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S.G.MATINYAN, G.K.SAVVIDY, N.G.TER-ARUTUNYAN-SAVVIDY

STOCHASTICITY OF YANG-MILLS CLASSICAL MECHANICS AND ITS ELIMINATION BY HIGGS MECHANISM

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С.Г.МАТИНЯН, Г.К.САВВИДИ,
Н.Г.ТЕР-АРУТЮНЯН-САВВИДИ.

СТОХАСТИЧНОСТЬ КЛАССИЧЕСКОЙ МЕХАНИКИ ЯНГА-МИЛЛСА И ЕЁ УСТРАНЕНИЕ МЕХАНИЗМОМ ХИГГСА

Показано, что механизм Хиггса устраняет обнаруженную в [I] стохастичность классической механики Янга-Миллса (Я.М.). При критическом значении параметра, характеризующего систему Я.М.-- Хиггса ($\mathcal{T}_c \simeq 0.15$), имеет место фазовый переход.

Ереванский физический институт
Ереван 1981

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STOCHASTICITY OF YANG-MILLS CLASSICAL MECHANICS AND ITS ELIMINATION BY HIGGS MECHANISM

The Higgs mechanism is shown to eliminate the discovered in [1] stochasticity of Yang-Mills (Y.M.) classical mechanics. The phase transition takes place at a critical value of the parameter characterizing the Y.M.-Higgs system ($\mathfrak{N}_c \simeq 0.15$).

Yerevan Physics Institute

Yerevan 1981

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YEREVAN PHYSICS INSTITUTE

S.G.MATINYAN, G.K.SAVVIDY, N.G.TER-ARUTUNYAN-SAVVIDY

STOCHASTICITY OF YANG-MILLS CLASSICAL MECHANICS AND ITS ELIMINATION BY HIGGS MECHANISM

Yerevan 1981

In our previous work [1] we have studied the Y.M. classical equations in the Minkowski space with no external sources, when the vector potential $A_\mu^a(x)$ in some coordinate system depends on time only (see [1, 2]). The system described by such potentials $A_\mu^a(t)$ reduces to the discrete non-linear mechanical system (Y.M. classical mechanics) with a hamiltonian [1]

$$H_{Y.M.} = \sum \frac{1}{2} (\dot{A}_i^a)^2 + \frac{g^2}{4} [(A_i^a A_i^a)^2 - (A_i^a A_j^a)^2] \qquad (1)$$

and constraint equations

$$M^a \equiv \varepsilon^{abc} A_i^b \dot{A}_i^c = 0 \qquad (2)$$

The system (1) for SU(2) group has nine degrees of freedom (a, i =1, 2,3) and four conserved integrals: $H_{Y.M.}$, $M_i \equiv \varepsilon_{ijk} A_j^a \dot{A}_k^a$. With increasing the gauge group dimensionality the number of "absent" integrals increases like $2N^2 - 6$ for SU(N) group.

The strong instability of trajectories in two- and three-dimensional subsystems (1) with the potentials

$$U_2 = \frac{g^2}{2}\left(A_1^1\ A_2^2\right)^2$$

$$U_3 = \frac{g^2}{2}\left[\left(A_1^1 A_2^2\right)^2 + \left(A_1^1 A_3^3\right)^2 + \left(A_2^2 A_3^3\right)^2\right]$$

brought us to the conclusion on their stochasticity [1]. Recently this conclusion was confirmed in [3]. A specific property of the studied in [1] system is the existence at least of enumerable set of periodical trajectories.

In the recent years great interest was attracted to the realization of one or another phase in gauge theories [4] (confinement phase - disorder, Higgs phase - order). By analogy we consider that to the absence of the total set of isolating integrals in a classical system (e.g. the system (1)) corresponds the disorder phase, while the systems with the total set of isolating integrals (when their number is equal to the number of degrees of freedom) the order phase corresponds to.

In connection with aforesaid it seems interesting to investigate phases of classical gauge systems with spontaneous breaking of symmetry.

