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SH.S.EREMYAN, A.E.NAZARYAN

ON RENORMALIZABILITY OF REGGEON FIELD THEORY WITH ACCOUNT OF THRESHOLDS AND "MASS" TERMS AT D = 2

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Ш.С.ЕРЕМЯН, А.Э.НАЗАРЯН

О ПЕРЕНОРМИРУЕМОСТИ РЕДЖЕОННОЙ ТЕОРИИ ПОЛЯ С УЧЕТОМ ПОРОГОВ И "МАССОВЫХ" ЧЛЕНОВ ПРИ D =2.

В настоящей работе показывается, что учет порогов рождения реджеонов $\xi_0 = \ln(M^2/s_0) \approx 2$ в реджеонной теории поля приводит к тому, что ε - разложение становится аналитическим при $\varepsilon = 2$, и появляется возможность одновременного предельного перехода к $\varepsilon \to 2$ и $E \to 0$, что соответствует физической размерности пространства в пределе асимптотических энергий. Введение порогов облегчает проведение теоретико-возмущенческих расчетов при D =2, устраняя ультрафиолетовые расходимости теории, и оказывается полезным для проведения гладкой сшивки теоретико-возмущенческого и асимптотического решений.

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ON RENORMALIZABILITY OF REGGEON FIELD THEORY WITH ACCOUNT OF THRESHOLDS AND "MASS" TERMS AT D = 2

It is shown that the account of the reggeon production thresholds $\xi_0 = \ln(M^2/S_0) \approx 2$ in the Reggeon field theory results in the fact that the ε-expansion becomes analytical at $\varepsilon = 2$ and the simultaneous limiting transition to $\varepsilon \to 2$ and $E \to 0$ becomes possible which is consistent with the physical space dimension in the asymptotic energy region. The introduction of thresholds facilitates the accomplishment of perturbation theory calculations at $D = 2$ eliminating the ultraviolet divergences of theory and proves useful for the accomplishment of smooth matching of the perturbation theory and asymptotic solutions.

Yerevan Physics Institute

Yerevan 1982

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ON RENORMALIZABILITY OF REGGEON FIELD THEORY WITH ACCOUNT OF THRESHOLDS AND "MASS" TERMS AT D = 2

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1. Introduction

The Reggeon field theory (RFT) has been first formulated in the works of Gribov and Migdal [1-3]. Later on the field theory techniques has been used to calculate the critical exponents of RFT in infrared limit [4-12] which define the behaviour of pomeron propagator at asymptotic energies.

All these calculations were carried out in a space with a dimension $D = 4 - \varepsilon$ after which the expansion in powers $\varepsilon \approx 0$ was made and later it was necessary to carry out an analytical continuation to the physical value $\varepsilon = 2$. It has however turned out that in RFT there is no analyticity in ε. Several works were devoted to the analysis of this question. For example, in refs. [7,8] the general structure of perturbation theory diagrams contributing in the proper self-energy part of pomeron was discussed. In the general case the interaction between pomerons will lead to the shift of their intercept. This may be compensated by adding a counterterm renormalizing the intercept directly into an interaction Lagrangian, just like adding a mass counterterm in usual field theories. When the pomeron intercept is below unit, one may so choose a counterterm that the intercept takes its physical value. However, when the intercept lies above unit, a branch point emerges in the counterterm at the zero value of coupling constant, which makes the intercept renormalization in each order of perturbation theory impossible. Another difficulty in the perturbation theory is the fact that when the pomeron intercept tends to unit, already in the one-loop approximation in the propagator there appears a tachyon (a pole in j-plane right of unit). This false pole vanishes only after summing an infinite number of diagrams. It is shown in this work that one can avoid all the above difficulties if $\xi_0 = \ln(M^2/S_0) \approx 2$ reggeon production thresholds are introduced in RFT. As shown in refs. [13,14], in the real Reggeon theory there should necessarily exist thresholds which are defined by the rapidity minimum possible difference along the reggeon, lower which it is impossible to speak about a reggeon. As shown in refs.[15-19] the consideration of thresholds must not affect the asymptotic behaviour of critical exponents, but their introduction proves to be useful for carrying out a smooth matching of perturbation-theory and asymptotic solutions. Besides, they facilitate the carrying out of perturbation theory calculations at $D = 2$ eliminating the ultraviolet divergences of the theory. And finally, the most important thing, they will result in the fact that the ε-expansion becomes analytical even at $\varepsilon = 2$ and a simultaneous limiting transition to $\varepsilon \to 2$ and $E \to 0$ becomes possible.

