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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

ЕФИ-517(4)-82

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DISCRETE SYMMETRIES IN SO(N) GRAND UNIFIED GROUPS AND THE SURVIVAL HYPOTHESIS

ԵՐԵՎԱՆ 1982 ЕРЕВАН

ЕФИ-5I7(4)-82

Г.М.АСАТРЯН

ДИСКРЕТНЫЕ СИММЕТРИИ В ГРУППАХ ВЕЛИКОГО ОБЪЕДИНЕНИЯ $SO(N)$ И ГИПОТЕЗА "ВЫЖИВАНИЯ"

Изучается возможность введения дискретной симметрии D , которая наряду с $SU^c(3)\times SU_L(2)\times U(1)$ - симметрией остается после нарушения группы симметрии великого объединения $SO(N)$ вакуумными средними полей Хиггса порядка 10^{15} Гэь. Квантовое число, отвечающее этой симметрии, отличает фермионы, связанные с W - бозонами через левые и правые токи. Тем самым обеспечивается наличие в теории фермионов с обычными массами.

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The possibility of introducing the discrete symmetry D is studied which along with $SU^c(3)\times SU_L(2)\times U(1)$ symmetry remains after the breaking SO(N) grand unified symmetry group by the Higgs fields vacuum expectation values of the order 10^{15} GeV. The quantum number corresponding to this symmetry distinguishes the fermions coupled with W-bosons via left and right currents. As a result the presence of low-mass fermions in the theory is provided.

Yerevan Physics Institute

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It is known that the grand unified model (GUM) based on the orthogonal group SO(10) [1] includes only one family of quarks and leptons and thus leaves open the family unification problem. For the solution of this problem orthogonal groups SO(N) with $N > 10$ are more convenient, since their spinor representation includes naturally different SO(10) families of fermions and is free from anomalies [2]. Recently there appeared many works dedicated to the SO(N) GUM [3-5].

In [6] the "rules" for the choice of the GUMs have been formulated. One of these "rules" is the so-called survival hypothesis. It states: low-mass fermions are those that cannot receive invariant with respect to group $G_0 = SU^c(3) \times SU_L(2) \times U(1)$ masses [6,7]. This means that they must form a complex representation of this group. The survival hypothesis looks reasonable enough, since the G_0-invariant vacuum expectation values (VEV) of Higgs fields breaking the grand unified group G down to G_0 must have the order of grand unification mass 10^{15} GeV (as we must give a mass of this order to the corresponding gauge bosons) and can give a mass of such order to fermions.

This hypothesis means however that in the SO(N) GUMs with $N>10$ there must be no low-mass fermions at all, since the spinor representation of this group is self-conjugate with respect to G_0 and all the fermions can receive a G_0-invariant superheavy mass $\sim 10^{15}$ GeV [7].

We shall consider here a modified variant of the survival hypothesis based on another scheme of breaking when SO(N) is broken by the Higgs fields VEVs $\sim 10^{15}$ GeV down to $D\times G_0$, where D is a certain discrete symmetry. Then the survival hypothesis must be formulated like this: only those fermions survive (i.e. are low-mass) that cannot receive $D\times G_0$-invariant masses, i.e. form a complex representation of this group [8]. The additional discrete symmetry D prevents the ordinary fermions from receiving a superheavy mass due to the fact that the self-conjugate with respect to SO(10) parts of the fermion representation transform differently with respect to D.

Among the SO(N) groups the SO(4n+2) groups only have complex representations [9]. In particular, they have a complex spinor representation $\underline{2}^{2n}$ which is commonly taken as a fermionic one. The SO(4n+2) groups contain for $n\geqslant 3$ the subgroup $SO(10)\times SO(4n-8)$, and the spinor representation $\underline{2}^{2n}$ may be decomposed with respect to this subgroup:

