ԵՐԵՎԱՆԻ ՖԻԶԻԿԱՅԻ ԻՆՍՏԻՏՈՒՏ
ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

ЕФИ-503(46)-81

K.Z.ATSAGORTSIAN, S.S.ELBAKIAN

MODULATIONAL INSTABILITY IN PLASMA-ELECTRON BEAM SYSTEM

ԵՐԵՎԱՆ 1981 ЕРЕВАН

ЕФИ-503(46)-81

К.З.АЦАГОРЦЯН, С.С.ЭЛБАКЯН

МОДУЛЯЦИОННАЯ НЕУСТОЙЧИВОСТЬ В СИСТЕМЕ ПЛАЗМА - ЭЛЕКТРОННЫЙ ПУЧОК

В статье методом Крылова-Боголюбова-Митропольского изучается поведение нелинейной волны малой, но конечной амплитуды, возникающей в системе плазма-пучок при движении через плазму моноэнергетического электронного пучка. Для амплитуды возбуждаемой волны получено нелинейное уравнение Шредингера, исходя из которого получено условие модуляционной устойчивости или неустойчивости плоской монохроматической волны.

Ереванский физический институт
Ереван 1981

ЕФИ-503(46)-8I

K.Z.ATSAGORTSIAN, S.S.ELBAKIAN

MODULATIONAL INSTABILITY IN PLASMA-ELECTRON BEAM SYSTEM

In the work the behaviour of the nonlinear wave of small but finite amplitude arising in the plasma-beam system when passing through the monoenergetic electron beam plasma is studied using the method of Krylov-Bogoliubov-Mitropolsky. For the excited wave amplitude the Schrodinger nonlinear equation is obtained proceeding from which the condition of modulational stability or instability of the plane monochromatic wave is obtained.

Yerevan Physics Institute

Yerevan 1981

ЕФИ-503(46)-8I

YEREVAN PHYSICS INSTITUTE

K.Z.ATSAGORTSIAN, S.S.ELBAKIAN

MODULATIONAL INSTABILITY IN PLASMA-ELECTRON BEAM SYSTEM

Yerevan 1981

In ref. [1] the foundations of the nonlinear theory of monoenergetic electron beam interaction with plasma for the case of the system stationary oscillations were laid down. It was shown that in such a system there may be produced under certain conditions a nonlinear electrostatic wave of sufficiently high amplitude, which is of practical interest for a number of problems in accelerator technique and plasma electronics. Refs [2-3] are the natural continuation and development of ref. [1]. In ref. [4] the authors took into account the electron beam thermal spread effect and indicated the possibility of the existence of solitary waves in the plasma-beam system. In ref. [5] the state of instable waves is investigated in such a system. In all the above-cited works the motion of plasma electrons is described by linear equations, whereas the occurance of nonlinear waves is due to the plasma influence on the motion of the beam electrons.

Here we investigated, using the asymptotic method of Krylov-Bogoliubov-Mitropolsky [6-7], the behaviour of the nonlinear wave of small finite amplitude occurred in the plasma-beam system at passage of the monoenergetic electron beam through the plasma. In this case from the very beginning the stationarity of the emerged oscillations is not assumed, and the motion of electrons of both the beam and plasma is described by nonlinear equations. This statement of a problem is of certain interest in connection with already available [8] and possible future experiments. For the excited-wave amplitude a nonlinear Schrodinger equation is obtained, following which the condition of modulational stability or instability of a plane monochromatic wave is obtained.

1. The initial equations that describe the interaction of the infinite cool electron beam of nonperturbated density n_{b_0} moving along the Z axis with the initial velocity u_{b_0} with the cool infinite plasma of electron density n_{e0} and immobile ions are: motion equations and equations of plasma and beam electron continuity, and the Poisson equation for field E of wave

$$\frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial z} = -E, \quad \frac{\partial n_e}{\partial t} + \frac{\partial}{\partial z}(n_e u_e) = 0,$$

$$\frac{\partial u_b}{\partial t} + u_b \frac{\partial u_b}{\partial z} = -E, \quad \frac{\partial n_b}{\partial t} + \frac{\partial}{\partial t}(n_b u_b) = 0 \qquad (1)$$

$$\frac{\partial E}{\partial z} = n_0 - n_e - n_b .$$

Here the electron densities of plasma and beam are measured in the units of the nonperturbated density n_{e0} of plasma electrons, velocities are measured in the units of the initial velocity of the electron beam u_{b0}, and field E - in the units of $\frac{m u_{b0} \omega_e}{e}$ where $\omega_e = \sqrt{\frac{4\pi e^2 n_{e0}}{m}}$; $t = \omega_e t, z = \frac{z}{q}$

where $\tau_d = \frac{u_{b0}}{\omega_e}$.