2. Perturbation theory and analytical continuation in $\mathcal{E}$

In RFT reggeons are treated as quasi-particles consistent with the following equation of motion

$$E = 1 - \alpha_0\left(-\vec{K}^2\right), \tag{1}$$

where $E = 1 - j$ and $\alpha_0(-\vec{K}^2)$ are the bare trajectory function of the reggeon. It is commonly assumed that the pomeron bare trajectory function is linear and has the form

$$\alpha_0(-\vec{K}^2) = \alpha(0) - \alpha'_0\,\vec{K}^2. \tag{2}$$

Then, according to [1-3], the free Lagrangian density is given in the form

$$\mathcal{L}_0(\vec{x},t) = \frac{i}{2}\psi_0^+(\vec{x},t)\overleftrightarrow{\frac{\partial}{\partial t}}\psi_0(\vec{x},t) - \alpha'_0\vec{\nabla}\psi_0^+(\vec{x},t)\vec{\nabla}\psi_0(\vec{x},t) - \delta_0\psi_0^+(\vec{x},t)\psi_0(\vec{x},t). \tag{3}$$

Here $\psi_0(\vec{x},t)$ is the pomeron bare field and $\delta_0 = 1 - \alpha_0(0)$ is the bare shift of the pomeron intercept. The density of the interaction Lagrangian is nonhermitian and is taken in the form

$$\mathcal{L}_I(\vec{x},t) = -\frac{1}{2}i\,r_0\left[\psi_0^+(\vec{x},t)\psi_0(\vec{x},t)^2 + \psi_0^+(\vec{x},t)^2\psi_0(\vec{x},t)\right] + \delta\Delta\,\psi_0^+(\vec{x},t)\psi_0(\vec{x},t), \tag{4}$$

where r_0 is the bare triple-pomeron vertex, $\delta\Delta$ is the intercept renormalization counterterm and it must have a structure to enable the shift of the intercept δ_0 to preserve its physical value.

Further on we shall need for comparison the results obtained in refs.[7,8] without consideration of the pomeron production thresholds. Therefore, in order to make the presentation complete, we shall give in the next chapter, along with our results, the basic results obtained in refs.[7,8] at $\xi_0 = 0$

We want to construct the Green function $G^{n,m}(E_i, \vec{K}_i)$ for n -incoming and m -outgoing pomerons. Each separate Feynman graph making a contribution in $G^{n,m}(E_i, \vec{K}_i)$ is defined in accord with the following rules:

1. To each pomeron with the momentum $\vec{K}$ and energy E corresponds a bare propogator

$$G_0(E, \vec{K}^2) = i\left[E - \alpha_0' \vec{K}^2 - \delta_0 + i\varepsilon\right]^{-1}. \tag{5}$$

2. For each triple-pomeron vertex put a factor of

$$\tau_0 / (2\pi)^{(D+1)/2}. \tag{6}$$

3. For each intercept renormalization counteterm put a factor of $i\delta\Delta$

4. Each two-pomeron loop with both momenta in the same direction is multiplied by ½.

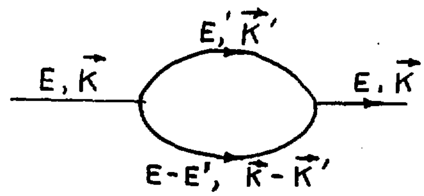
5. Energy and momentum at all the vertices are conserved.

