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REGGEON FIELD THEORY AT $D=2$
IN TWO-LOOP APPROXIMATION

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РЕДЖЕОННАЯ ТЕОРИЯ ПОЛЯ ПРИ $D = 2$
В ДВУХПЕТЛЕВОМ ПРИБЛИЖЕНИИ

Разработан общий метод конструирования явного представления для пропагатора померона при наличии дополнительных параметров, таких как порог рождения померона ξ_0 , переданный импульс \vec{K} или смещение интерсекта δ_0 . Показано, что метод применим и в однопетлевом, и в двухпетлевом приближениях. Полученные общие формулы позволяют рассмотреть пропагатор померона и в асимптотической области и в области применимости теории возмущений, и, кроме того, обеспечивают гладкую шивку обеих областей. Вычислены наблюдаемые величины и обсуждаются результаты, связанные с асимптотически большими энергиями.

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A general method of constructing an explicit representation is developed for the pomeron propagator in the presence of additional parameters, such as the pomeron production threshold ξ_0 , momentum transfer \bar{K} or the intercept shift δ_0 . The method is shown to be applicable in both one-loop and two-loop approximations. The obtained general formulae allow to consider the pomeron propagator in both asymptotic region and the region of the perturbation theory applicability. Besides, they provide the smooth matching of both these regions. The observed values are calculated, and the results connected with asymptotically high energies are being discussed.

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1. Introduction

The Reggeon field theory (RFT) has been first formulated by Gribov and Migdal in refs. [1-3]. A further progress in this field has been achieved in works by Migdal, Polyakov and Ter-Martirosyan [4], Abarbanel and Bronzan [5,6]. In these works the infrared behaviour of RFT was investigated using the technique of renormalization group and ϵ -expansion. It was shown that in so doing one may obtain the asymptotic behaviour of the Green function and other values of physical interest. Later on this technique has been developed in refs. [7-13]. Further followed a large series of works dealing with this problem (see reviews [10,13] and the works cited there).

However, all these works contained one essential shortcoming. All the calculations were carried out at the space dimension $D=4-\epsilon$, after which they were expanded in the powers of $\epsilon \approx 0$ and later an analytical continuation to the physical value $\epsilon = 2$ was carried out. In refs. [8,9] the difficulties arising at this approach were analyzed and the analytical transition from $\epsilon = 0$ to $\epsilon = 2$ was shown to be lacking. Besides, after

calculations in the two-loop approximation [14-21] it turned out that the expansion in powers of ϵ was a very poorly converging series, and, therefore, the results obtained for critical exponents gave rise to serious doubts [13].

The alternative to this is the direct calculation of all the diagrams of the theory immediately at $D=2$, which, however, encounters serious difficulties connected with the infrared divergence of theory. This opportunity was considered in detail in refs. [8-10, 15-17, 20-23] and in our previous papers on this subject [24-26]; we shall, therefore, not dwell on the grounds of this approach.

Besides the infrared problem an ultraviolet one appears at any D connected with the integrals divergence over the momentum transfer in the upper limit which is eliminated in the theory with $D=4-\epsilon$ by means of dimensional regularization. However, this problem has a purely formal character, since it is well known [27,28] that in the real RFT there exist reggeon production thresholds. The minimum possible rapidity difference along the reggeon is determined by the production threshold

$\xi_0 = \ln(M^2/s) \approx 2$. Therefore, there appears a need to consider the threshold effects in RFT. As shown in [22,24-26,29,30], the consideration of thresholds must not affect the asymptotic behaviour of critical exponents, but their introduction proves useful for carrying out the smooth matching of the perturbation theory and asymptotic solutions, they facilitate the perturbation theory calculations at $D=2$, eliminating the ultraviolet divergences of theory and, the most important, they provide the analyticity of ϵ -expansion at $\epsilon=2$. More details on this will

be found in our work [26].

In the present work we shall keep to the following scheme grounded in detail in refs. [8-10, 24-26]. In the one-loop approximation at $D=2$ the pomeron renormalization-group propagator $\Gamma^{1,1}$ is calculated and the obtained propagator is expanded in series of perturbation theory. The expansion term $\sim \tau_0^2$ will contain the sum of contributions of the first order "bubble" and intercept renormalization counterterm. Doing the calculations in the two-loop approximation, in all diagrams where such a "bubble" occurs as a block, we shall use instead of it the expression containing the sum of the "bubble" and intercept renormalization counterterm. Then no problems connected with the divergences will arise and this procedure may be repeated any number of times. It is indeed possible to construct in this way a complete two-dimensional theory, and that is what we are going to do in the present work.

In the second section some definitions and basic results obtained in the one-loop approximation in ref. [24] will be given. The third section deals with the pomeron Green function in the two-loop approximation. In the fourth section we have derived the integral representation for the pomeron propagator in the two-loop approximation. In the fifth section the observed values are calculated and the results connected with the asymptotically large energies are discussed.

2. Basic results of the one-loop approximation

We shall further follow the plan and the notations presented in refs. [24-26]. The pomeron bare inverse propagator has the

form

$$i \Gamma_0^{1,1}(E, k^2) = i [G_0^{1,1}]^{-1} = (E - \alpha'_0 k^2 - \delta_0 + i\epsilon) \mathcal{L}^{-\xi_0(E - \alpha'_0 k^2 - \delta_0)} \quad (1)$$

The bare pomeron trajectory is given as

$$\alpha_0(k^2) = \alpha_0(0) - \alpha'_0 k^2 = 1 - \delta_0 - \alpha'_0 k^2, \quad (2)$$

where

$$\delta_0 = 1 - \alpha_0(0) \quad (3)$$

is the bare shift of the pomeron intercept. The rules of the Reggeon diagram technique are described in detail in ref. [24]. The pomeron production threshold ξ_0 in eq.(1) is taken into account according to ref. [27]. The pomeron complete non-renormalized propagator has the form [24]

$$i \Gamma^{1,1}(E, k^2) = i [G^{1,1}(E, k^2)]^{-1} = (E - \delta_0 - \alpha'_0 k^2) \mathcal{L}^{-\xi_0(E - \delta_0 - \alpha'_0 k^2)} + \delta\Delta - \Sigma(E, k^2) \quad (4)$$

where Σ is the pomeron proper self-energy part, and $\delta\Delta$ is the intercept renormalization counterterm.

It can be shown [8,9,24,26] that the RPT Lagrangian has a renormalization-group invariance. Let's introduce the renormalization constants Z_i for the corresponding parameters of theory

$$\Gamma_R^{n,m}(E_i, \vec{k}_i; \alpha', \delta, \tau; E_N, k_N^2) = Z_3^{(n+m)/2} \Gamma^{n,m}(E_i, \vec{k}_i; \alpha_0, \delta_0, \tau_0), \quad (5)$$

$$\alpha' = Z_2^{-1} \alpha'_0; \quad \delta = Z_5 \delta_0; \quad \tau = Z_3^{3/2} Z_1^{-1} \tau_0. \quad (6)$$

The renormalization constants Z_i are defined proceeding from the conditions on the renormalized propagator and the vertex function, which, when the threshold ξ_0 is taken into account, have the following form [24]

$$i \Gamma_R^{1,1}(E, K^2) \Big|_{E=K^2=0} = 0; \quad \frac{\partial}{\partial E} i \Gamma_R^{1,1}(E, K^2) \Big|_N = a_1;$$

$$\frac{\partial}{\partial K^2} i \Gamma_R^{1,1}(E, K^2) \Big|_N = -\alpha' a_1; \quad \delta \frac{\partial}{\partial \delta} i \Gamma_R^{1,1}(E, K^2) \Big|_N = -\delta a_1; \quad (7)$$

$$a_1 = \left[1 + \xi_N \left(1 + h + \frac{p}{4} \right) \right] e^{\xi_N \left(1 + h + \frac{p}{4} \right)};$$

$$\Gamma_R^{1,2}(E, E_1, E_2; \bar{K}, \bar{K}_1, \bar{K}_2) \Big|_{E=2E_1=2E_2=-E_N; \bar{K}=2\bar{K}_1=2\bar{K}_2=\bar{K}_N} = \tau / (2\pi)^{3/2}.$$

Here and later on we shall work in the generalized renormalization point N ($E = -E_N$ and $K^2 = K_N^2$). The first condition requires that the renormalized pomeron intercept should lie at $j = 1$. The second condition defines the renormalization constant Z_3 , the rest serve as definitions of renormalized α' , δ and τ .