2. To investigate the nonlinear behaviour of wave solutions of the set of equations (1) of small finite amplitude slowly varying in time and space, we shall use, as stated above the Krylov-Bogoliubov-Mitropolsky method [6-7]. The amplitude smallness means that the distances passed by the electron in the wave field are small as compared with the characteristic ones. This leads to the condition that the electric field amplitude must be far less than $\frac{m u_{b0} \omega_e}{e}$. Let us introduce "a small parameter" ε that characterizes the ratio of the typical length of the wave or period to the typical length or interval of time for modulations, and look for the solution of the equation set (1) in the form of expansion in powers ε:

$$\begin{bmatrix} n_e \\ n_b \\ u_e \\ u_b \\ E \end{bmatrix} = \begin{bmatrix} 1 \\ n_{b0} \\ 0 \\ 1 \\ 0 \end{bmatrix} + \varepsilon \begin{bmatrix} n_e^{(1)} \\ n_b^{(1)} \\ u_e^{(1)} \\ u_b^{(1)} \\ E^{(1)} \end{bmatrix} + \varepsilon^2 \begin{bmatrix} n_e^{(2)} \\ n_b^{(2)} \\ u_e^{(2)} \\ u_b^{(2)} \\ E^{(2)} \end{bmatrix} + \varepsilon^3 \begin{bmatrix} n_e^{(3)} \\ n_b^{(3)} \\ u_e^{(3)} \\ u_b^{(3)} \\ E^{(3)} \end{bmatrix} + \dots \qquad (2)$$

Since the wave amplitude is assumed small, the wave will not differ much from the sinusoidal one, i.e. higher harmonics that are in equilibrium with the main one are small. Then the wave may be characetrized by the wave number K and frequency ω of the main harmonic, and the main averaged nonlinear effect for such a wave is the dependence of frequency on amplitude [9]. As following from this we choose $E^{(1)}$ as a plane monochromatic wave

$$E^{(1)} = a e^{i\psi} + \bar{a} e^{-i\psi}, \tag{3}$$

where a is the complex amplitude normalized for $\frac{m u_{e_0} \omega_e}{e}$ i.e. $a \ll 1$, $\bar{a}$ is the complex-conjugated of a, $\psi = K z - \omega t$ is the phase where the frequency ω and the wave number K satisfy the linear dispersion equation

$$1 - \frac{1}{\omega^2} - \frac{n_{b0}}{(\omega - K)^2} = 0, \tag{4}$$

where $\omega \equiv \frac{\omega}{\omega_e}$, $K \equiv K r_d$.

Each of the values $n_e^{(i)}$, $n_b^{(i)}$, $u_e^{(i)}$, $u_b^{(i)}$ and $E^{(i)} (i = 1,2,3)$ being in expansion (2) in powers ε depends on z and t through a, $\bar{a}$ and ψ, and the complex amplitude a as a slowly varying function of time and coordinates, is determined by the following differential equations:

$$\frac{\partial a}{\partial t} = \varepsilon A_1(a, \bar{a}) + \varepsilon^2 A_2(a, \bar{a}) + \cdots, \tag{5}$$

$$\frac{\partial a}{\partial z} = \varepsilon B_1(a, \bar{a}) + \varepsilon^2 B_2(a, \bar{a}) + \cdots. \tag{6}$$

The unknown functions A_1, B_1, A_2, B_2 are determined, as usual, from the requirement of disappearance of secularly growing terms in expansions (2) in powers ε. Substituting expansions (2) into expansion set (1) and equating the terms at equal powers ε we obtain a set of equations for the first, second and third approximations of the unknowns. In the first approximation we have a dispersion relation (4) and the following expressions for densities and velocities of plasma and beam electrons:

$$n_e^{(1)} = -\frac{iK}{\omega^2}\left(ae^{i\psi} - \bar{a}e^{-i\psi}\right), \tag{7}$$

$$n_b^{(1)} = \frac{iK}{\omega^2}(1-\omega^2)\left(ae^{i\psi} - \bar{a}e^{-i\psi}\right), \tag{8}$$

$$u_e^{(1)} = -\frac{i}{\omega}\left(ae^{i\psi} - \bar{a}e^{-i\psi}\right), \tag{9}$$

$$u_b^{(1)} = \frac{i(1-\omega^2)(\omega-K)}{n_{b0}\,\omega^2}\left(ae^{i\psi} - \bar{a}e^{-i\psi}\right) \tag{10}$$