6. Integration $\int d^D K\, dE$ is carried out round each closed loop.

Let us start with the study of the pomeron propagator structure in the perturbation theory. We have

$$i\Gamma^{1,1}(E, K^2) = iG^{1,1}(E, K^2)^{-1} = E - \alpha_0' K^2 - \delta_0 - \Sigma(E, K^2) + \delta\Delta \tag{7}$$

where $\Sigma(E, K^2)$ is the irreducible proper self-energy part of the pomeron. A lower order diagram of perturbation theory contributing in $\Sigma(E, K^2)$ has the form

$$\Sigma_2(E,K^2) = $$ 

(8)

According to the above rules of Reggeon diagram technique we have

$$\Sigma_2(E,K^2) = -\frac{i}{2}\frac{r_0^2}{(2\pi)^{D+1}}\int d^DK'dE'\left[E'-\alpha'_0K'^2-\delta_0+i\varepsilon\right]^{-1}\cdot \tag{9}$$
$$\cdot\left[E-E'-\alpha'_0(\vec K-\vec K')^2-\delta_0+i\varepsilon\right]^{-1}$$

Integrating by E' and substituting the variable we shall have

$$\Sigma_2(E,K^2) = -\frac{r_0^2}{2(2\pi)^D}\int\frac{d^DK'}{E-\frac{\alpha'_0}{2}K^2-2\delta_0-2\alpha'_0K'^2}\;. \tag{10}$$

For a further integration the dimensional regularization [20] should be used. All the further occuring integrals are determined at $\varepsilon > 2$. After carrying out the integration the answer holds for every value of ε that we are interested in. From [20] we have

$$\int d^Dq\left[Aq^2+B\right]^{-N} = \pi^{D/2}A^{-D/2}B^{D/2-N}\frac{\Gamma(N-D/2)}{\Gamma(N)}\;. \tag{11}$$

Substituting (11) in (10) we shall finally obtain

$$\Sigma_2(E,K^2) = c\,r_0^2\left(\frac{1}{2}\alpha'_0K^2+2\delta_0-E\right)^{1-\varepsilon/2} \tag{12}$$

where

$$C = \frac{\Gamma(-1+\varepsilon/2)}{2(8\pi\alpha'_0)^{D/2}} \tag{13}$$

From the requirement that the pomeron intercept should remain in $\alpha(0)$ it follows obviously that

$$\Gamma^{1,1}(\delta_0, 0) = 0, \tag{14}$$

which leads to the condition

$$\delta\Delta_2 = c\tau_0^2\,(2\delta_0)^{1-\varepsilon/2} + \delta_0 \tag{15}$$

Many of the difficulties of perturbation theory arise already in the one-loop approximation. Let's first of all mention that the limits $\delta_0 \to 0$ and $\varepsilon \to 0$ do not commute. Indeed at $\delta_0 = 0$ and $\varepsilon \geqslant 2$ it is impossible to choose such a $\delta\Delta_2$ that eq. (15) be satisfied. Besides, if we substitute the obtained solution (15) in eq. (7), it will turn out that it has two more false solutions for equation

$$\Gamma^{1,1}(E, 0) = 0 \tag{16}$$

one at $E < 2\delta_0$, when $(2\delta_0)^{\varepsilon/2} < (-1 + \varepsilon/2)\,c\,\tau_0^2$ and a second one at $E < 0$ when $(2\delta_0)^{\varepsilon/2} < -c\tau_0^2(2^{1-\varepsilon/2} - 1)$. The second solution corresponds to a tachyon, i.e. to a pole in j-plane lying right of j = 1. It was shown (see e.g. [19]) that such a tachyon is absent in the complete perturbation theory solution of the field theory. However its presence in the one-loop approximation points to the fact that the tendency $\delta_0 \to 0$ should be carried out with great care. In the higher orders of perturbation theory there will appear tachyon-tachyon and tachyon-pomeron cuts and only after summing an infinite number of such diagrams the tachyon pole will vanish from the complete solution.

