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STOCHASTIC PROCESSES AND THE DYNAMICS OF MASSIVE BLACK HOLES

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СТОХАСТИЧЕСКИЕ ПРОЦЕССЫ И ДИНАМИКА МАССИВНЫХ ЧЕРНЫХ ДЫР

Рассматривается роль стохастических процессов в динамике массивной черной дыры, расположенной в центре компактного звездного скопления. Показано, что в определенных условиях черная дыра может совершать броуновское движение по скоплению под влиянием звездных флуктуирующих возмущений.

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STOCHASTIC PROCESSES AND THE DYNAMICS OF MASSIVE BLACK HOLES

The role of stochastic processes in the dynamics of massive black hole situated at the centre of a compact star cluster is discussed. It is shown that under certain conditions the black hole can escape from the centre of mass of the system and has to execute Brownian motion influenced by fluctuating stellar perturbations.

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Yerevan 1982

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1. *Introduction*

The problem of the existence of a massive black hole at the centre of a compact stellar system is a subject of intense investigation during the last decade. Important consequences of the interaction of stars with the black hole are understood: the influence of the central hole on the dynamics of stellar system including the redistribution of surrounding stars by the hole and the stellar consumption rate (among reviews and papers see [1-4, 11-13]), accretion of the gas liberated at the tidal disruption of stars by the black hole [5-10] etc.

In 1976 Bahcall and Wolf [11] mentioned that the massive black hole of a mass M cannot be localized exactly at the centre of a star cluster (i.e. at the centre of mass) due to the interaction with stars and should oscillate with an amplitude

$$\sim r_c \left(\frac{m}{M} \right)^{1/2} , \qquad (1)$$

where r_c is the cluster core radius, m is the mean stellar mass. Later on it has been shown that this effect does not alter essentially either the rate of tidal disruption of stars inside the Roche lobe, or the distribution law of bound stars nearby the hole [12, 13]. The role of various processes on the displacement of the black hole from the centre was discussed recently by Lin and Tremaine[14]. Particularly, they showed that the interaction of unbound stars with the hole is more important as compared with bound ones in the case of globular clusters.

In the present paper we reinvestigate the character of stellar perturbations on the dynamics of the massive black hole at the centre of star cluster The problem in our consideration is analogous to that of the behaviour of a massive test particle in a gas of light particles investigated in the classical theory of Brownian motion. Therefore the results obtained here may be equally applied to compact objects of other nature situated at the centres of globular clusters or galactic nuclei. Below, to be brief, we shall speak about a black hole.

2. Solution of Stochastic Equation

In order to describe the behaviour of a black hole under the influence of a rapidly changing force we shall proceed from a well-known method in classical theory of Brownian motion based on the solving a stochastic equation in the simplest case in Langevin form

$$\frac{M}{2}\frac{d^2x}{dt^2} = F(t) + f + u\,, \tag{2}$$

where X is the stochastic variable (one-dimensional). The force acting on the test particle from the right-hand side of this equation is decomposed, as usual, to the fluctuating component $F(t)$, the dynamic friction f and the external force u.

For $F(t)$ we have by definition

$$\langle F(t) \rangle = 0 \tag{3a}$$

and

$$\langle F(t)F(t') \rangle = \beta\,\delta(t-t')\,, \tag{3b}$$

where β is a constant. The second condition reflects the Markovian character of the considered chance process.

The resistance force affecting the black hole at the motion inside the Maxwellian stellar gas of a temperature ϑ and mean concentration n, as it is shown in the Appendix, can be represented in the form

$$f = -n\kappa\vartheta\sigma\left[(2\xi^2+1)\frac{e^{-\xi^2}}{\sqrt{\pi}\,\xi} + \Psi(\xi)\left(2\xi^2 - 2 - \frac{1}{2\xi^2}\right)\right], \tag{4}$$

where ξ = (velocity of black hole - V)/(mean velocity of stellar gas - v).

Here we approximate the black hole with a model of a sphere of a cross section $\sigma \simeq \pi R_t R_h$ ($R_t = \left(\frac{6M}{\pi\rho_*}\right)^{1/3}$ is the tidal radius, $R_h = 2GM/\langle v^2\rangle$ – the radius of gravitational influence) moving in the rare gas of light particles.