Making use of the first approximation (3), (7)-(10) we obtain in the second approximation a set of differential equations for $n_e^{(2)}$, $n_b^{(2)}$, $u_e^{(2)}$, $u_b^{(2)}$ and $E^{(2)}$ from which the differential equation for $E^{(2)}$ follows:

$$\frac{d^2E^{(2)}}{d\psi^2} + E^{(2)} = \frac{2i}{\omega^3}\left\{\frac{\omega^3-K}{\omega-K}A_1 - \frac{\omega(1-\omega^2)}{\omega-K}B_1\right\}e^{i\psi} + \tag{11}$$
$$+\frac{3iKa^2}{\omega^4}\left(1+\frac{(1-\omega^2)}{n_{b0}}\right)e^{2i\psi} + \text{c.c.} \quad ,$$

where c.c. denotes complex conjugation. In order that $E^{(2)}$ would contain no resonance terms we have to require the coefficients before $e^{\pm i\psi}$ vanish, i.e.

$$A_1 + v_g B_1 = 0 \tag{12}$$

where $v_g = \frac{d\omega}{dK} = \frac{\omega(\omega^2-1)}{\omega^3-K}$ is the wave group velocity. With the help of (5) and (6) we can in the first order over ε

represent A_1 and B_1 respectively as $\frac{\partial a}{\partial t}$ and $\frac{\partial a}{\partial z_1}$ where $t_1 = \varepsilon t$, $z_1 = \varepsilon z$. Then eq.(12) will be written as

$$\frac{\partial a}{\partial t_1} + v_g \frac{\partial a}{\partial z_1} = 0 . \tag{13}$$

Eq.(13) shows that in the lower approximation over ε the wave amplitude a remains invariable in the system of reference moving with the group velocity v_g. Keeping in mind (13) and linear dispersion relation (4) we have for the unknowns in the second approximation the following free from resonance terms expressions:

$$E^{(2)} = -\frac{i K a^2}{\omega^4}\left[1 + \frac{(\omega^2-1)^2}{n_{b0}}\right] e^{2i\psi} + b e^{i\psi} + c.c. , \tag{14}$$

$$n_b^{(2)} = -\frac{K^2 a^2}{2\omega^4}\left[\frac{3(\omega^2-1)^2}{n_{b0}} + \frac{n_{b0}}{(\omega-K)^2} + \frac{(\omega^2-1)^2}{(\omega-K)^2}\right] e^{2i\psi} - \tag{15}$$

$$-\left[\frac{(\omega^2-1)(\omega^3+K)}{\omega^2(\omega^3-K)} B_1 + \frac{i n_{b0} K b}{(\omega-K)^2}\right] e^{i\psi} + c.c. + C_1 ,$$

$$n_e^{(2)} = -\frac{K^2 a^2}{2\omega^4}\left[3 + \frac{1}{\omega^2} + \frac{(\omega^2-1)^2}{\omega^2 n_{b0}}\right] e^{2i\psi} + \tag{16}$$

$$+\left\{\frac{B_1}{\omega^2}\left[\frac{2K(\omega^2-1)}{\omega^3-K} - 1\right] - \frac{i K b}{\omega^2}\right\} e^{i\psi} + c.c. + C_2 ,$$

$$U_b^{(2)} = -\frac{K a^2}{2\omega^4}\,\frac{(\omega^2-1)(2\omega^2-1) + n_{b0}}{n_{b0}(\omega-K)} e^{2i\psi} + \tag{17}$$

$$+\left[\frac{1-\omega^2}{n_{b0}\,\omega^2}\;\frac{\omega-K}{\omega^3-K} B_1 - \frac{i b}{\omega-K}\right] e^{i\psi} + c.c. + C_3 ,$$

$$u_e^{(2)} = -\frac{\kappa a^2}{2\omega^3}\left[1 + \frac{1}{\omega^2} + \frac{(\omega^2-1)^2}{\omega^2 n_{b_0}}\right]e^{2i\psi} + \left[\frac{\omega^2-1}{\omega(\omega^3-\kappa)} B_1 - \frac{ib}{\omega}\right]e^{i\psi} + c.c. + C_4 , \qquad (18)$$

where complex functions b and $\tilde{b}$ and real functions C_1, C_2, C_3 and C_4 are independent of ψ and dependent on a and $\tilde{a}$ only. They will be determined from the requirement of absence of secularly growing terms in the equations of a higher approximation.