Let's see now what will change in this picture after introducing a pomeron production threshold ξ_0, after, following refs. [13,19], taking it into account. Then the rule 1 of the above reggeon diagram technique (RDT) will have the following form:

1. To each pomeron with the momentum $\vec{K}$ and energy E corresponds a bare propagator

$$G_0^{1,1} = i\,\frac{e^{\xi_0(E-\delta_0-\alpha_0' K^2)}}{E-\delta_0-\alpha_0' K^2+i\varepsilon}\,. \qquad (17a)$$

And, besides, the complete non-renormalized inverse propagator of the pomeron will look like

$$i\Gamma^{1,1}(E,K^2)=(E-\delta_0-\alpha_0' K^2)\,e^{-\xi_0(E-\delta_0-\alpha_0' K^2)}+\delta\Delta-\Sigma(E,K^2). \qquad (17b)$$

At the same time in the theory there will appear new dimensionless values

$$\xi_N=-\xi_0 E\,;\quad \xi_K=\xi_0\alpha_0' K^2;\qquad \xi_\delta=\xi_0\delta. \qquad (18)$$

According to the new rule, the diagram (8) will be written not in the form (9) but in the form

$$\Sigma_2(E,K^2)=-\frac{i r_0^2}{2(2\pi)^{D+1}}\int\frac{d^D K' dE' \exp\{\xi_0[E-2\delta_0-\alpha_0'(K'^2+(\vec{K}-\vec{K}')^2)]\}}{(E'-\alpha_0' K'^2-\delta_0+i\varepsilon)(E-E_0'-\alpha_0'(\vec{K}-\vec{K}')^2-\delta_0+i\varepsilon}. \qquad (19)$$

Integrating by E' and making the corresponding substitution of variables we shall obtain

$$\Sigma_2(E,K^2)=-\frac{r_0^2}{2(2\pi)^D}\,e^{\xi_0(E-2\delta_0-\frac{1}{2}\alpha_0' K^2)}\int\frac{d^D K_1\, e^{-2\xi_0\alpha_0' K_1^2}}{2\delta_0-E+\alpha_0'(\frac{1}{2}K^2+2K_1^2)} \qquad (20)$$

To use the dimensional regularization method we shall now need instead of the integral (11), the integral

$$\int\frac{d^D K e^{-zK^2}}{(AK^2+B)^N}=\pi^{D/2}A^{-D/2}B^{-N+\frac{D}{2}}\,\psi\left(\frac{D}{2},-N+\frac{D}{2}+1;\frac{zB}{A}\right), \qquad (21)$$

where $\psi(a, b; x)$ is the second type confluent hypergeometric function. Using (21) we shall obtain

$$\Sigma_2(E,K^2) = -\frac{\tau_0^2\, e^{\xi_0(E-2\delta_0-\frac{1}{2}\alpha_0' K^2)}}{2\,(8\pi\alpha_0')^{D/2}}\left(2\delta_0 - E + \frac{1}{2}\alpha_0' K^2\right)^{\frac{D}{2}-1} \cdot$$

$$\cdot\, \psi\left(\frac{D}{2}, \frac{D}{2};\ \xi_0\left(2\delta_0 - E + \frac{1}{2}\alpha_0' K^2\right)\right). \tag{22}$$

The function ψ at $D \geq 2$ may be presented in the following form

$$\psi\left(\frac{D}{2}, \frac{D}{2}; x\right) = e^x\, \Gamma\left(-\left(\frac{D}{2}-1\right); x\right) =$$
$$= \frac{(-1)^{D/2}}{\left(\frac{D}{2}-1\right)!}\left[e^x Ei(-x) - \sum_{K=0}^{\frac{D}{2}-2} \frac{(-1)^K K!}{x^{K+1}}\right]. \tag{23}$$