According to [11, 12, 22, 24], the theory contains the following dimensionless parameters

$$g = \frac{\tau}{\sqrt{\alpha' E_N}}; \quad h = \frac{\alpha' K_N^2}{E_N}; \quad p = \frac{4\delta}{E_N}; \quad \xi_N = \xi_0 E_N; \quad \xi_K = \xi_0 \alpha' K_N^2; \quad \xi_\delta = \xi_0 \delta(\theta)$$

Here g is the renormalized dimensionless coupling constant. It is connected with the non-renormalized constant g_0 by the following relation

$$g = Z_g g_0; \quad g_0 = \frac{\tau_0}{\sqrt{\alpha'_0 E_N}}; \quad Z_g = Z_3^{3/2} Z_2^{1/2} Z_4^{-1}. \quad (9)$$

The renormalization constants are calculated by means of the following equations:

$$Z_3^{-1} = \alpha_1^{-1} \frac{\partial}{\partial E} i \Gamma^{1,1} \Big|_N; \quad Z_2^{-1} = -\frac{\alpha_1^{-1}}{\alpha_0'} Z_3 \frac{\partial}{\partial K^2} i \Gamma^{1,1} \Big|_N; \quad Z_\delta = \alpha_1^{-1} Z_3 \frac{\partial}{\partial \delta} i \Gamma^{1,1} \Big|_N$$

$$Z_1^{-1} = [(2\pi)^{3/2}/z_0] \Gamma^{1,2} \Big|_{E=2E_1=2E_2=-E_N; K^2=2K_1^2=2K_2^2=K_N^2}. \quad (10)$$

To obtain the renormalization-group equations in multiparameter RFT, one should differentiate $\Gamma^{n,m}$ with respect to E_N , K_N^2 and δ [11,12]. In so doing one obtains a set of three renormalization-group equations, each one of which will be determined by four renormalization-group functions of the following form [11,12,24]:

$$\gamma_E = \frac{\partial \ln Z_3}{\partial \ln E_N} \Big|_{B_E}; \quad \gamma_K = \frac{\partial \ln Z_3}{\partial \ln K_N^2} \Big|_{B_K}; \quad \gamma_\delta = \frac{\partial \ln Z_3}{\partial \ln \delta} \Big|_{B_\delta}, \quad (11a)$$

$$\tau_E = -\frac{\partial \ln Z_2}{\partial \ln E_N} \Big|_{B_E}; \quad \tau_K = -\frac{\partial \ln Z_2}{\partial \ln K_N^2} \Big|_{B_K}; \quad \tau_\delta = -\frac{\partial \ln Z_2}{\partial \ln \delta} \Big|_{B_\delta}, \quad (11b)$$

$$\xi_E = \frac{\partial \ln Z_5}{\partial \ln E_N} \Big|_{B_E}; \quad \xi_K = \frac{\partial \ln Z_5}{\partial \ln K_N^2} \Big|_{B_K}; \quad \xi_\delta = \frac{\partial \ln Z_5}{\partial \ln \delta} \Big|_{B_\delta}, \quad (11c)$$

$$\beta_E = \frac{\partial g}{\partial \ln E_N} \Big|_{B_E}; \quad \beta_K = \frac{\partial g}{\partial \ln K_N^2} \Big|_{B_K}; \quad \beta_\delta = \frac{\partial g}{\partial \ln \delta} \Big|_{B_\delta}, \quad (11d)$$

where

$$B = \{\tau_0, \alpha_0, \xi_N, \xi_K, \xi_\delta\}; \quad B_E = \{B, K_N^2, \delta_0\}; \\ B_K = \{B, E_N, \delta_0\}; \quad B_\delta = \{B, E_N, K_N^2\}.$$

The renormalization constants can depend on the dimensionless parameters g, h, ρ and ξ_N only. The calculations of Z_i are presented in detail in [24].

As a result we have obtained

$$Z_3 = \exp \left\{ \int_0^g dg' \bar{\gamma}(g') / \bar{\rho}(g') \right\}; \quad Z_2 = \exp \left\{ - \int_0^g dg' \bar{\alpha}(g') / \bar{\rho}(g') \right\}; \\ Z_5 = \exp \left\{ \int_0^g dg' \bar{\xi}(g') / \bar{\rho}(g') \right\}; \quad Z_g = \exp \left\{ \int_0^g dg' \bar{\beta}(g') / \bar{\rho}(g') \right\}, \quad (12)$$

where \bar{y}_i and $\bar{\beta}$ are the generalized renormalization-group functions in multiparameter RFT. They have the following form^[24]

$$\bar{\beta} = \beta_E [(1 + \tau_K)(1 + \zeta_S) - \tau_S \zeta_K] + \beta_K [(1 - \tau_E)(1 + \zeta_S) - \tau_S(1 - \zeta_E)] + \beta_S [\zeta_K(\tau_E - 1) - (1 + \tau_K)(\zeta_E - 1)], \quad (13)$$

$$\bar{y} = y_E [(1 + \tau_K)(1 + \zeta_S) - \tau_S \zeta_K] + y_K [(1 - \tau_E)(1 + \zeta_S) - \tau_S(1 - \zeta_E)] + y_S [\zeta_K(\tau_E - 1) - (1 + \tau_K)(\zeta_E - 1)], \quad (14)$$

$$\bar{y}_\alpha = (\tau_E + \tau_K)(1 + \zeta_S) + \tau_S(1 - \zeta_E - \zeta_K), \quad (15)$$

$$\bar{y}_S = (\zeta_E + \zeta_S)(1 + \tau_K) + \zeta_K(1 - \tau_E - \tau_S), \quad (16)$$

$$\bar{y}_g = \frac{1}{g} \bar{\beta} + \frac{1}{g} [(1 + \tau_K)(1 + \zeta_S) - \tau_S \zeta_K]. \quad (17)$$

Later on we shall keep to the following scheme^[11,12,24]:

1. Calculate the values $\Gamma^{1,1}$ and $\Gamma^{1,2}$ by the perturbation theory.
2. By means of eq.(10) calculate the perturbation theory Z_i in the finite type form in powers of g_0 .
3. By means of these perturbation theory Z_i calculate the renormalization-group functions (13)-(17), again in the finite type form in powers of g_0 .
4. Determine the relation between g_0 and g by means of the formulae (9), and substitute it in the renormalization-group functions (13)-(17) calculated according to point 3. They will then adopt the form of finite series already in powers of g .

5. And, finally, by the formulae (12) calculate the renormalization-group Z_i and not the perturbation theory ones, and make no approximations. The series of exact perturbation theory Z_i must transform into formulae (12). New, renormalization-group Z_i are infinite power series which are singular at those values of g where $\bar{\beta}$ is zero. These are the singularities that define the infrared behaviour of the theory. The transition from the finite series of perturbation theory to the infinite ones is simply a more complete use of information contained in the perturbation theory.

For the further advance and in order to obtain the integral representation for the pomeron propagator, we shall need the Z_i obtained from the perturbation theory. Let's now turn to the calculation of Z_i in the one-loop approximation.