3. Consider now the equations in the third approximation. Making use of the solution of the first (3), (7)-(10) and second (14)-(18) approximations we obtain a set of differential equations for $n_e^{(3)}$, $n_b^{(3)}$, $u_e^{(3)}$, $u_b^{(3)}$ and $E^{(3)}$ which contain secular terms of two types: secular constants and secular-resonance terms. After averaging over rapidly oscillating phase ψ we obtain the number of equations necessary to determine the functions C_1, C_2, C_3, C_4 which are given by the following formulae:

$$C_1 = -C_2 = \frac{a\tilde{a}\kappa^2(\omega^2+1)(\omega^3-\kappa)(3\omega^3-6\omega-2\kappa\omega^2+3\kappa)}{\omega^6(\omega-\kappa)^2(\omega^4-3\omega^2+3)} + d_1, \qquad (19)$$

$$C_3 = -\frac{a\tilde{a}\kappa}{(\omega-\kappa)^3}\left[2 + \frac{\kappa(3\omega^3-6\omega-2\kappa\omega^2+3\kappa)}{\omega^4(\omega^4-3\omega^2+3)}\right] + d_3 , \qquad (20)$$

$$C_4 = -\frac{a\tilde{a}\kappa}{\omega^3}\left[2 + \frac{\kappa(\omega^2-1)^2(3\omega^3-6\omega-2\kappa\omega^2+3\kappa)}{\omega^2(\omega-\kappa)^2(\omega^4-3\omega^2+3)}\right] + d_4 , \qquad (21)$$

where d_1, d_3 and d_4 are constants independent of ψ, a and $\tilde{a}$

The requirement as to expression for $E^{(3)}$ not to contain resonance terms brings to the following equation:

$$i(A_2 + v_g B_2) + P\left(B_1 \frac{\partial B_1}{\partial a} + \tilde{B}_1 \frac{\partial B_1}{\partial \bar{a}}\right) = Q|a|^2 a + Ra, \qquad (22)$$

where

$$p = -\frac{1}{2}\frac{d v_g}{dK} = \frac{3}{2}\frac{\omega^3(\omega^2-1)(\omega-K)}{(\omega^3-K)^3},$$

$$(23)$$

$$Q = \frac{n_{60}}{4\omega^3(\omega^3-K)(\omega-K)(\omega^4-3\omega^2+3)}\left\{-3K^2(\omega^4-3\omega^2+3)\left(1-\frac{\omega^2}{(\omega-K)^2}\right)^2 + 2\left[(\omega^3-3K)\left(1-\frac{\omega^2}{(\omega-K)^2}\right) + 2K\omega^2\right]^2\right\},$$

$$R = \frac{\omega(\omega-K)}{2(\omega^3-K)}\left[\frac{2K(\omega^2-1)}{\omega-K}d_3 + \frac{2K}{\omega}d_4 - \left(1-\frac{\omega^2}{(\omega-K)^2}\right)d_1\right].$$

Using (5) and (6) eq. (22) can be written as a nonlinear Schrodinger equation for the amplitude a :

$$i\frac{\partial a}{\partial \tau} + p\frac{\partial^2 a}{\partial \xi^2} = Q|a|^2 a + Ra \qquad (24)$$

where

$$\xi = \varepsilon(z - v_g t), \quad \tau = \varepsilon^2 t. \qquad (25)$$

Arbitrary constants d_1, d_3 and d_4 that involve in the expression for R must be determined from initial or boundary conditions. However, since the term Ra describes the linear interaction, it brings but the wave phase shift and may be excluded from eq. (25) by replacing [10]

$$a \to a\exp\left(-i\int^{\tau} R(\tau')\,d\tau'\right). \qquad (26)$$

Thus, as it follows from eq.(24), the consideration of electron nonlinearity in the Langmuir oscillations at the electron beam interaction with plasma leads to the dependence of oscillation frequency on the amplitude.

At $n_{b_0} = 0$ ("switching off" the beam) $Q = 0$ (one must take into account the conditions of hydrodynamic application of the problem $\frac{|\omega - k|}{k} \gg \frac{V_{Tb}}{u_{b_0}}$ consideration where V_{Tb} is the beam thermal spread), i.e. the nonlinear correction to the Langmuir oscillation frequency vanishes, which is in agreement with the well-known results [11,12].

The solutions of the Schrodinger nonlinear equation are intensively studied [13,15] in connection with the available appendices of theory of waves in plasma and also in connection with the appendices in other fields of physics. As it follows from the Schrodinger nonlinear equation theory [10,13] the plane wave is modulatingly stable if $PQ > 0$ and instable, if $PQ < 0$ Here the maximum increment of the modulational instability increase is given by the expression

$$\gamma_m = Q \rho_0^2 , \tag{27}$$

where ρ_0 is the wave initial amplitude.

Assuming that frequency ω is a real number and solving the dispersion relation (4) with respect to the wave vector k we obtain:

$$K^{(1)} = \omega\left(1 + \frac{n_{b_0}^{1/2}}{\sqrt{\omega^2 - 1}}\right), \quad K^{(2)} = \omega\left(1 - \frac{n_{b_0}^{1/2}}{\sqrt{\omega^2 - 1}}\right). \tag{28}$$

Substituting (28) into the product PQ we shall find the regions of modulational instability over frequency ω in the frequency region ($\omega > 1$) where both the waves $K^{(1)}$ and $K^{(2)}$ are stable in the linear approximation (Figs 1-2). For the first wave $K^{(1)}$ there are two regions of modulational instability (I and III) (Fig. 1) practically for all densities n_{b_0} up to unity (II is the modulational instability region) whereas the second wave $K^{(2)}$ is instable in the region I (Fig. 2).