Substituting (23) in (22) we shall obtain

$$\Sigma_2(E,K^2) = -C_0\, e^{-\xi_0 X} X^{1-\varepsilon/2}\, \frac{(-1)^{D/2}}{\left(1-\frac{\varepsilon}{2}\right)!} \cdot$$
$$\cdot \left\{ e^{\xi_0 X} Ei(-\xi_0 X) - \sum_{K=0}^{\frac{D}{2}-2} \frac{(-1)^K K!}{(\xi_0 X)^{K+1}} \right\}, \tag{24}$$

where

$$C_0 = \frac{\tau_0^2}{2\,(8\pi\alpha_0')^{D/2}}, \qquad D = 4 - \varepsilon \tag{25}$$

$$X = 2\delta_0 - E + \frac{1}{2}\alpha_0' K^2. \tag{26}$$

The direct calculations at $D = 2$ and $\xi_0 = 0$ give

$$\Sigma_2^{D=2}(K, E^2) = -\frac{\tau_0^2}{16\pi\alpha_0'}\, \ln(\Lambda/X), \tag{27}$$

where Λ is the cut-off parameter at integrating by the transfered momentum in the integral (10). When $\xi_0 \neq 0$, at $D=2$ we obtain

$$\Sigma_2^{D=2}(\varepsilon, \kappa^2; \xi_0) = \frac{\tau_0^2}{16\pi \alpha_0'} Ei(-\xi_0 X) \cdot \tag{28}$$

Using expansion

$$Ei(-x) = \ln(\gamma x) + \sum_{\kappa=1}^{\infty} \frac{(-x)^{\kappa}}{\kappa \cdot \kappa!} , \tag{29}$$

and making a substitution in (28)

$$\xi_0 = 1/\Lambda \tag{30}$$

we shall obtain to within the powers by X terms the formula (27). Besides, it is seen from (24) that if we let ε tend to 2, then eq. (24) exactly transforms into eq. (28) from where it is possible to make later a transformation to eq. (27), whereas eq. (12) at limiting transformation $\varepsilon = 2$ becomes senseless due to the pole appearing in Γ -function. Thus in the absence of thresholds there exists no analytic continuation in ε , and the introduction of thresholds results in analyticity in ε .

This result is the consequence of the fact that function ψ is the analytic function of its indices at $D \geq 1$ and $\xi_0 X \geq 0$, i.e. Σ_2 from (22) may be determined at any reasonable value of D , and among them at $D=2$. When there are no thresholds, we obtain for Σ_2 the formula (12) which indicates that at $\varepsilon = 2$ there exists a branch point which doesn't allow to produce analytic continuation from $D=4$ to the physical dimension $D=2$.

Let us return to eq. (14), when $\xi_0 \neq 0$. Instead of eq.

(15) we shall obtain

$$\delta\Delta_2 = -C_0\delta_0^{1-\varepsilon/2}\psi\left(\frac{D}{2}, \frac{D}{2}; \xi_0\delta_0\right). \tag{31}$$

At $\delta_0 \to 0$ we have

$$\text{at } \varepsilon < 2 \quad \psi\left(\frac{D}{2}, \frac{D}{2}; \xi_0\delta_0\right) \approx \frac{\Gamma(1-\frac{\varepsilon}{2})}{\Gamma(D/2)}(\xi_0\delta_0)^{\varepsilon/2-1}, \tag{32}$$

$$\text{at } \varepsilon = 2 \quad \psi(1, 1; \xi_0\delta_0) \approx \ln(\gamma\xi_0\delta_0). \tag{33}$$

At $\varepsilon < 2$ and $\delta_0 \to 0$ there occurs a curious result

$$\delta\Delta_2 = -C_0\frac{\Gamma(1-\frac{\varepsilon}{2})}{\Gamma(D/2)}\xi_0^{-1+\varepsilon/2} \tag{34}$$

i.e. it turns out that the pomeron intercept renormalization counterterm is independent of the intercept bare shift δ_0 and is proportional to the threshold value ξ_0.