The lower order diagram contributing into the proper self-energy part of the pomeron propagator (4) has the form

$$\Sigma_2(E, k^2) = \text{---} \circlearrowleft \text{---} \quad (18)$$

The calculations of the contribution of this graph are given in [24,26]. Making use of these results we obtain for the perturbation theory Z_i in the one-loop approximation

$$Z_3^{-1} = 1 - \frac{g_0^2}{16\pi} a; \quad Z_2^{-1} = Z_3 \left(1 - \frac{g_0^2}{32\pi} a \right); \quad (19)$$

$$Z_6 = Z_3 \left(1 - \frac{g_0^2}{8\pi} a \right),$$

where

$$a_4 = 1 + \frac{p}{2} + \frac{h}{2}; \quad a = \frac{e^{-\xi_N(2 + \frac{3}{2}h + \frac{3}{4}p)}}{a_4(1 + \xi_N(1 + h + \frac{p}{4}))}; \quad a(0) = 1 \quad (20)$$

The vertex function in the one-loop approximation has the form²⁴

$$\Gamma_3^{1,2}(E_N, K_N^2) = -\frac{z_0 g_0^2}{(2\pi)^{5/2}} b = \text{diagram 1} + \text{diagram 2} \quad (21)$$

where b is calculated in^[24] and we are not going to present it here. At $\xi_0 = K_N^2 = \delta_0 = 0$ $b = \ln 2$. From (10) we obtain

$$Z_1^{-1} = 1 - g_0^2 b / (2\pi). \quad (22)$$

To within the accuracy of terms of the order of g^2 we have

$$g_0 = g \left[1 + (g^2 / 32\pi) \cdot (16b - \frac{5}{2}a) \right]. \quad (23)$$

Let's now pass on to the calculation of the renormalization-group functions. Differentiating Z_i from (19) and (22), replacing g_0 by g by eq. (23), we obtain generalized renormalization-group functions

$$\begin{aligned} \bar{\gamma} &= -\bar{\gamma}_\delta = -\frac{g^2}{16\pi} a; & \bar{\gamma}_\alpha &= -\frac{g^2}{32\pi} a; \\ \bar{\gamma}_g &= \frac{1}{2} + \frac{1}{g} \bar{p} - \frac{g^2}{64\pi} \frac{a}{a_4} \left(\frac{1}{2} - p \right); & \bar{p} &= -\frac{1}{2} g \left(1 - \frac{g^2}{g_1^2} \right), \end{aligned} \quad (24)$$

where g_1^2 determines the position of the zero of \bar{p} -function.

$$g_1^2 / (16\pi) = \left[16b - \frac{5}{2}a + \frac{a}{2a_4} \left(\frac{1}{2} - p \right) \right]^{-1}. \quad (25)$$

At the point $g^2 = g_1^2$ the \bar{p} -function is zero and $\bar{p}'(g_1^2) > 0$ i.e. is the infrared stable point of the theory in the one-loop approximation that we have been seeking. In the case of $\xi_0 = K_N^2 = \delta_0 = 0$ we obtain

$$g_1^2 / (16\pi) = (16 \ln 2 - 5/2)^{-1} = 0.116 \quad (25a)$$

That is $g_1^2 / (16\pi)$ is a fairly small value and it can serve as an effective parameter of expansion. g_1^2 is the function of the

parameters h , ρ and ξ_N . Its behaviour, depending on these parameters, is considered in detail in [24].

Let's finally calculate the renormalization-group Z_i by eq.(12). We shall obtain

$$\begin{aligned} Z_3 = Z_8^{-1} &= (1 - g^2/g_1^2)^{\bar{\gamma}(g_1^2)}; & Z_2 &= (1 - g^2/g_1^2)^{-\bar{\gamma}_\alpha(g_1^2)}; \\ Z_g &= (1 - g^2/g_1^2)^{\frac{1}{2}} [1 + \tau_\kappa(g_1^2) + \zeta_\delta(g_1^2)] \end{aligned} \quad (26)$$

It can be shown that if we expand eq.(26) in series of g and again replace g by g_0 , we shall obtain the exact expressions for Z_i in the one-loop approximation of eqs.(19) and (22). Which means that all the perturbation theory series are exactly restored from the renormalization-group results (26), at least in the first order by g_0^2 .

Let's now pass on to the construction of integral representation for the pomeron propagator. As already mentioned [8, 9, 11, 12, 22, 24], it is the specific character of RFT that namely the bare values, and not the normalization ones, are observable in it. Therefore, later on we shall need the expressions of Z_i versus g_0^2 . In the one-loop case the problem becomes much simpler [22], since

$$g = g_0 (1 - g^2/g_1^2)^{C_g}; \quad C_g = \frac{1}{2} + \frac{1}{2} [\tau_\kappa(g_1^2) + \zeta_\delta(g_1^2)]. \quad (27)$$

τ_κ and ζ_δ are small [24] and we can assume to a fairly good accuracy that

$$C_g = 1/2. \quad (28)$$

It has been shown in [24] that at the variations of h and ρ from 0 to 10, the exponent C_g changes by less than 1%. This case has been first considered in [22], and in [11, 12] a me-

thod of obtaining the representation for the propagator in the case of arbitrary C_g has been suggested. In [24] both these methods were used and they were shown to give the same results. From eqs. (27) and (28) we immediately obtain

$$g_0 = g(1 - g^2/g_1^2)^{-1/2}; \quad g = g_0(1 + g^2/g_1^2)^{-1/2}. \quad (29)$$

As already mentioned in [10-12, 22,24], all Z_i are finite values at fixed $Z_0, \alpha_0', \delta_0, E_N$ and K_N^2 , therefore, there appears an opportunity to calculate the pomeron propagator $\Gamma^{1,1}$, though, in fact, it is independent of E_N and K_N^2 . Such a manipulation seems somewhat doubtful. However, after some analysis [10-12, 22,24] it becomes obvious that all the values ($Z_i, \Gamma_R, E_N, K_N^2$ etc.) related to the renormalization group may be used to define the derivatives of $\Gamma^{1,1}$, if we simply replace in them $E_N \rightarrow -E$, $K_N^2 \rightarrow K^2$.

The inverse non-renormalized total propagator, with an imposed condition (7) that the intercept is a unity, is determined by integrating eq.(10) over E' in the limits from 0 to E at fixed p and h replacing everywhere $-E_N \rightarrow E$ and $K_N^2 \rightarrow K^2$ [10-12, 22,24]. We shall obtain

$$i\Gamma^{1,1}(E, K^2) = \int_0^E dE' \frac{\partial}{\partial E'} i\Gamma^{1,1}(E', h, p) \Big|_{h,p}, \quad (30)$$

where

$$\begin{aligned} \frac{\partial}{\partial E} i\Gamma^{1,1}(E, h, p) \Big|_{h,p} &= \frac{\partial i\Gamma^{1,1}}{\partial E} \Big|_{K^2, S} + \frac{\partial i\Gamma^{1,1}}{\partial K^2} \Big|_{E, S} \frac{\partial K^2}{\partial E} \Big|_{h,p} + \\ &+ \frac{\partial i\Gamma^{1,1}}{\partial \delta_0} \Big|_{E, K^2} \frac{\partial \delta_0}{\partial E} \Big|_{h,p} \end{aligned} \quad (31)$$

All the derivatives of $i\Gamma^{L,L}$ are determined from eq.(10). If we make in them an inverse replacement $-E_N \rightarrow E$ and $\kappa_N^2 \rightarrow \kappa^2$ then

$$\frac{\partial}{\partial E} i\Gamma^{L,L}(E, h, \rho) \Big|_{h, \rho} = a_1 z_3^{-1} \left\{ 1 - \alpha'_0 z_2^{-1} \frac{\partial \kappa^2}{\partial E} \Big|_{h, \rho} - z_8 \frac{\partial \delta_0}{\partial E} \Big|_{h, \rho} \right\} \quad (32)$$

Since h and ρ are now fixed values, it follows from eq.(18) that κ^2 and δ_0 must change in such a way, that h and ρ remain fixed [10,22,24].