$$\underline{2}^{2n}=(16,2^{2n-5})+(\overline{16},(2^{2n-5})') \qquad (1)$$

where $\underline{2^{2n-5}}$ and $\underline{(2^{2n-5})'}$ are two different real spinor representations of the SO(4n-8) group. This means that the fermion representation $\underline{f}$ (if identified with the spinor one) will contain an equal number of $\underline{16}$ and $\overline{\underline{16}}$ SO(10) families, i.e. families with left V - A and right V + A couplings with W-bosons [5]. Then the fermion mass operator

$$\underline{f} \times \underline{f} = (16 \times 16,\ 2^{2n-5} \times 2^{2n-5}) + (\overline{16} \times \overline{16},\ (2^{2n-5})' \times (2^{2n-5})') + 2 \cdot (16 \times \overline{16},\ 2^{2n-5} \times (2^{2n-5})') \quad (2)$$

contains SO(10)-singlets. $\underline{16}$ and $\overline{\underline{16}}$ SO(10) families mixing with each other can produce particles with SO(10)-invariant (and therefore G_0-invariant, since $SO(10) \supset G_0$) masses which must be superheavy.

We shall choose a scheme of the SO(4n+2)-symmetry breaking by the Higgs fields VEVs $\sim 10^{15}$ GeV so that along with the G_0 the discrete symmetry D remains. The following Higgs fields are taken: totally symmetric tensor of 2n-4 rank $\varphi_{i_1 \ldots i_{2n-4}}$ and the fifth rank totally antisymmetric tensor χ_{ijknm} $(i_1, \ldots, i_{2n-4}, i, j, k, n, m = 1, 2, \ldots, 4n+2)$. The distinct from zero VEVs of the symmetric tensor $\sim 10^{15}$ GeV are the following

$$\langle \varphi_{1 \ldots 1} \rangle = \langle \varphi_{2 \ldots 2} \rangle = \cdots = \langle \varphi_{6 \cdots 6} \rangle \quad (3a)$$

$$\langle \varphi_{7 \ldots 7} \rangle = \langle \varphi_{8 \ldots 8} \rangle = \langle \varphi_{9 \ldots 9} \rangle = \langle \varphi_{10 \cdots 10} \rangle \quad (3b)$$

$$\langle \varphi_{i_1 \ldots i_{2n-4}} \rangle ,\ i_1, \ldots i_{2n-4} = 11, \ldots, 4n+2 \quad (3c)$$

and they break the SO(4n+2) symmetry down to $SU^c(4) \times SU_L(2) \times SU_R(2)$. The decomposition of the χ_{ijknm} with respect to $SO(10) \times SO(4n-8)$ is

$$(4n+2)^5_{as} = (1,(4n-8)^5_{as}) + (10,(4n-8)^4_{as}) + \\ +(45,(4n-8)^3_{as}) + (120,(4n-8)^2_{as}) + \\ +(210, 4n-8) + (126,1) + (\overline{126},1) \,. \tag{4}$$

where $(4n-8)^K_{as}$ is the K rank totally antisymmetric tensor of SO(4n-8) group. By means of the G_o-singlet VEVs (which also are $\sim 10^{15}$ GeV) contained in the last two terms of the decomposition (4) one may break the $SU^c(4) \times SU_L(2) \times SU_R(2)$ symmetry down to G_o.

In the above mechanism of breaking along with G_o there may remain an additional discrete symmetry D – the subgroup of SO(4n-8). The question, what type of symmetry D must be chosen to provide the survival of fermions, will be discussed on concrete examples of grand unified groups SO(14), SO(18) and SO(22).

1. SO(14) group. The VEVs (3a)-(3c) of the Higgs fields φ_{ij} along with the VEV of the <u>126</u> (or $\underline{\overline{126}}$) representation of SO(10) (from the expansion (4) χ_{ijknm}) break SO(14) down to $D \times G_o$, where the generating element T of discrete symmetry D has the following form for the vector representation <u>14</u> SO(14)

$$T_v = \begin{pmatrix} \mathbf{1} & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & -1 \end{pmatrix} \tag{5}$$

where $\mathbf{1}$ is the 10×10 unit matrix. For the SO(4) ($SO(14) \supset SO(10) \times SO(4)$) spinor representations $\underline{2_s}$ and $\underline{2'_s}$ we have the followi: , expressions for T [1]:

$$T_s = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad T_s' = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \tag{6}$$