At $\varepsilon = 2$ and $\delta_0 \to 0$ we obtain

$$\delta\Delta_2 = -C_0\delta_0^{1-\varepsilon/2}\ln(\gamma\xi_0\delta_0). \tag{35}$$

Thus the simultaneous limiting transition $\delta_0 \to 0$ and $\varepsilon \to 2$, impossible in the case of $\xi_0 = 0$, turns here possible. Hence at $\xi_0 \neq 0$ the renormalization of the pomeron intercept may be carried out in any order of the perturbation theory, as distinct from the results of refs.[7,8] where it is impossible due to $\xi_0 = 0$. We have shown in [19] that such a renormalization of the intercept may be carried out at least in the first two orders of perturbation theory. We have obtained an integral representation for the pomeron propagator allowing to produce an expansion both in the perturbation theory series and asymptotic series, and providing the smooth matching of these two solutions.

Let's now substitute eqs. (35) and (28) in eq. (7). It turns out that now equation

$$\Gamma(E,0)=0$$

has one basic solution $E=\delta_0$. Other singularities lie at $E>\delta_0$ and are not essential for the asymptotics. Thus, as distinct from the case $\xi_0=0$ there are no false poles and tachyons.

Consider now the higher terms in the expansion $\Sigma(E,K^2)$ and to do this we shall use the Raleigh-Schrodinger perturbation theory [7,8,13] which is a counterpart of carrying out all the integrations over E. There is conservation of momentum in this theory, but no conservation of energy in each vertex and the integration is carried out only over the loop momentum. The answer obtained in the Raleigh-Schrodinger theory will coincide with the covariant perturbation theory with an accuracy of logarithmic factors. In each triple-pomeron vertex the factors of (17) are written and for each n-pomeron intermediate state an n-pomeron Green function is written

$$G_n(E,\vec{K}_1,\ldots,\vec{K}_n)=i\left[E-n\delta_0-\alpha_0'\sum_{i=1}^{n}K_i^2+i\varepsilon\right]^{-1}. \qquad (36)$$

Consider a $2n$ order diagram on a triple-pomeron constant. Using the Feynman identity, according to refs. [7,8], one may write for the combination of the Green function denominators (neglecting the interference terms in the exponent):

$$\sum_n(E,K^2)=(-1)^{n+1}\left(r_0^2/(2\pi)^D\right)^n\frac{1}{2}\int\prod_{i=1}^{n}d^DK_i\int_0^1\prod_{i=1}^{2n-1}dz_i\,\delta\Big(1-\sum_{i=1}^{2n-1}z_i\Big)\cdot$$

$$\cdot \exp\left\{-\xi_0 \alpha'_0 \sum_{i,j=1}^{n} A_{ij}(z)\vec{K}_i \vec{K}_j\right\}\Big[\sum_{i,j=1}^{n} A_{ij}(z)\alpha'_0 \vec{K}_i \vec{K}_j + {} + a(z)K^2 + b(z)\delta_0 - E - i\varepsilon\Big]^{-(2n-1)}. \qquad (37)$$

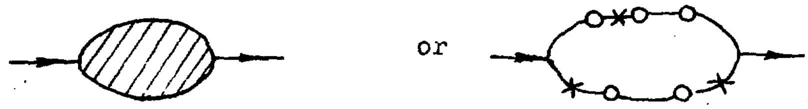
Here $A_{ij}(z)$ is the positive symmetrical matrix, $a(z)>0$ and $b(z)\geq 2$ at all permissible values of z. Depending on the topology of the diagram there may exist here additional factors $\frac{1}{2}$ arising due to closed loops. Note that since at $\varepsilon>0$ RFT is a superrenormalizable theory, then $\Sigma_n(E,K^2)$ as a function of ε has no singularities caused by the ultrared region of integration ($K_i^2 \approx 0$) even when $\delta_0 = 0$. Such infrared singularities arise in relativistic field theories with massless scalar particles which makes the analysis of such theories more complicated [7,8]. In our case the only singularities of $\Sigma_n(E,K^2)$ as a function of ε will be the poles arising from the ultraviolet divergence at $\xi_0 = 0$.