Let's again return to z_i which we shall now consider not as renormalization constants but merely as derivatives of $\Gamma^{L,L}$, and to do this we shall make in z_i the replacement $E_N \rightarrow -E$ and $\kappa_N^2 \rightarrow \kappa^2$. Making use of eqs.(26) and (29) we obtain

$$z_3^{-1} = z_8 = (1 - E_0(p, h)/E)^{c_3}; \quad z_2 = (1 - E_0(p, h)/E)^{-c_2} \quad (33)$$

where

$$c_3 = \bar{y}(g_1^2); \quad c_2 = -\bar{y}_\alpha(g_1^2); \quad E_0(p, h) = z_0^2 / (\alpha'_0 g_1^2(p, h, \xi_N)) \quad (34)$$

As shown in [24], all c_i are very weak functions of h, ρ and ξ_N , therefore, we shall later consider that

$$c_3(p, h, \xi_N) = c_3(0) = -c_8(0) = -2c_2(0) = -0.116 \quad (35)$$

Making use of eqs.(6), (8) and (33) let's calculate the derivatives of κ^2 and δ_0 . Substituting them in eq.(32) we finally obtain [24]

$$i\Gamma^{L,L}(E, \kappa^2) = \int_0^E dE' (-E_0(p, h)/E')^{c_3} (1 - E'/E_0(p, h))^{c_3} a_1 \cdot \left. \left\{ 1 + h \left(1 + \frac{c_2}{1 - E'/E_0(p, h)} \right) + \frac{\rho}{4} \left(1 - \frac{c_5}{1 - E'/E_0(p, h)} \right) \right\} \right|_{h, \rho} \quad (36)$$

This is the desirable integral representation for the pomeron propagator in the one-loop approximation. Though the subintegral expression in eq.(36) cannot be expanded in series of perturbation theory [8,9,22], the expression for $\Gamma^{1,1}$, obtained after integration, already can be expanded in such a series, and moreover, it correctly reproduces the initial series of perturbation theories. As it will be shown in the next section, not only the terms $\sim \tau_0^2$, but also the terms of the order τ_0^4 , lacking in the initial series, are reproduced correctly. In the case when $\xi_0 E \ll 1$ we have

$$i \Gamma^{1,1}(E, k^2) = - \frac{E_0(p, h)}{1 - c_3} \left(- \frac{E}{E_0(p, h)} \right)^{1 - c_3} \left\{ \left(1 + h + \frac{p}{4} \right) \cdot \right. \\ \cdot {}_2F_1 \left(1 - c_3, 1 - c_3; 2 - c_3; \frac{E}{E_0(p, h)} \right) + \left(c_2 h - c_3 \frac{p}{4} \right) \cdot (37) \\ \left. \cdot {}_2F_1 \left(1 - c_3, 1 - c_3; 2 - c_3; \frac{E}{E_0(p, h)} \right) \right\},$$

where ${}_2F_1(a, b; c; z)$ is the ordinary hypergeometrical Gaussian function. The infrared asymptotics of eq.(37) has the form

$$i \Gamma^{1,1}(E, k^2) = - \frac{E_0(p, h)}{1 - c_3} \left(- \frac{E}{E_0(p, h)} \right)^{1 - c_3} \left\{ 1 + h(1 + c_2) + \frac{p}{4}(1 - c_3) \dots \right\} (38)$$

In the perturbation theory limit $E/E_0 \gg 1$ we obtain

$$i \Gamma^{1,1}(E, k^2) = E - \alpha'_0 k^2 - \delta_0 - \frac{\tau_0^2}{16\pi\alpha'_0} \left[\ln \left(\frac{16\pi\alpha'_0 c_3}{\tau_0^2} \left(E - \frac{\alpha'_0 k^2}{2} - 2\delta_0 \right) \right) + \Psi(2) - \Psi(1 - c_3) \right] + \frac{\tau_0^4}{2(16\pi\alpha'_0)^2} \frac{1 - c_3}{c_3} \cdot \\ \cdot \left(E - \frac{\alpha'_0 k^2}{2} - 2\delta_0 \right)^{-1}. (39)$$

The term $\sim \tau_0^2$ is an exact expression for the value $\sum_2 -\delta \Delta$

and the term $\sim \tau_0^4$, as it will be shown later, is numerically equal to the contribution of Σ_4 .

3. Two-loop approximation

Let's continue the investigation of the pomeron propagator structure in the perturbation theory. Consider the two-loop contribution in the pomeron Green function $\Gamma^{l,l}$. It has the form

$$\Sigma_{4a} = \text{diagram a} ; \Sigma_{4b} = \text{diagram b} \quad (40)$$

$$\Sigma_4 = 2(\Sigma_{4a} + \Sigma_{4b})$$

The contribution of these diagrams has been calculated in detail in ref. [25].

In accord with the rules of Reggeon diagram technique

$$2 \Sigma_{4a} = -\frac{\tau_0^2}{4\pi} \int \frac{d^2 k_1 dE_1}{2\pi^2 i} \frac{\ell}{(-E_1 + \delta_0 + \alpha'_0 k_1^2)^2} \frac{\xi_0(E+E_1-3\delta_0) - \xi_0 \alpha'_0 (2k_1^2 + (\vec{k} - \vec{k}_1)^2)}{\ell} \cdot \frac{\Sigma_2(E_1, k_1^2) - \delta \Delta}{-E + E_1 + \delta_0 + \alpha'_0 (\vec{k} - \vec{k}_1)^2} \quad (41)$$

From (4) we obtain

$$\Sigma_2(E, k^2) - \delta \Delta = (E - \alpha'_0 k^2 - \delta_0) \ell \frac{-\xi_0(E - \delta_0 - \alpha'_0 k^2)}{-i \Gamma^{l,l}} \quad (42)$$

where $i \Gamma^{l,l}$ from eq.(36) should be expanded in series up to the terms of the order τ_0^2 . The obtained expression should be substituted in eq.(41) and be integrated.

Consider now the two-loop vertex function $\Gamma_{\varepsilon}^{l,2}$. It has

All the integrals of J_i and $\Gamma_{V1}^{1,2}$ are determined and calculated in ref. [25].

Consider the renormalization constants Z_i which will be written in the two-loop approximation in the form

$$\begin{aligned} Z_3^{-1} &= 1 - g_0^2 a^{(0)}/16\pi + g_0^4 f; & Z_2^{-1} &= Z_3 (1 - g_0^2 a^{(0)}/32\pi + g_0^4 f_2); \\ Z_8 &= Z_3 (1 - g_0^2 a^{(0)}/8\pi + g_0^4 f_8); & Z_1^{-1} &= 1 - g_0^2 b^{(0)}/2\pi + g_0^4 d. \end{aligned} \quad (46)$$

The index (0) ($a^{(0)}, b^{(0)}$) in (46) implies that they contain h_0 and p_0 , and when passing to the final expressions, these values should be renormalized (i.e. transform from $h_0 \rightarrow h$ and $p_0 \rightarrow p$). This operation wasn't performed in refs. [15-17].