In Figs 3-5 the dependences of the modulational instability increase increments determined by the expression (27) upon frequency ω for various values of n_{b_0} whence it is seen that the maximum value of the increase increment decreases as the density $n_{b_0}^{*}$ of the beam electrons grows. For sufficiently large frequencies $\omega \gg 1$ the increase increment γ_m / ρ_0^2 tends to the constant value $1/4 n_{b_0}^{3/2}$ Besides there have been quantitatively found the solutions of the dispersion relation (4) with respect to ω and substituted in the expressions for the product PQ and the regions of modulational instability for the dispersion relation four branches have been found which are presented in Figs (7-10). The curve 1 in Figs 8 and 9 corresponds to the minimum value of the wave number $K_m = \left(1 + n_{b_0}^{1/3}\right)^{3/2}$ above which these two branches of the dispersion relation are stable in linear approximation.

In the modulational instability regions the Schrodinger nonlinear equation (24) admits the following soliton solution for the envelopes [16]:

$$P(\xi - u_g \tau) = \sqrt{2}\rho_0 \, \mathrm{sech}\left[K_m(\xi - u_g \tau)\right], \qquad (29)$$

where ρ is coupled with the complex amplitude by the relation $a=\rho(\xi,\tau)e^{i\theta(\xi,\tau)}$ (ρ and θ are real). In the formula (29) $U_g = 2PK$, $K_m = \left(\frac{|Q|}{|P|}\right)^{1/2}\rho_0$ is the maximum value of the modulation wave vector. The soliton given by the formula (29) is a stable formation. It is a solitary wave packet propagating with velocity $v = U_g$ without form distortion. The soliton amplitude maximum value is $\sqrt{2}\,\rho_0$. Let us give a typical numerical example. At $n_{e0} = 10^{11}cm^{-3}$ and electron energy $\frac{mU_e^2}{2} = 10\,keV$ the electrostatic fields in plasma owing to the modulational instability development can reach the values of the order of $E^{(1)} \sim 10^4 V/cm$ at $\rho_0 = 0.1$ and $E^{(1)} \sim 10^3 V/cm$ at $\rho_0 = 0.01$.

In conclusion the authors would like to thank A.Ts.Amatuni and A.A.Rukhadze for discussing the results.

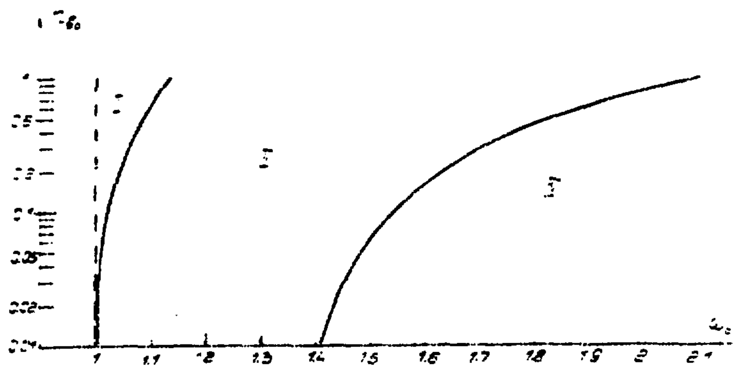

Fig. 1 Regions of modulational instability (I and III) as functions of the frequency ω_c for the wave $K^{(1)}$ (in the region II the wave is stable).

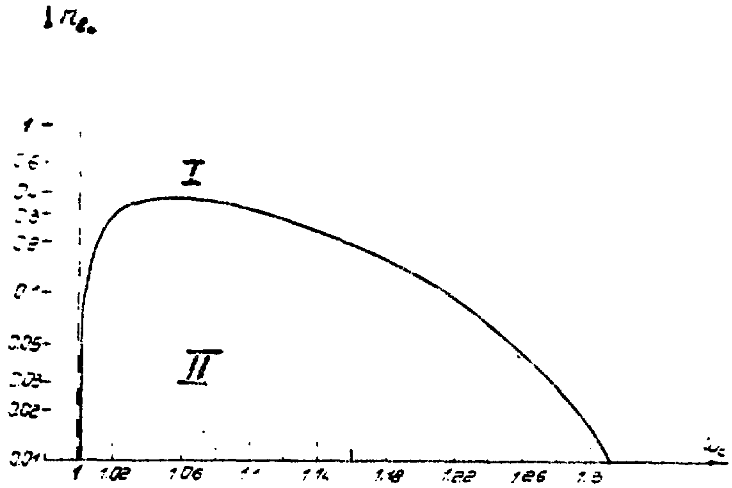

Fig. 2 Region of the modulational instability (I) as a function of critical frequency ω_c for the wave $K^{(2)}$ (in the region II the wave is stable).