Consider eq. (37) first at $\xi_0 = 0$. According to [7,8], after the integration we obtain

$$\Sigma_n(E,K^2) = (-1)^{n+1}\frac{1}{2}\left(r_0^2/(4\pi\alpha'_0)^{D/2}\right)^n \frac{\Gamma(-1+n\varepsilon/2)}{\Gamma(2n-1)}\cdot \qquad (38)$$

$$\cdot\int_0^1 \prod_{i=0}^{2n-1} dz_i\,\delta\Big(1-\sum_{i=1}^{2n-1} z_i\Big)\left[\det A(z)\right]^{-D/2}\left[a(z)K^2+b(z)\delta_0-E-i\varepsilon\right]^{1-\frac{n\varepsilon}{2}}$$

Due to the general ultraviolet divergence of the diagram at $\xi=0$ in (38) appears a pole contained in its explicit form in Γ-function at $\varepsilon = 2/n$. If the topology of the diagram is such that one of the inner lines contains a self-energy insert of the order $2m$, then there will still be an additional pole at $\varepsilon = 2/m$ arising due to $\det A(z)$ when one or more Feynman parameters are zero. In refs. [7,8] the contributions of more complicated graphs of the ladder and bubble chain types were discussed, e.g.

 (39)

where the cross on the line signifies the contribution of the intercept renormalization counterterm. As it has turned out, graphs of such type do not give essentially new results as compared with the discussed ones, therefore, we shall not specifically dwell on them.

A simple count of powers shows that at $\varepsilon > 0$ only ultraviolet divergences can appear in the proper self-energy part. Therefore, after the intercept renormalization all the Green functions must become finite. Such a renormalization may be carried out in every order of perturbation theory at $\delta_0 > 0$ but not at $\delta_0 = 0$, which is of practical interest for us. In order to make sure in this, let's substitute $\delta_0 = 0$ in (38). We shall have

$$\sum\nolimits_n (E,0) = (-1)^n (\tau_0^2/\alpha_0'^{D/2})^n (-E)^{1-\frac{n\varepsilon}{2}} C_n (1 - n\varepsilon/2)^{-1}. \quad (40)$$

It is clear [7,8] that at $n \geq 2/\varepsilon$ it is here impossible to choose such $\delta\Delta_n$ as to have $\sum_n (0,0) - \delta\Delta_n = 0$. This is a generalization of the difficulty we have encountered in the one-loop approximation at $\varepsilon \geq 2$. The problem lies in the fact that when $\delta_0 = 0$, $\delta\Delta$ has a branch point at

$\tau_o = 0$. This follows from the fact that at $\delta_o = 0$ the only remaining value having energy dimension is $(\tau_o^2/\alpha_o'^{\,D/2})^{2/\varepsilon}$

Therefore, $\delta\Delta$ is to have the form

$$\delta\Delta = (\tau_o^2/\alpha_o'^{\,D/2})^{2/\varepsilon}\, f(\varepsilon), \tag{41}$$

where $f(\varepsilon)$ is a dimensionless value independent of τ_o and α_o'

Let's now return to the case when $\xi_o \neq 0$.

After somewhat complicated calculations, successively using eq. (21), one may obtain that at fairly small δ_o and E eq. (37) is reduced to the form

$$\sum\nolimits_n(E,K^2) = (-1)^{n+1}\left(\frac{\tau_o^2}{(4\pi\alpha_o')^{D/2}}\right)^n \frac{1}{2}\,\frac{\Gamma(1+\frac{\varepsilon}{2}(n-1))}{\Gamma(2n-1)}\, e^{D(z)K^2\xi_o}\cdot$$

$$\cdot\int_0^1 \prod_{i=1}^{2n-1} dz_i\,\delta\Big(1-\sum_{i=1}^{2n-1} z_i\Big)\left[\det A(z)\right]^{-D/2}\left[a(z)K^2+b(z)\delta_o-\right. \tag{42}$$

$$\left.-E-i\varepsilon\right]^{1-\frac{n\varepsilon}{2}}\,\psi\Big(\frac{D}{2},\,2-\frac{n\varepsilon}{2};\,\xi_o\alpha_o'\left[a(z)K^2+b(z)\delta_o-E\right]\Big)\cdot$$