Consider the gauge theory with isodublet breaking of SU(2) group in the gauge $A_0^a = 0$. The corresponding to (1) hamiltonian has now the form:

$$H = H_{Y.M.} + \frac{1}{2}\left(\dot{B}_a^2 + \dot{\sigma}^2\right) + \frac{g^2}{4}\left(A_i^a A_i^a\right) \times \left[\frac{B_a^2}{2} + \left(\frac{\sigma}{\sqrt{2}} + \eta\right)^2\right] + \lambda^2\left[\frac{B_a^2}{2} + \left(\frac{\sigma}{\sqrt{2}} + \eta\right)^2 - \eta^2\right] \quad (3)$$

and constraint equations are as follows:

$$\varepsilon^{abc} A_i^b \dot{A}_i^c - \frac{\eta}{\sqrt{2}}\dot{B}_a + \frac{1}{2}\left[\sigma \dot{B}_a - B_a \dot{\sigma} - \varepsilon^{abc} B_b \dot{B}_c\right] = 0 \quad (4)$$

where η is the vacuum expectation value of scalar field

$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i B_1 + B_2 \\ \sqrt{2}\eta + \sigma - i B_3 \end{pmatrix}$$

Let us study in detail the two-dimensional case (see [1]), when $A_1^1 \equiv$ $\equiv x$, $A_2^2 \equiv y$, and all the rest components A_i^a, B_a, and σ are zero:

$$H \equiv \mu^4 = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) + \frac{g^2}{2}(xy)^2 + \frac{g^2\eta^2}{4}(x^2 + y^2) \tag{5}$$

The behaviour of the system (5) is characterized by one parameter

$$\Pi = \left(\frac{g}{2}\right)^2 \left(\frac{\eta}{\mu}\right)^4 \tag{6}$$

which can be shown easily using the transformation $x \to \alpha x$, $y \to \alpha y$, $t \to \beta t$.

The purpose of this work is to calculate the critical value of parameter Π_c at which the phase transition occurs in the following sence: at large values of Π the system comes close to the integrated one, and the motion in the phase space ($x\ \dot{x}\ y\ \dot{y}$) represents the tore winding [5] (the ergodic trajectory measure is equal to zero [6]), i.e. the phase of order realizes, while for small but finite values of Π ($\Pi < \Pi_c = 0.15$) the motion just like at $\Pi = 0$ [1] is stochastic, i.e. the phase of disorder realizes.

We describe briefly the computer experiment by means of which Π_c is determined. (The first experiments of this type were done by Contopoulos, Henon, Heiles, Ford et al. [7]). The computer was programmed to solve motion equations of system (5) at given $\mathfrak{N}$, as well as to light out the points of intercept phase trajectory in space ($x\ \dot{x}\ y\ \dot{y}$) with ($y\ \dot{y}$) plane at $\dot{x} > 0$. If the motion is periodical, then the intercept occurs in the finite number of points; if it is restricted to the tore surface, then points fall down on a closed curve in ($y\ \dot{y}$) plane, and, finally, at ergodic motion the point wanders chaotically in ($y\ \dot{y}$) plane covering densely the finite area.

Fig.1 illustrates the picture in ($y\ \dot{y}$) plane for $\mathfrak{N}$ = 4.84; one can see that the points make closed curves in accord with KAM-theorem [6]. The stable periodical trajectories correspond to the centres of three small closed curves, while to two points of contact of closed curves correspond unstable periodical trajectories (separatrices [5]). Just in the vicinity of the latters occur initially "macroscopic" regions of ergodic motion of nonzero measure (see Fig.2, $\mathfrak{N}$ = 0.35, cf. [7]). When decreasing parameter $\mathfrak{N}$ the area occupied by stochastic motion increases rapidly and at a critical value $\mathfrak{N}_c \simeq 0,15$ comes to be almost equal to the whole admissible region of motion on ($y\ \dot{y}$) plane. This is illustrated by Fig.3 (all the points in this figure represent one trajectory).

It is not clear so far to what extent the above phenomenon is connected with the quantum theory of phase transitions [4]; however, to our mind, there seems to exist a close connection between them.

In conclusion the authors would like to express their gratitude to Yu.P.Mozharov and the other members of the EPI Computer centre for assistance in computing.

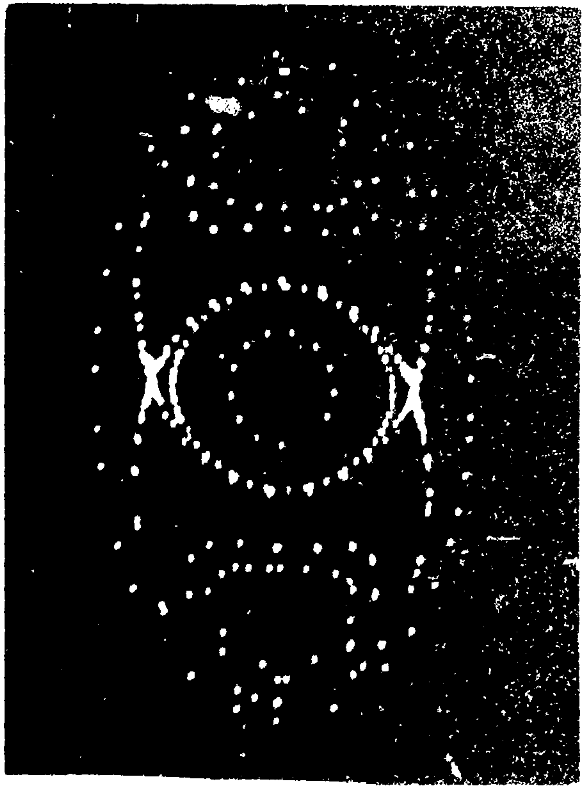

Fig.1.