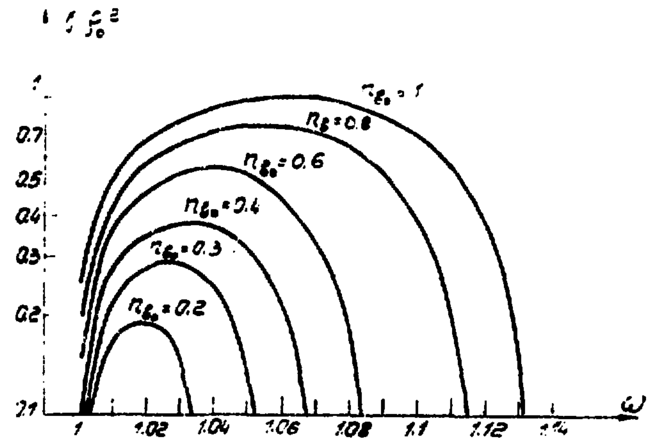

Fig. 3 Dependence of the maximum increase increment γ_m/ρ_0^2 of the modulational instability on the frequency ω for various values of n_{b_0} for the wave $K^{(1)}$ in the region I.

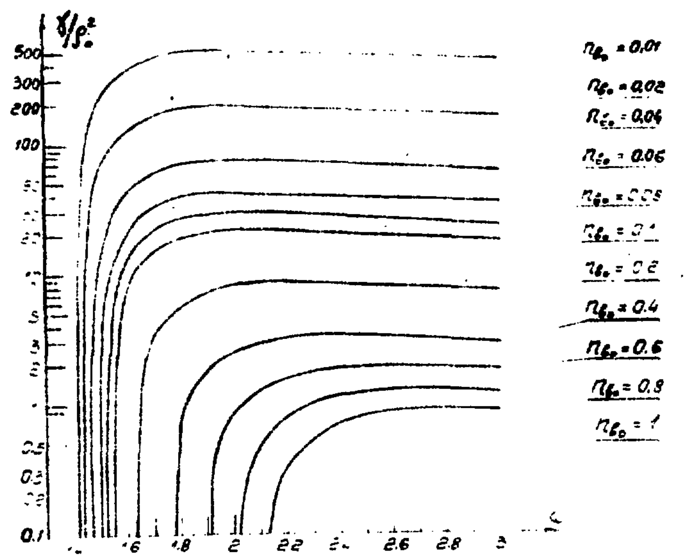

Fig. 4 Dependence of the maximum increase increment γ_m/ρ_0^2 of the modulational instability on the frequen ω for the wave $K^{(1)}$ in the region III.

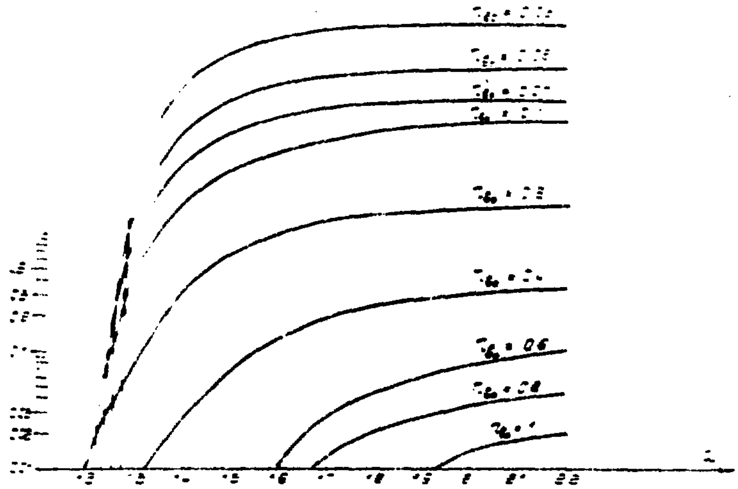

Fig. 5 Dependence of the maximum increase increment δ_m/ρ_0^2 of the modulational instability on the frequency for the wave $K^{(2)}$