Comparing eqs. (38) and (42) we see that there is no pole in eq. (42) at $\varepsilon = 2/n$. It may also be shown that no addings of self-energy will make them appear. The intercept renormalization may be carried out in every order of perturbation theory, even at $\delta_o = 0$. Substituting $\delta_o = 0$ and $K^2 = 0$ in (42), we shall have

$$\sum\nolimits_n(E,0) = (-1)^n\,(\tau_o^2/\alpha_o'^{\,D/2})^n\,(-E)^{1-\frac{n\varepsilon}{2}}\, C_n\,\Gamma\Big(1+\frac{\varepsilon}{2}(n-1)\Big)\cdot$$
$$\cdot\psi\Big(\frac{D}{2},\,2-\frac{n\varepsilon}{2};\,\xi_o\alpha_o'(-E)\Big)\cdot \tag{43}$$

At $E \to 0$ and $\frac{n\varepsilon}{2} > 1$ from eq. (43) we have

$$\sum_n (E,0) = (-1)^n (\tau_0^2/\alpha_0'^{D/2})^n (-E)^{1-\frac{n\varepsilon}{2}} \Gamma(\tfrac{n\varepsilon}{2}-1)\, C_n \, . \qquad (44)$$

It is obvious that at $n \geq 2/\varepsilon$ one may always choose such $\delta\Delta_n$ as to have $\Sigma_n(0,0) - \delta\Delta_n = 0$. As distinct from the previous case, here at $\delta_0 = 0$ there remains one more dimensional parameter ξ_0, therefore, eq. (41) can no longer be written in such a simple form, and the new $\delta\Delta_n(\xi_0)$ will have no branch point at $\tau_0 = 0$.

It follows from the aforesaid, that it is sufficient to carry out the renormalization of the graph from eq. (8), and, when calculating higher order diagrams, where such bubbles occur, use the already renormalized value in them instead of eq. (12) (at $\xi_0 = 0$) or eq. (22) (at $\xi_0 \neq 0$). And on so doing the renormalization group techniques may be applied. The sequence of actions will then be as follows:

1. In the perturbation theory all one-loop diagrams are calculated.

2. Using the renormalization group, the renormalized $\Gamma^{1,1}$ in the one-loop approximation is calculated.

3. The obtained renormalization group $\Gamma^{1,1}$ is expanded in powers of perturbation theory. And the obtained term $\sim \tau_0^2$ will determine the renormalized contribution of the one-loop diagram from eq. (8).

4. In all following calculations (in two-loop approximations and higher), wherever a one-loop "bubble" occurs in the diagram as a block, the value obtained in the point 3 should be inserted. The then obtained answer will be free from any divergences even at $\xi_0 = 0$ and $\varepsilon = 2$.

In ref. [19] this program was accomplished for the two-loop approximation. It has turned out, that though at first sight the integral approximation for the propagator does not allow any expansion in powers of perturbation theory, nevertheless, the already integrated expression may be expanded in such a series. And the then obtained expression for the counterterm coincides well with the ones obtained in ref. [4], as distinct from the claim [7,8] that it is impossible to determine the counterterm in every order of the perturbation theory. For example, if we expand the pomeron Green function obtained from the renormalization group calculations in ref. [19], in the perturbation theory series in the limit $\xi_0 \to 0$, then we shall obtain for the intercept renormalization counterterm the following expression

$$\delta \Delta_2 = \ln\left(\frac{-\tau_0^2}{16\pi\alpha_0' C_3} \right) + \psi(1 - C_3) \tag{45}$$

where C_3 is the critical exponent of the pomeron Green function and $C_3 = -0.116$.

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