The values f_i and d are determined from the following equations

$$\begin{aligned} f &= -\frac{a_1^{-1}}{g_0^4} \frac{\partial}{\partial E} \Sigma_4|_N; & f_2 &= \frac{a_1^{-1}}{a_0' g_0^4} \frac{\partial}{\partial \kappa^2} \Sigma_4|_N; & f_8 &= \frac{a_1^{-1}}{g_0^4} \frac{\partial}{\partial \delta} \Sigma_4|_N; \\ d &= d_{VG} + d_S = \frac{(2\pi)^{3/2}}{z_0} \Gamma_S^{1,2} \Big|_{\substack{E=2E_1-2E_2=-E_N \\ \kappa=2\kappa_1-2\kappa_2=\kappa_N}} \\ d_{VG} &= (2\pi)^{3/2}/z_0 (\Gamma_V^{1,2} + \Gamma_G^{1,2}); & d_S &= (2\pi)^{3/2}/z_0 \Gamma_S^{1,2}. \end{aligned} \quad (47)$$

All the values of (47) are calculated in ref. [25]. The following feature turns out: if we plot the terms of the order z_0^4 obtained due to expansion of the one-loop Z_i from eq. (26) in series of perturbation theory, on the one and the same graph, they will exactly coincide with the graphs for f, f_2, f_8 and d_{VG} at all values of the parameters h, p and ξ_N .

The analysis shows that this is exactly what should have

been in RFT at $D=2$. In fact, there were diagrams (18) and (21) in one-loop approximation, and their contribution was calculated exactly, without any approximations. They were then inserted into the renormalization group and the non-perturbation-theory answer (26) was calculated, which later was expanded back in series of perturbation theory. At the same time, besides the one-loop contribution, the contribution of all two-loop diagrams, containing the graphs from (18) and (21) as compound blocks, should have been reproduced as well. The Green function $\Gamma^{L,1}$ then is reproduced completely, since in the two-loop approximation there are only two graphs (40) completely constructed of diagrams (18) and (21). The vertex graphs $\Gamma_V^{L,2}$ from (44a) are composed of diagrams (21), and $\Gamma_G^{L,2}$ are constructed of graphs (18), hence, the renormalization group must correctly reproduce the sum $\Gamma_V^{L,2} + \Gamma_G^{L,2}$, i.e. d_{VG} . It is impossible to construct the diagrams $\Gamma_S^{L,2}$ from (44c) by means of the graphs (18) and (21), therefore, in the one-loop approximation the renormalization group does not reproduce their contribution, and, hence, we could have not calculated the contributions of graphs (40) and (44a,b) at all, but limit ourselves to the calculation of $\Gamma_S^{L,2}$ from (44c) and add its contribution to the perturbation theory expansion for \bar{Z}_1^{-1} , which can be obtained from (26) making use of eqs.(9) and (29). The obtained total reversibility of the renormalization-group transformations demonstrates the main advantage of the work at $D=2$, namely a direct connection with the perturbation theory.

Later for f_i and d_{VG} the expressions should be used

obtained after expanding the formulae (26) in series of perturbation theory:

$$f = \frac{a}{2(16\pi)^2} \left[16b - \frac{5}{2}a - \frac{a}{2a_u} \left(\frac{h}{2} - P \right) \right], \quad (48a)$$

$$f_2 = \frac{a}{4(16\pi)^2} \left[16b - 2a - \frac{a}{2a_u} \left(\frac{h}{2} - P \right) \right], \quad (48b)$$

$$f_8 = \frac{a}{(16\pi)^2} \left[16b - \frac{a}{2} - \frac{a}{2a_u} \left(\frac{h}{2} - P \right) \right], \quad (48c)$$

$$d_s = \frac{b}{(8\pi)^2} \left[24b - \frac{5}{2}a - \frac{a}{2a_u} \left(\frac{h}{2} - P \right) \right] + d_s, \quad (48d)$$

where d_s is calculated by the formula (45c).

At last, calculating all the necessary derivatives, performing the renormalizations of all the values with indices (0) and substituting all this in eqs.(13)-(17), after fairly long calculations, we finally obtain the following expressions for generalized renormalization-group functions in the two-loop approximation

$$\bar{\gamma} = -\bar{\gamma}_8 = -\frac{g^2}{16\pi} a + \frac{g^4}{(16\pi)^2} a^2 \left[\frac{1}{a_u} \left(P - \frac{h}{2} \right) - \xi_N P \right], \quad (49a)$$

$$\bar{\gamma}_\alpha = -\frac{g^2}{32\pi} a + \frac{g^4}{(16\pi)^2} a^2 \left[\frac{1}{2a_u} \left(P - \frac{h}{2} \right) - \xi_N \left(P - \frac{h}{4} \right) \right] \quad (49b)$$

$$\bar{\gamma}_g = \frac{g^2}{32\pi} \left(16b - \frac{5}{2}a \right) + \frac{g^4}{(16\pi)^2} a^2 \left[\frac{1}{a_u} \left(P - \frac{h}{2} \right) - \xi_N \left(P - \frac{h}{4} \right) \right] - 2g^4 d_s, \quad (49c)$$

$$\bar{\beta} = -\frac{1}{2}g(1 + g^2\beta_1 + g^4\beta_2), \quad (50)$$

where

$$\beta_1 = -\frac{1}{16\pi} \left[16b - \frac{5}{2}a + \frac{a}{2a_4} \left(\frac{1}{2} - p \right) \right], \quad (51)$$

$$\beta_2 = 4d_s - \frac{2a^2}{(16\pi)^2} \left[\frac{1}{a_4} \left(p - \frac{1}{2} \right) - \xi_N \left(p - \frac{1}{4} \right) \right] - T, \quad (52)$$

$$T = -\frac{g^4}{2(16\pi)^2} \frac{a}{a_4} \left\{ \left(\frac{1}{2} - p \right) \left[16b + \frac{a}{2} \left(3 + \frac{2}{2_4} \left(\frac{1}{2} - p \right) + \frac{1}{2} \xi_N \right) \right] - \frac{3}{4}ah \right\} - g^4 E_N^3 \left[\frac{1}{\alpha'} \frac{\partial}{\partial \kappa^2} (f-f_2)_{B_\kappa} + \frac{p}{4} \frac{\partial}{\partial \delta_0} (f-f_1)_{B_\delta} \right] \quad (53)$$

At $\xi_0 = \kappa^2 = \delta_0 = 0$ we obtain

$$\begin{aligned} \bar{\gamma}_0 &= -\bar{\gamma}_s = -\frac{g^2}{16\pi} a; & \bar{\gamma}_{\alpha 0} &= \frac{-g^2}{32\pi} a; \\ \bar{\gamma}_{g_0} &= \frac{g^2}{32\pi} (16b - \frac{5}{2}a) - 2g^4 d_s; & & \\ \bar{\beta}_0 &= -\frac{1}{2}g \left\{ 1 + g^2\beta_{10} + 4d_s g^4 \right\}. \end{aligned} \quad (54)$$

Hence, when all the additional parameters of theory are equal to zero $\bar{\gamma}$, $\bar{\gamma}_\alpha$ and $\bar{\gamma}_s$ are equal to their one-loop values (24), and $\bar{\gamma}_g$ and $\bar{\beta}$ gain an addition due to the contribution of nonplanar graphs $\Gamma_g^{1,2}$. Namely due to the fact that $\bar{\gamma}_0$ etc. are equal to their one-loop values, and the additional terms (except d_s) in eq.(49) occur only due to the renormalizations of additional parameters, the expansion of one-loop results in series of perturbation theory coincides with the two-loop calculations with an accuracy to the terms containing

Consider now the obtained two-loop $\bar{\beta}$ -function of Gell-Mann-Low. Let's first investigate it at zero values of the parameters $\bar{\xi}_0 = k^2 = \delta_0 = E$. We have

$$\bar{\beta}_0 = -\frac{1}{2}g(1 - g^2 \cdot 0.172 + g^4 \cdot 0.0158) \quad (55)$$

which is in fairly good agreement with the previous results in [15-17]. (In [15,16] a bit larger value was obtained for d_s , since the authors had multiplied the contribution of the third diagram from $\Gamma_s^{4,2}$ not by $\frac{1}{2}$ but by 2. A similar error was committed in refs. [14,18]).