Then from the decomposition (1) we shall easily obtain the decomposition of the spinor representation $\underline{64}$ of SO(14) with respect to $D \times SO(10)$

$$\underline{64} = 2 \cdot \underline{16}^1 + 2 \cdot \underline{\overline{16}}^{-1}, \tag{7}$$

where we have written down the quantum number T as a top index. Due to the fact that the $\underline{16}$-s and $\underline{\overline{16}}$-s have different quantum numbers T, they will not be mixed until the $D \times G_0$-symmetry is broken (it is broken by the Higgs fields VEVs of the order of W,Z-boson masses) and, hence, they will not get a superheavy mass. Therefore in this model there are two $\underline{16}$ and two $\underline{\overline{16}}$ SO(10) families of low mass fermions.

It is well-known that the number of ordinary fermion families (with V - A coupling with W-bosons) must be at least three, therefore we must consider orthogonal groups of higher orders.

2. SO(18) group. The spinor representation of this group $\underline{256}$ contains eight $\underline{16}$-s and eight $\underline{\overline{16}}$-s of SO(10). If they all are low-mass ones, then the condition of the $SU^c(3)$ colour group asymptotic freedom which requires that the number of quarks should be not more than 16 will be violated. Therefore we shall choose the Higgs field VEVs in the following manner. We shall assume as distinct from zero the VEVs (3a), (3b) of

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1) As a matter of fact T_s and T_s' are determined through T_V with an accuracy of common multiplication by -1, but this is not essential for what follows.

φ_{ijkn} as well as

$$\langle \varphi_{ijkn} \rangle \quad \begin{array}{l} i,j,k,n = \begin{cases} 11,\dots,14 \\ 15, \quad 16 \\ 17, \quad 18 \end{cases} \\ i,j = 15,16; \quad k,n = 17,18 \\ i,j = 11,\dots,14; \; k = 15,16; \quad n = 17,18 \end{array} \tag{8}$$

As seen from (4), χ_{ijknm} includes the third rank antisymmetric tensor ϑ_{abc} $(a, b, c = 1,\dots,8)$ of the SO(8) group, which is a G_0-singlet at the same time. Let the following VEVs of ϑ_{abc} be distinct from zero:

$$\langle \vartheta_{abc} \rangle \quad \begin{array}{l} a = 1,\dots,4; \; b, c = 5,6 \\ a = 1,\dots,4; \; b, c = 7,8 \end{array} \tag{9}$$

Then the VEVs (3a), (3b), (8), (9) along with the VEV of <u>126</u> (or $\overline{126}$) of SO(10) from (4) break SO(18) down to symmetry $D \times G_0$. The generating element T of discrete symmetry D for the vector $\underline{8_v}$ and the spinor $\underline{8_s}$, $\underline{8_s'}$ representations of SO(8) has the form of 8 x 8 diagonal matrices

$$T_v = \mathrm{diag}\,(-1,-1,-1,-1,\; i,\; i,\; -i,\; -i,)$$

$$T_s = \mathrm{diag}\,(1,\; 1,\; 1,\; 1,\; i,\; i,\; -i,\; -i) \tag{10}$$

$$T_s' = \mathrm{diag}\,(-1,-1,-1,-1,\; i,\; i,\; -i,\; -i)$$

With regard to (10) the fermion representation <u>256</u> is decomposed with respect to D×SO(10) group as follows:

$$256 = 4\cdot(\underline{16}^{1} + \overline{16}^{-1}) + 2\cdot(\underline{16}^{i} + \underline{16}^{-i} + \overline{16}^{i} + \overline{16}^{-i}) \tag{11}$$

The self-conjugate with respect to D×SO(10) part of fermion representation $2\cdot(\underline{16}^{i} + \underline{16}^{-i} + \overline{16}^{i} + \overline{16}^{-i})$ receives a superheavy mass through the VEVs of ϑ_{abc} contained in the mass operator (2). Four <u>16</u> and four $\overline{16}$ SO(10)-families of fermions remain low-mass and the asymptotic freedom at our energies is not violated.