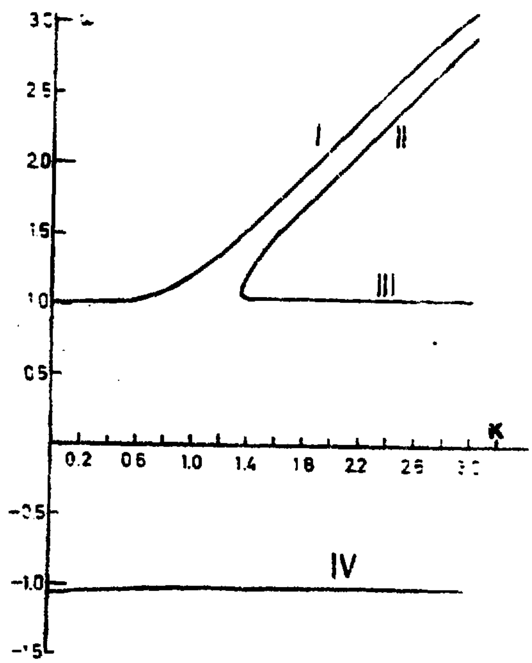

Fig. 6 Dispersion curves for $n_{\ell_0} = 0,01$

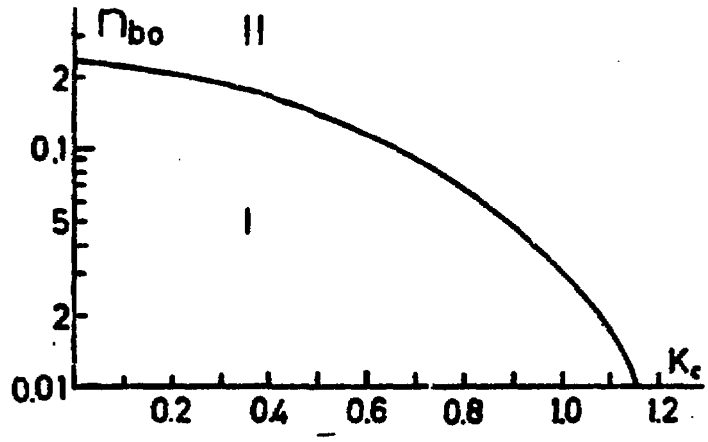

Fig. 7 Region of modulational instability (II) as a function of the wave number K_c for the dispersion curve first branch (see Fig. 6).

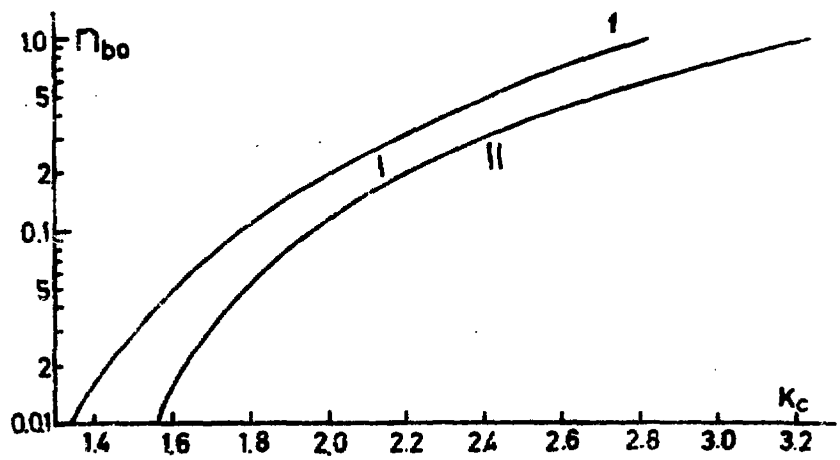

Fig. 8 Region of modulational instability (region II) as a function of the wave number K_c for the dispersion curve second branch.

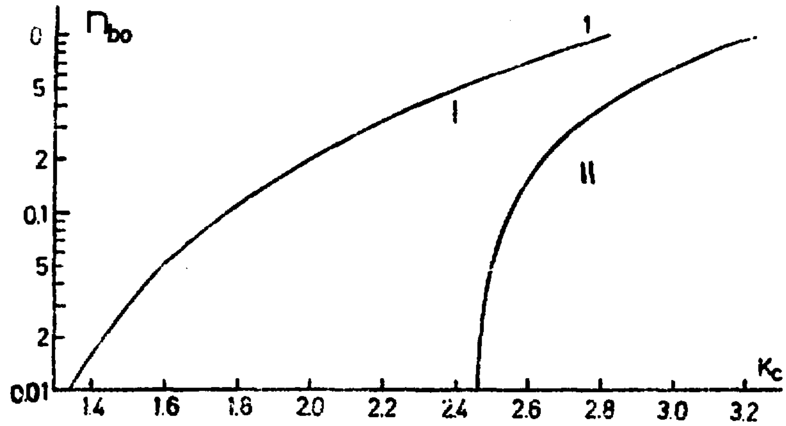

Fig. 9 Region of modulational instability (region II) as a function of the wave number K_c for the dispersion curve third branch.