From eq.(55) we see that in the two-loop approximation the $\bar{\beta}$ -function does not have a real zero at $g^2 > 0$. This mustn't be a surprise, since in all previous works both at $D=2$ [15-17] and in ϵ -expansion [14,18,19] (at $\epsilon = 2$) the two-loop $\bar{\beta}$ -function had no zero. In ref. [17] it was obtained that in the three-loop approximation the $\bar{\beta}$ -function had no zero at the point $g_1^2/16\pi = 0.13$. Besides, it was shown in [20,21] that when taking into account the higher order perturbation theories, the RFT at $D=2$ should have a stable infrared point $\bar{\beta}(g_1)$ with $\bar{\beta}'(g_1) > 0$. Therefore, following ref. [16], we shall look for the zero of $\bar{\beta}$ -function with the help of Pade approximation. The Pade approximant for the $\bar{\beta}$ -function fixed by eq.(55) has the form

$$\bar{\beta}^{[1,1]} = -\frac{1}{2}g \frac{\beta_1 + g^2(\beta_1^2 - \beta_2)}{\beta_1 - g^2\beta_2} \quad (56)$$

with a zero at the point

$$g_1^2 = -\frac{\beta_1}{\beta_1^2 - \beta_2} \quad (57)$$

At the zero values of parameters we obtain

$$g_1^2/16\pi = 0.25 \quad (58a)$$

which is in excellent agreement with the value

$$g_1^2/16\pi = 0.26 \pm 0.02 \quad (58b)$$

obtained in Cardy's works [20,21] when higher terms of perturbation theory at $D=2$ are taken into account. Besides, this value is in good agreement with the number

$$g_1^2/16\pi = 0.2 \div 0.25 \quad (59)$$

obtained in ref. [19] in ϵ -expansion using Pade-Borel approximation. The values $g_1^2/16\pi$ obtained by means of three so different methods turned out to be very close to each other. It follows from here that the present quantity of this value apparently must not differ strongly from the quantity (58).

The fact that the two-loop $\bar{\beta}$ -function has no infrared stable point is apparently the reflection of the fact that beginning with the two loops the character of the perturbation theory series is changed: in the one-loop case the Green function has a logarithmic behaviour in \bar{E} , and all the other orders of perturbation theory make contributions exponential with respect to L/E , therefore, due to the change of mode of the propagator behaviour one cannot limit oneself to two-loop approximations only. The three-loop calculations in [17] have shown that the zero in $\bar{\beta}$ -function has again been restored. Besides, Cardy has shown in [20,21] that the contributions of the perturbation theory higher orders in $\bar{\beta}$ -function

at $D=2$ form a sign-stable series, which provides the existence of infrared stable point in RFT at $D=2$. And the fact that by means of Padé approximation namely Cardy's result^[20,21] has been obtained, and not that of Harrington^[17], shows that the Padé approximant (56) describes the structure of $\bar{\beta}$ -function in much more exact way than the perturbation theory, even at the three-loop level. Therefore, in our further calculations we shall use the $\bar{\beta}$ -function having the form (56).

Let's calculate with its help the renormalization constants Z_i . Substituting (49) and (56) in eq.(12), we shall have after integration

$$\begin{aligned} Z_3 = Z_8^{-1} &= (1 - g^2/g_1^2)^{c_3} \varphi_3(g^2); \quad Z_2 = (1 - g^2/g_1^2)^{c_2} \varphi_2(g^2); \\ Z_g &= (1 - g^2/g_1^2)^{c_g} \varphi_g(g^2), \end{aligned} \quad (60)$$

where the following notations are introduced

$$c_3 = \bar{\gamma}(g_1^2)/\bar{\beta}'(g_1^2); \quad c_2 = -\bar{\gamma}_\alpha(g_1^2)/\bar{\beta}'(g_1^2); \quad c_g = \bar{\gamma}_g(g_1^2)/\bar{\beta}'(g_1^2) \quad (61)$$

and

$$\begin{aligned} \varphi_i(g^2) &= \exp \left\{ g^2 \frac{g_1^2}{\beta_1} [\gamma_{i1} \beta_2 + g_2^2 \gamma_{i2} \beta_1^2] + \frac{1}{2} g^4 \frac{g_1^2}{\beta_1} \gamma_{i2} \beta_2 \right\}, \\ \bar{\gamma}_i(g_1^2) &= -g_1^2 \gamma_{i1} - g_2^4 \gamma_{i2}; \quad \bar{\beta}'(g_1^2) = -\frac{1}{g_1^2 \beta_1}. \end{aligned} \quad (62)$$

All the values of γ_i and β_i are defined in the formulae (49)-(53).

In fig. 1 the results are given of numerical calculations on a computer for the critical exponents C_i and $g_1^2/16\pi$

in the two-loop approximation at various values of parameters ξ_N , h and ρ . It can be seen that at reasonably small values of ξ_N all C_i depend very weakly on h and ρ , therefore, we shall assume later that

$$C_i(h, \rho, \xi_N) = C_i(h = \rho = \xi_N = 0). \quad (63)$$

Let's present here the values of C_i that we are going to use in the future

$$C_3 = -0.555; \quad C_2 = -C_3/2 = 0.277; \quad C_g = 0.973. \quad (64)$$

$$g_1^2/16\pi = 0.254; \quad \beta_1 = -0.171; \quad \beta_2 = 0.0158; \quad \bar{\beta}(g_1^2) = 0.458.$$

If we expand the renormalization-group constants Z_i in series in g and pass from g to g_0 and from a_i to $a_i^{(0)}$ then the perturbation theory initial series will be restored with absolute exactness, which is a verification of the validity of the carried out calculations.

4. Integral representation for the pomeron propagator in the two-loop approximation

We shall further follow the technique described in detail in refs. [11,12,22,24,25] and in section two of this paper. Consider now eq.(60) at small h , ρ and ξ_N . We can write that in this case

$$\Psi_g \approx 1; \quad C_3 = -C = -0.555; \quad \bar{g}_g = \frac{1}{2}; \quad C_g = 0.973 \approx 1 \quad (65)$$

Hence

$$g^2 = g_1^2 \left(1 + \frac{g_1^2}{2g_0^2} G\right) \quad \text{where} \quad G = 1 - \left(1 + \frac{4g_0^2}{g_1^2}\right)^{1/2} \quad (66)$$

This formula is the analog of eq.(29) in the two-loop approximation. Expressing g_o^2 via E we shall obtain

$$g_o^2 = g_l^2 \left(1 - \frac{E}{2E_o} G\right); \quad G = 1 - \left(1 - \frac{4E_o}{E}\right)^{1/2}, \quad (67)$$

where

$$E_o \equiv E_o(\xi_N, h, p) = z_o^2 / [\alpha_o' g_l^2(\xi_N, h, p)]. \quad (68)$$

Substituting (67) in (60) we obtain the following expression for the renormalization constants Z_i via E and E_o :

$$Z_3 = X^{-c} \exp[-c\alpha(1-X)]; \quad Z_2^{-1} = X^{-c_2} \exp[-c_2\alpha(1-X)] \quad (69)$$

where

$$X = \frac{E}{2E_o} G; \quad \alpha = \beta_2 / \beta_1^2. \quad (70)$$

We shall carry out all further calculations at $\delta_o = 0$ and small h and ξ_N . From eq.(39) we have

$$\frac{\partial i\Gamma^{4,1}}{\partial E} \Big|_h = Z_3^{-1} \left[1 - \alpha_o' z_2^{-1} \frac{\partial K^2}{\partial E} \Big|_h\right] a_1. \quad (71)$$