When $\xi \ll 1$, (4) takes a form which in a first approximation is proportional to V :

$$f = -n\kappa\vartheta\frac{8\sigma}{3\sqrt{\pi}}\,\frac{V}{v}\left(1 + O\left(\frac{V^2}{v^2}\right)\right). \tag{5}$$

The gravitational field within a stellar system of density $\rho(r)$ is

$$\vec{g}(r) = \frac{4\pi G\vec{r}}{r^3}\int_0^r \rho(\eta)\eta^2 d\eta = \frac{4\pi G\rho_0 r_c^2\vec{r}}{r^2}, \tag{6}$$

where ρ_0 is the central density. In deriving this expression we used the isothermal density distribution law $n(r) \sim r^{-2}$, which is in accord with the observed density run of realistic stellar systems, too. The existence of density cusp of finite stars cannot significantly alter the results, since in this case the law $n(r) \sim r^{-7/4}$ does not differ much from the isothermal one. Note that in their investigation of the displacement of the black hole Lin and Tremaine neglected the decreasing of the density by radius (adopting

ρ = const., potential $\sim r^2$), which appears rather crucial.

Using expressions (5) and (6) the initial equation (2) after averaging takes the form

$$\frac{M}{2}\frac{d}{dt}\left\langle \frac{d}{dt}x^2\right\rangle - \kappa\vartheta + \alpha = -\frac{1}{\gamma}\left\langle \frac{d}{dt}x^2\right\rangle , \tag{7}$$

where

$$\gamma = \frac{3\sqrt{\pi}}{8}\frac{\langle v\rangle}{n\kappa\vartheta\sigma} \quad ; \quad \alpha = 4\pi G \rho_0 r_c^2 M . \tag{8}$$

In (7) we have taken into account the fact that x and $F(t)$ are not correlated:

$$\langle x F(t)\rangle = 0 .$$

The solution of eq.(7) can be represented in the form

$$\langle x^2\rangle = x_0^2 \, e^{-\frac{t}{\gamma m}} + 2\gamma(\kappa\vartheta - \alpha)t , \tag{9}$$

where

$$x \to x_0 \qquad \text{when} \qquad t \to 0 .$$

The second member in (9) $2\gamma\kappa\vartheta t = 2Dt$ is the well-known member derived originally by Einstein while constructing the theory of Brownian motion (D is the coefficient of diffusion)

3. Discussion

One can see from (9) that when

$$\kappa\vartheta > \alpha \tag{10}$$

the effective gravitational potential of the system cannot confine the black hole at the centre of the cluster and the former will have to execute Brownian motion within the system.

The condition (10) rewritten in the form of

$$M < \frac{\langle v^2 \rangle}{8\pi G n_0 r_c^2} \quad ; \quad \langle v \rangle \propto \left(\frac{r_h}{r}\right)^{1/2} \tag{11}$$

limits the mass of the hole which is able to escape from the potential well

For the last condition with characteristic values of physical parameters we have approximately:

for globular clusters

$$M < 5 \cdot 10^6 \left(\frac{n_0}{5 \cdot 10^4 pc^{-3}}\right)^{-3} \left(\frac{r_c}{0.5\ pc}\right)^{-6} M_\odot \ ; \tag{12a}$$

for galactic nuclei

$$M < 7 \cdot 10^2 \left(\frac{n_0}{10^6 pc^{-3}}\right)^{-3} \left(\frac{r_c}{0.5\ pc}\right)^{-6} M_\odot \ . \tag{12b}$$

Now omitting the non-stationary part in (9) for the mean squared displacement of the hole of a mass determined by (12) at time t, we have

$$\langle x^2 \rangle = \frac{3}{4} \frac{\sqrt{\pi}}{n_0 \sigma} \langle v \rangle t$$

or

$$\langle r^2 \rangle^{1/2} = \frac{3}{2\pi^{1/4}} \left(\frac{\langle v \rangle}{n_0 R_t R_h}\right)^{1/2} t^{1/2} \tag{13}$$

Respectively, we have:

for globular clusters

$$\langle r^2 \rangle^{1/2} \simeq 1\,pc \left(\frac{t}{10^5 yr}\right)^{1/2} \left(\frac{M}{10^4 M_\odot}\right)^{-1/2} \left(\frac{V_0}{10\ \frac{km}{s}}\right) \left(\frac{n_0}{5 \cdot 10^4\ pc^{-3}}\right)^{-1/2} ; \tag{14a}$$

for galactic nuclei

$$\langle r^2 \rangle^{1/2} \simeq 4.5\,pc \left(\frac{t}{10^5 yr}\right)^{1/2} \left(\frac{M}{10^4 M_\odot}\right)^{-1/2} \left(\frac{V_0}{200\ \frac{km}{s}}\right) \left(\frac{n_0}{10^6\ pc^{-3}}\right)^{-1/2} . \tag{14b}$$

In order the continual transition made at the description of the dynamics of the black hole be correct one should become sure that the characteristic time scales of variation of physical parameters exceed the mean period of fluctuating force $F(t)$. The latter value approximately equals to [15]

$$n_o^{-1/3} \langle v^2 \rangle^{-1/2}$$

and for considered values of n and V yields less than $10^{-2} - 10^{-3}$ yr. As it follows from (14), during this time the hole is able to cover only a distance which does not exceed the statistical uncertainty of the localization at the cluster centre. This shows the correctness of the derived results.