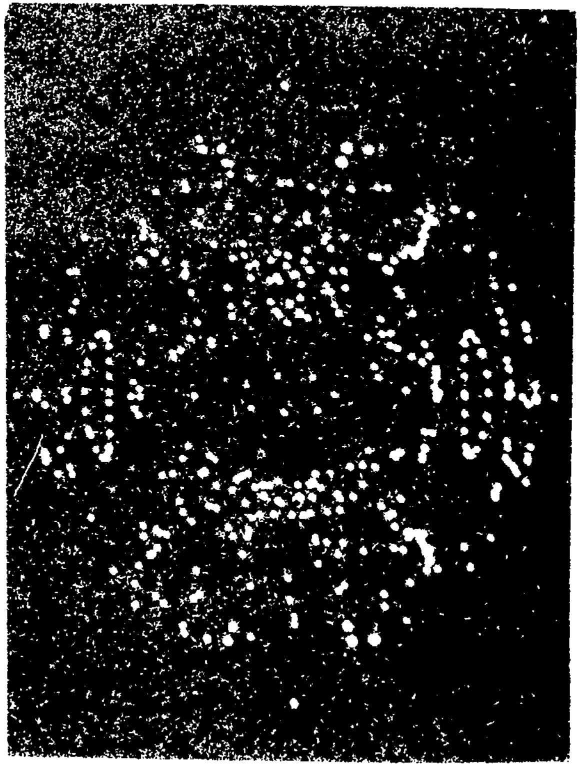

Fig.2

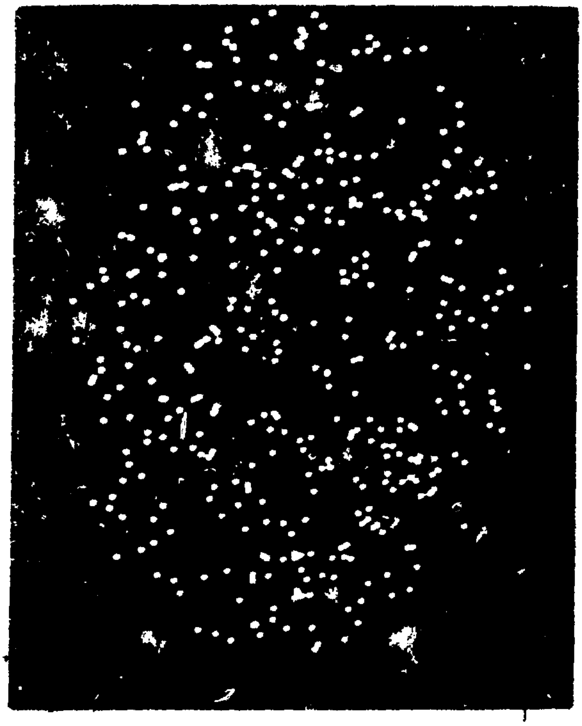

Fig.3

REFERENCES

1 S.G.Matinyan, G.K.Savvidy, N.G.Ter-Arutunyan-Savvidy. JETP, 80, 830, 1981.

2 G.Z.Baseyan, S.G.Matinyan, G.K.Savvidy. Pis'ma v JETP, 29, 641, 1979.

3 B.V.Chirikov, D.L.Shepelyansky. Pis'ma v JETP, 34, 171, 1981.

4 K.Wilson. Phys.Rev., D10, 2445, 1974;
G.t'Hooft. Nucl.Phys., B138, 1, 1978;
S.Mandelstam. Phys.Rev., D19, 2391, 1979.

5 V.I.Arnold. Mathematical Methods in Classical Mechanics, in Russian, M., "Nauka", 1979.

6 A.N.Kolmogorov. Dokl.Akad.Nauk, 98, 527, 1954;
V.I.Arnold. Usp. Mat. Nauk, 18, 13, 1963;
J.Moser. Nachr. Acad. Wiss. Gottingen, No.1, 1962.

7 G.Contopoulos. Astron.J., 68, 14, 1963;
M.Henon, C.Heiles. Astron.J., 69, 73, 1963;
G.N.Walker, J.Ford. Phys.Rev., 188, 416, 1969.

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(на английском языке , перевод З.Н.Асланян)

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