Besides

$$K^2 = -\frac{h}{\alpha_o'} E Z_2 = -\frac{h}{\alpha_o'} E X^{c_2} \exp[c_2\alpha(1-X)]. \quad (72)$$

Differentiating (72) with respect to E we obtain

$$-\alpha_o' Z_2^{-1} \frac{\partial K^2}{\partial E} \Big|_h = h \left\{1 + \frac{1}{2} c_2 \frac{1-X}{1-\frac{1}{2}X} (1-\alpha X)\right\}. \quad (73)$$

Substituting eqs.(69) and (73) in (71) we obtain the desired integral representation for $i\Gamma^{4,1}$ in the two-loop approximation

$$i \Gamma^{1,1}(E, k^2) = \int_0^E dE' X^c a_1 e^{c\alpha(1-X)} \left\{ 1 + h + \frac{h}{2} c_2 \frac{1-X}{1-\frac{1}{2}X} (1-\alpha X) \right\} \quad (74)$$

At $\xi_0 \approx 0$ and fixed h, E_0 is independent of E , and eq.(74) can be integrated. We shall have

$$i \Gamma^{1,1}(E, k^2) = -\frac{2E_0}{2+c} X^{2+c} e^{c\alpha} \left\{ (1+h) [\Phi_1(2+c, 2, 3+c; X, -c\alpha X) - \frac{2+c}{2(3+c)} X \Phi_1(3+c, 2, 4+c; X, -c\alpha X)] + \right. \quad (75)$$

$$+ \frac{h}{4} c_2 [\Phi_1(2+c, 1, 3+c; X, -c\alpha X) - \alpha X \frac{2+c}{3+c} \Phi_1(3+c, 1, 4+c; X, -c\alpha X)] \left. \right\}$$

where $\Phi_1(a, b, c; x, y)$ is the confluent hypergeometric function of two second order variables. At $F/E_0 \ll 1$ we have at asymptotics

$$X = 2x^{1/2} (1 - x^{1/2} + \frac{1}{2}x - \frac{1}{8}x^2); \quad x = -E/4E_0. \quad (76)$$

Expanding all Φ_1 in (75) in series in X , which in turn should be expanded in series in powers of X , we shall obtain the following asymptotic expression for the pomeron propagator in the two-loop approximation with an accuracy to the preasymptotic terms of the order

$$i \Gamma^{1,1}(E, k^2) = -\frac{E_0}{2+c} 2^{3+c} e^{c\alpha} x^{1+c/2} \left\{ 1 + h(1 + \frac{c_2}{2}) + \right. \quad (77)$$

$$+ \frac{2+c}{3+c} B_1 x^{1/2} + \frac{2+c}{4+c} B_2 x + \frac{2+c}{5+c} B_3 x^{3/2} + \frac{2+c}{6+c} B_4 x^2 \left. \right\}$$

The main difference of eq.(77) from the one-loop eq.(38) lies in the fact that in eq.(77), besides the constants C_i , there

have appeared half-integral powers $\bar{E}/4E_0$. The coefficients B_i in eq.(77) are algebraic functions of h and c_i . Their form is given in ref. [25] (see eq.(4.77)).

Consider now the limit of perturbation theory when $E_0/E \ll 1$. Then

$$X = 1 - \left(-\frac{E_0}{E}\right) + 2\left(-\frac{E_0}{E}\right)^2 - 5\left(-\frac{E_0}{E}\right)^3 \quad (78)$$

Hence, the small value, by which the expansion may be carried out, is $1-\beta$. Making some transformations we obtain (in more detail see [25])

$$\begin{aligned} i\Gamma^{L,1}(E) = E - \frac{z_0}{16\pi\alpha_0} \left[\ln\left(\frac{E}{z_0^2} 16\pi\alpha_0' c_0\right) + \Psi(2) - \Psi(1+c_0) \right] + \\ + \frac{z_0^4}{2(16\pi\alpha_0')^2} \frac{1}{E} \frac{1}{c_0} (c_0 + 3 - \alpha(4 + c_0(2-\alpha))) \end{aligned} \quad (79)$$

where $C_0 = 0.116$ is the critical exponent in the one-loop case. The numerical coefficient before the term $\sim z_0^2$ is exactly the same as the one obtained in eq.(39) and before the term z_0^4 it is 8.81; in the one-loop case they have been 9.6. This difference is due to the approximation made in eq.(65) where we have replaced C_g by a unit. As already mentioned, the series (39) exactly coincides with the bare perturbation-theory series. Hence the series (79) too is a fairly good approximation to the exact answer. Therefore, we shall later make use of the approximation (65) and propagator (74).

5. Values under observation

In order to obtain the experimentally observed values, one should have a pomeron propagator in (y,t) -representation,

where $Y = \ln(\mathcal{S}/\mathcal{S}_0)$ is the rapidity. The transition from (E, k^2) -representation to (Y, t) -representation is carried out by means of the Sommerfeld-Watson transformation from the propagator in (E, k^2) -representation at fixed $k^2 = -t$. It has the form

$$F(Y, k^2) = -\frac{1}{2\pi i} \int_{\uparrow} dE e^{-EY} [i \Gamma^{1,1}(E, k^2)]^{-1} \quad (80)$$

Let's now define the behaviour of total cross section at asymptotic values of energy. Substituting the propagator (77) in (80), we obtain at $\xi_0 = \delta_0 = 0$

$$\begin{aligned} \sigma_{tot}(Y) \propto F(Y) &= \frac{1+c/2}{\Gamma(1+c/2)} e^{-c\alpha} (\bar{E}_0 Y)^{c/2} \left\{ 1 + \frac{2+c}{3+c} \cdot \right. \\ &\cdot c(1+2\alpha) \frac{\Gamma(1+c/2)}{\Gamma(\frac{1}{2}+c/2)} (4\bar{E}_0 Y)^{-1/2} + \frac{c}{2} D_1 (4\bar{E}_0 Y)^{-1} - \frac{1}{2} (1-c) \cdot \\ &\cdot \left. \frac{\Gamma(1+c/2)}{\Gamma(4/2+c/2)} D_2 (4\bar{E}_0 Y)^{-3/2} - \frac{c}{2} (1-\frac{c}{2}) D_3 (4\bar{E}_0 Y)^{-2} \right\} \quad (81) \end{aligned}$$

where

$$\bar{E}_0 = \frac{r_0^2}{16\pi\alpha_0' c_0} (1-\alpha) \quad (82)$$

The values D_i are rather bulky combinations of the values C_i , β_i and α . Therefore, we do not bring them here (see [25]).

The expression (81) is very much like the one-loop cross section [24], only now the rate of the growth of total cross section has increased: if in the one-loop case the cross section has been growing as $Y^{0.116}$, now it grows as $Y^{0.277}$, which is in good agreement with Cardy's results [20,21].

The analytic structure of the propagator (77) singularities in complex E -plane at $\xi_0 = k^2 = \delta_0 = 0$ is quite

simple: there is only one cut beginning at the point $E = 0$ and directed along the positive part of the real axis.

Consider now various particular cases of propagator (77).