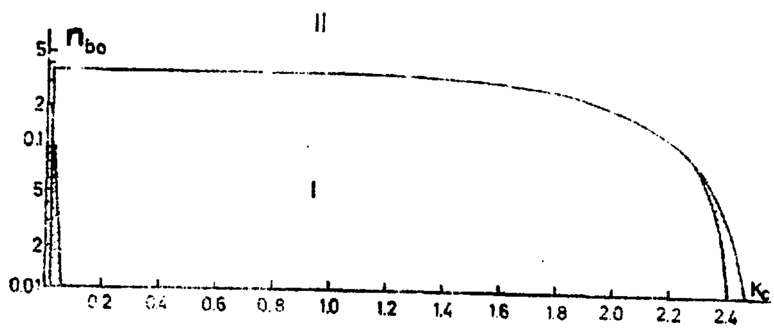

Fig. 10 Region of modulational instability (region II) as a function of the wave number K_c for the dispersion curve fourth branch.

References

1. Kovtun R.I., Rukhadze A.A. On the Theory of Nonlinear Interaction of the Low Density Relativistic Electron Beam with Plasma.- JETP, 1970, vol. 58, p. 1709; On the Nonlinear Interaction of Electron Beam with Plasma.- Radiotekhnika i elektronika, 1970, vol. 15, p. 2124.
2. Thode L.E., Sudan R.N. Two-stream instability of plasmas by relativistic electron beams.- Phys. Rev. Lett., 1973, vol. 30, N.16, p. 732.
3. Ivanov S.T., Reshetnikova K.A., Filipova N.A. On the Nonlinear Theory of Electron Beam Interaction with Plasma.- Preprint JINR-9-11370, Dubna, 1978.
4. Bliokh Yu.P., Lubarsky M.G. Nonlinear Potential Oscillations of Cool Infinite Plasma with Charged Particle Beam.- Fizika Plasmi, 1979, vol. 5, p. 339.
5. Shapiro V.D., Shevchenko V.I. The Wave-Particle Interaction in Non-equilibrium Media.- Izv. Vuzov/Radiofizika 1976, vol. 19, p. 767.
6. Bogoliubov N.N., Mitropolsky Yu.A. Asymptotic Methods in the Theory of Nonlinear Oscillations.-Fizmatgiz, Moscow, 1974, chapters 1,5.
7. Kakutani T., Sugimoto N. Krylov-Bogoliubov-Mitropolsky method for nonlinear wave modulation.- Physics of Fluids, 1974, vol. 17, N.8, p. 1617.
8. Antipov S.V., Nezlin M.V., Snezhkin E.N., Trubnikov A.S. Excitation of Langmuir Solitons with Monoenergetic

Electron Beam.- JETP, 1979, vol. 76, p. 1571.

9. Kadomtsev B.B. Cooperative Phenomena in Plasma.-Fizmatgiz, Moscow, 1976, p. 130.

10. Taniuti T., Jajima N. Perturbation method for a nonlinear wave modulation I.- J.Math. Phys., 1969, vol. 10, N.8, p. 1369.

11. Akhiezer A.I., Lubarsky G.Ya. On the Nonlinear Theory of Electron Plasma Oscillations.- Dokl. Akad. Nauk SSSR, 1951, vol. 80, p. 193.

12. Sturrock P.A. Non-linear effects in electron plasmas.- Proc. Roy. Soc., 1957, series A, vol. 242, N.5, p. 277.

13. Zakharov V.E., Shabat A.B. Exact Theory of Two-Dimensional Self-Focusing and One-Dimensional Self-Modulational of Waves in Nonlinear Media.- JETP, 1971, vol. 61, p.118.

14. Asano N. Modulation for nonlinear wave in dissipative or unstable media.- J.Phys.Soc. Japan, 1974, vol. 36, N.3, p. 861.

15. Petviashvili V.I. Theory of Strong Nonlinear Waves and Solitons in Plasma.- Dis. for the degree of Doctor of Phys. and Math., Moscow, Kurchatov Inst. of Atom. Ener., 1978, p. 123.

16 Rasmussen J.J. Finite amplitude electron plasma waves in a cylindrical waveguide.- Plasma Phys., 1978, vol. 20, N.10, p. 997.

The manuscript was received 30 April 1980.
After retreatment 6 October 1981.

К.З.АЦАГОРЦЯН, С.С.ЭЛБАКЯН

МОДУЛЯЦИОННАЯ НЕУСТОЙЧИВОСТЬ В СИСТЕМЕ
ПЛАЗМА - ЭЛЕКТРОННЫЙ ПУЧОК

(на английском языке, перевод Л.Н.Багдасаряна)
Ереванский физический институт

Тех.редактор А.С.Абрамян

Заказ 538 ВФ-04925 Тираж 299

Препринт ЕФИ Формат издания 60x84/16
Подписано к печати 26/X-81г. 1,5 уч.изд.л. Ц.10 к.

Издано Отделом научно-технической информации
Ереванского физического института, Ереван-36, пер.Маркаряна 2

индекс 3624