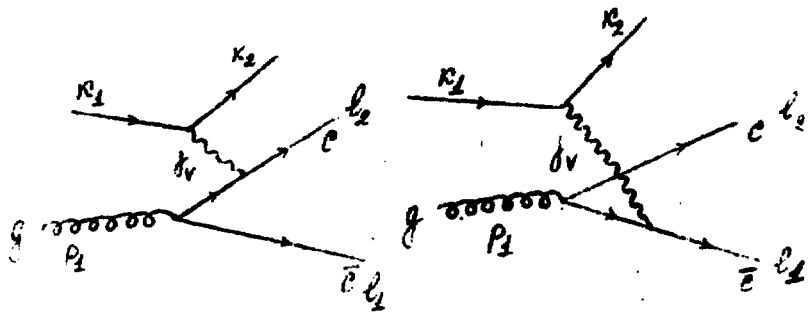


FIG. 2

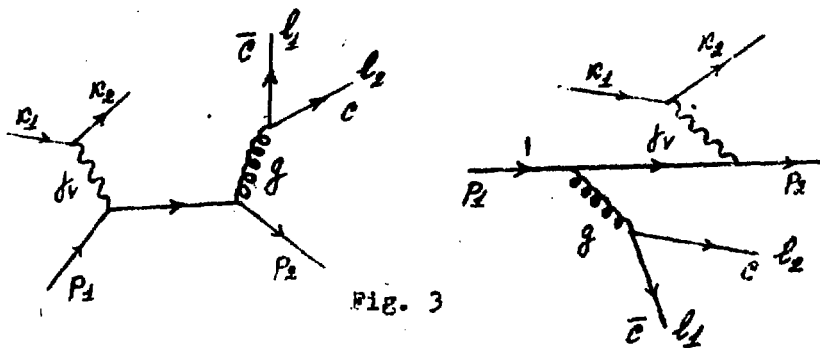


FIG. 3

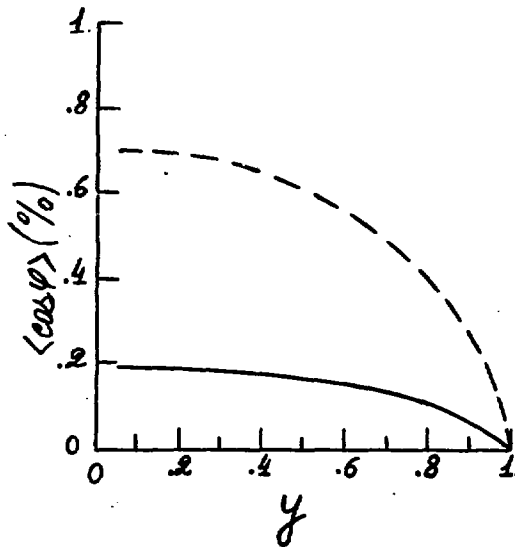


Fig. 4

Dependence of  $\langle \cos \psi \rangle$  on  $y$  without consideration of  $K_1$ , at fixed  $\chi_H$  and  $Q^2$  with  $G(\xi) = G^c(\xi)$ . Distribution  $G^B(\xi)$  for large  $\chi_H$  gives a value  $R$  times smaller. The solid line corresponds to  $\chi_H = 0.4$ ,  $Q^2 = 27 \text{ GeV}^2$ ; the dotted one - to  $\chi_H = 0.1$ ,  $Q^2 = 30 \text{ GeV}^2$ .

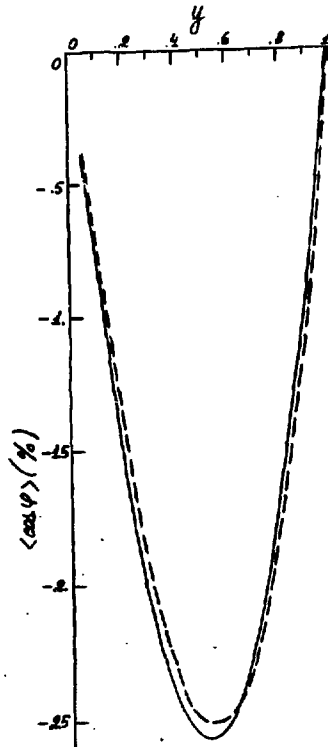


Fig. 5

Dependence of  $\langle \cos \psi \rangle$  on  $Y$  at singled out contribution to intrinsic transverse motion for fixed  $X_H$  and  $Q^2$ ,  $G(\xi) = G^c(\xi)$ . The solid line corresponds to  $X_H = 0.5$ ,  $Q^2 = 27 \text{ GeV}^2$ . The dotted one - to  $X_H = 0.1$ ,  $Q^2 = 30 \text{ GeV}^2$ . At other fixed values of  $X_H$  and  $Q^2$  we have analogous distributions.

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ВОЗМОЖНАЯ ПРОВЕРКА КХД ПАРТОННОЙ МОДЕЛИ  
В ПОЛУИНКЛУЗИВНОМ  $\mu^-$  - РОЖДЕНИИ  $J/\psi$  - МЕЗОНА  
( на английском языке, перевод Л.Н.Багдасаряна )

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