3. SO(22) group. In much the same way as the previous case, making use of VEVs of the Higgs fields φ_{ijknme} and χ_{ijknm} one may break the SO(22) symmetry up to $D \times G_0$ where the generating element T has the following form for the vector representation 12 of SO(12):

$$T_V = diag(-1,-1,\alpha,\alpha,\alpha,\alpha^*,\alpha^*,\alpha^*,\alpha^2,\alpha^2,\alpha^{2*},\alpha^{2*}) \quad (12)$$
$$\alpha = e^{i\frac{2\pi}{6}}$$

The fermion representation 1024 is decomposed with respect to $D \times SO(10)$ 1)

$$\underline{1024} = 6(\underline{16}^{\alpha}+\underline{16}^{\alpha *}) + 5(\underline{16}^{\alpha^2}+\underline{16}^{\alpha^2 *}) + 4\cdot\underline{16}^{-1} + 6\cdot\underline{16}^{1} + 5(\overline{\underline{16}}^{\alpha}+\overline{\underline{16}}^{\alpha *}) + 6(\overline{\underline{16}}^{\alpha^2}+\overline{\underline{16}}^{\alpha^2 *}) + 6\cdot\overline{\underline{16}}^{-1} + 4\cdot\overline{\underline{16}}^{1} \quad (13)$$

Only the self-conjugate part of (13) receives a superheavy mass, and again four 16 and four $\overline{16}$ SO(10)-families of fermions remain low-mass.

An analogous discussion may be carried out for orthogonal groups of a higher order.

The discussed mechanism of the symmetry breaking leads to the fact that a particle in low-mass families, for instance the right neutrino in 16 (if the VEV of 126 SO(10) is distinct from zero) receives a superheavy Mayorana mass [10]. The left neutrino may receive the Mayorana mass only at the next stage of breaking, and it will be fairly small [10].

In our discussion, besides the ordinary SO(10) fermion families $\underline{16}$, the $\overline{16}$-s as well remain low-mass. They receive masses via the same Higgs field VEVs that give masses to the W and Z-bosons. It is possible to make the particles in $\overline{16}$ heavier than ordinary ones: if the ordinary particles masses are of the order 1 GeV, the masses of the particles in $\overline{16}$ may be $\sim$ 100 GeV [5] . These questions will be discussed in details elsewhere.

The discussed examples show that the survival hypothesis in its modified form may be successfully used for the SO(N) GUM.

As to the SU(N) groups, it is well-known that the SO(4n+2) contains SU(2n+1) as a subgroup, and the spinor representation may be decomposed with respect to this subgroup. If we take the fermion representation of unitary group just in this form then all the aforesaid about orthogonal groups may extend also to these groups, the survival hypothesis will have here the same consequences as for corresponding orthogonal groups.

A similar discussion for the unitary groups SU(N) has been carried out in Ref. [8] but the mechanism of the SU(2n+1) breaking chosen there differs from the one described above. Therefore, the results in the specific case of the SU(9)-group are different: in our case there are no problems with asymptotical freedom violation as it is in [8] .

In conclusion I would like to express my gratitude to S.G.Matinyan and A.G.Sedrakyan for fruitful discussions.

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The manuscript was received 8 December 1981.

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SO(N) И ГИПОТЕЗА "ВЫЖИВАНИЯ"

(на английском языке, перевод Л.Н.Багдасаряна)

Ереванский физический институт

Тех.редактор А.С.Абрамян

Заказ 033 ВФ-03626 Тираж 299

Препринт ЕФИ Формат издания 60х84/16
Подписано к печати I/II-82г. 0,5 уч.изд.л.Ц. 4 к.

Издано Отделом научно-технической информации
Ереванского физического института, Ереван-36, пер.Маркаряна 2

индекс 3624