Thus, a black hole situated at the centre of globular cluster may escape from the centre of mass of the system. This conclusion will make one to reinvestigate two important problems related with globular clusters widely discussed at present.

Already in 1975 it has been suggested [16, 17] that those of observed X-ray sources which were identified with globular clusters may be massive black holes situated at the centres of these systems. Up to the present time much progress has been made towards understanding the nature of these X-ray sources - both from experimental and theoretical positions. From at least fifteen globular clusters with X-ray sources for eight clusters the precise positions of the latter are determined (up to September 1980 [18]). Theoretical investigation of experimental data seriously supports the model of close binary system generating the observed X-ray luminocity via accretion onto a neutron star accompanied with nuclear burning on its surface (e.g. [19]). Photometric data do not show the necessity of a central black hole, too [20], as had been previously suggested, particularly, for globular cluster M15 [21]. However one important remark should be made here: the masses of X-ray sources are usually determined from the relation (1) by known core radius and potential offset. Now, in view of the presented results one should be cautious in derivation of the source mass by its displacement from the cluster centre only (cf. [22]).

The second problem to be reinvestigated is the one related with the dynamical evolution of the globular cluster containing a central massive black hole. As it was originally shown by Shapiro [23], the black hole, in principle, may not only halt the core collapse, but even get to reexpand the cluster up to its complete dissolution.

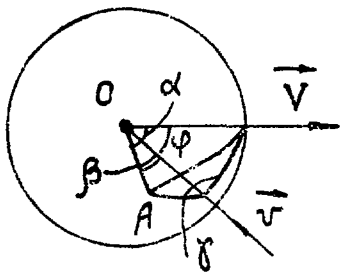
Thus, our qualitative analysis allows us to formulate the following conclusions:

a) a massive black hole practically may not be localized at the centre of a globular cluster and has to move within the cluster core influenced by stochastic perturbations of stars;

b) the stochastic behaviour of the black hole is suppressed in compact galactic nuclei, but may become sufficient in nuclei with mean stellar concentration less than $10^4 - 10^5 pc^{-3}$.

APPENDIX

Let us obtain the force of dynamical friction affecting a sphere of a radius R moving with a velocity $\vec{V}$ in a rare gas of light particles of a temperature ϑ. Assume that the particles of a mean velocity v and mass m lose their all initial momentum colliding with the sphere.



In the polar system of coordinates, as it is shown in the figure, the angle α between $\vec{OA}$ (A is the encounting point of the particle with the sphere surface) and $\vec{V}$ is determined by the expression

$$\cos\alpha = \cos\varphi\cos\beta + \sin\varphi\sin\beta\cos\gamma ,$$

where β, γ are the angular coordinates of the encounting point, φ is the angle between $\vec{V}$ and $\vec{v}$.

The momentum got by the sphere in the direction $\vec{OA}$

$$m\,|v-V|\cos\beta$$

has a projection on

$$-m\,|v-V|\cos\beta\cos\alpha .$$

The number of collisions of the particles with sphere's surface element

$$dS = R^2\sin\beta\, d\beta\, d\gamma$$

in a unit of time will be

$$|v-V|\,R^2\sin\beta\cos\beta\, d\beta\, d\gamma .$$

Assuming that the velocity distribution of particles is given by the Maxwell law, we shall have for the force affecting the sphere

$$f = -mn\left(\frac{m}{2\pi k\vartheta}\right)^{3/2}\sigma\int_0^\infty (v-V)^4 dv\int_0^\pi 2\pi\sin\varphi\exp\left[-\frac{m}{2k\vartheta}(\vec{v}+\vec{V})^2\right]d\varphi$$

$$\int_0^{\pi/2}\cos^2\beta\sin\beta\, d\beta\int_0^{2\pi}(\cos\varphi\cos\beta+\sin\varphi\sin\beta\cos\gamma)\,d\gamma .$$

After integrating we obtain

$$f = -nk\vartheta\sigma\left[(2\xi^2+1)\frac{e^{-\xi^2}}{\sqrt{\pi}\,\xi} + \Psi(\xi)\left(2\xi^2+2-\frac{1}{2\xi^2}\right)\right]$$

where

$$\Psi(\xi) = \frac{2}{\sqrt{\pi}}\int_0^\xi e^{-x^2}dx \quad ;$$

$$\xi = V/v .$$

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