At $h = \xi_0 = 0$ and $p \neq 0$ we have

$$E_0(p) = \frac{\bar{E}_0}{1+p/2}, \quad p = -\frac{4\delta_0}{E} \left(-\frac{E}{E_0(p)}\right)^{c_5} \left[1 + \left(-\frac{E}{E_0(p)}\right)\right]^{-c_5} \quad (83)$$

In the first approximation over $E/E_0 \ll 1$ the propagator (77) has a pole at $P_{\text{pole}} = -4/(1-c_5)$. At small E and δ_0 we may assume that $p \approx p_0 = -4\delta_0/E$, since $c_5 \ll 1$. Then the position of the pole in E -plane will be

$$E_{\text{pole}} \approx \delta_0(1-c_5) \quad (84)$$

besides, the propagator will have a branch point at

$$E_{\text{cut}} \approx 2\delta_0 \quad (85)$$

and the structure of singularities in E -plane will have the form shown in fig. 2. Hence, when the pomeron intercept is smaller than a unit, the main singularity is the simple pole to the left of unit in j -plane [11,12,23-25]. The total cross section will have the form

$$\sigma_{\text{tot}}(y, \delta_0) \propto e^{-\delta_0 y(1-c_5/2)} f(c_i) \quad (86)$$

That is, with the increase of energy the total cross section exponentially decreases at $\delta_0 > 0$.

At $\xi_0 \neq 0$ and $p = h = 0$ the total cross section at asymptotics has the form

$$\sigma_{\text{tot}}(y, \xi_0) \propto (E_0 y)^{c/2} \left\{ 1 + \frac{2+c}{3+c} c(1+2\alpha) \frac{\Gamma(1+c/2)}{\Gamma(1/2+c/2)} \cdot (4\bar{E}_0 y)^{-1/2} \left[1 + (2\xi_0 \bar{E}_0)^{1/2} \right] \right\} \quad (87)$$

Thus, like the one-loop case [24], the introduction of the pomeron production threshold affects only the rate of approaching the scale limit, but doesn't alter the limit itself.

In fig. 3 the relation $\sigma_{tot}(y)/\sigma_{tot}(0)$ is given for the asymptotic solution (81) and for the perturbation theory series taken up to terms of the order γ_0^6 . Besides, for comparison, the one-loop result from [24] is given. The one-loop solution begins to coincide with that of perturbation theory already at $y \approx 5$. The two-loop solution is exactly equal to that of perturbation theory at the point $y = \bar{E}_0^{-1} \approx 17.6$, which corresponds to the energy $S = 4.8 \cdot 10^7 \text{ GeV}^2$, nevertheless, already at fairly small energies these two curves come very close to each other. We may apparently consider that at the energies accessible today we are at the very beginning of transition region, and a 2 order energy increase should already lead to asymptotic behaviour of cross sections corresponding to eq.(81). At numerical calculations the following values were taken for the constants

$$\gamma_0^2 = 0.36; \quad \alpha'_0 = 0.5; \quad \bar{E}_0 = 0.123 \quad (88)$$

The most optimistic value was taken for the constant γ_0 from ref. [31-33]. The answer proved to be not very sensitive to the value γ_0^2 .

Let's now obtain the differential cross section in the asymptotic limit. We have

$$h = -\frac{\alpha'_0 k^2}{E} X^{-c_2} e^{c_2 \alpha(1-X)}; \quad E_0(h) = \frac{\bar{E}_0}{1+Bh} \quad (89)$$

where $B = 0.31$. The further plan is stated in detail in

refs. [24,25]. Using the iteration method we solve eq.(89) with respect to $-E/\bar{E}_0$. Replacing in the integral (80) the variable, we make a transition to the h -plane instead of integrating over E . It turned out that at $E=0$ there is already no singularity, since $h \sim E^{-1}$ at $E \rightarrow 0$. Besides, the singularity at $h = -B^{-1}$ is mapped into the point $E = \infty$. The main singularities at the E -plane at $\kappa^2 > 0$ will be a moving (as a function of κ^2) pair of complex-conjugate branch points. In fig. 4 the complex E -plane is shown at $h \neq 0$. Its mapping into h -plane is shown in fig. 5. Finally we obtain in the first approximation by E/\bar{E}_0 that

$$F(y, \kappa^2) = \frac{1+c/2}{\Gamma(1+c/2)} (\bar{E}_0 y)^{c/2} F_1(x) \quad (90)$$

where $F_1(0) = 1$ and

$$F_1(x) = -\frac{h_0 \Gamma(2+c)}{2+c_2} \left\{ \frac{1}{\Gamma} P \int_{-B^{-1}}^0 dh \frac{\text{Im} f_0(h, x)}{h-h_0} - \text{Re} f_0(h_0+i\epsilon, x) \right\} \quad (91)$$

and

$$f_0(h, x) = \frac{1}{h} (\xi \varphi_1(h))^{-c/2} (1+Bh)^{-c/2} \varphi_3(h) e^{\xi \varphi_1(h)} \quad (92)$$

$$\varphi_1(h) = h^{-\frac{2}{2+c_2}} (1+Bh)^{-\frac{c_2}{2+c_2}}; \quad \varphi_2(h) = \frac{c_2(1-\alpha)}{2+c_2} \varphi_1^{3/2}(h) (1+Bh)^{1/2} \quad (93)$$

$$P = \left(\frac{\alpha_0' \kappa^2}{\bar{E}_0} 2^{c_2} e^{\alpha c_2} \right)^{\frac{2}{2+c_2}}; \quad h_0 = -\frac{1}{1+c_2/2} \quad (94)$$

where P denotes the integral as a basic value, and h_0 denotes the position of the propagator pole in h -plane. Be-

sides, following [11,12,22,24] we have introduced the following notation

$$x = (\rho \bar{E}_0 y)^{\frac{2+c_2}{2}}; \quad \xi = x^{\frac{2}{2+c_2}}; \quad \varphi_3 = 1 + \frac{c_2}{2} \frac{Bh}{1+Bh} \quad (95)$$

The computer-calculated function $F_1^2(x)$ is shown in fig.6. The second term in the asymptotic expansion $F(y, \kappa^2)$ may be obtained in the same way [22,24] but its calculation doesn't make much sense, since RPT gives information only on the imaginary part of the amplitude. The quantity of the real part contribution in the differential cross section may reach 10% and even be more than the second term.

The contribution of the pomeron propagator in the differential cross section of the process $2 \rightarrow 2$ has the form

$$d\sigma/dt = (16\pi)^{-1} [\beta_1(t) \beta_2(t)]^2 F^2(y, -t) \quad (96)$$

where $\beta_i(t)$ is the residue of the pomeron. If we take constants instead of β_i , and instead of \bar{E}_0 , C_i and α'_0 substitute the obtained numbers, it will turn out that the calculated differential cross section is in excellent agreement with experimental data at maximum attainable energies $\sqrt{s} = 53$ GeV (fig. 7). In spite of such an agreement, we mustn't attach a special importance to it, since we have here neglected many effects which should have been taken into account in successive phenomenological analysis [33,34]. Besides the real part of the amplitude, we have not taken into account the contributions of nonamplified [35] and semiamplified graphs [36], of secondary poles and possible four- (and more) pomeron vertices, which must make at such energies a substantial contri-

bution. However, in spite of all the above stipulations, the fact that the theoretical differential cross section has a dip in the required place $-t \approx 1.25$ and manages to drop by six orders to $-t \approx 1$ indicates that the asymptotic limit may come not at $y \approx \bar{E}_0^{-1}$ but much earlier. This is supported by the behaviour of total cross sections presented in fig. 3.

6. Conclusion

In the present work as well as in refs. [24,26] a general method of constructing an explicit representation is developed for the pomeron propagator, when there are complementary dimensionless parameters, such as the pomeron production threshold ξ_0 or the intercept shift δ_0 . Besides, the contribution of the pomeron propagator in the angular distribution of elastic cross section is calculated. The developed method is shown to be applicable in both one-loop and two-loop cases.

The problems connected with K^2 and the subcritical pomeron (the renormalized intercept $\alpha_{\mathbb{P}}(0) < 1$) essentially differ from the problem of introducing a threshold. The threshold leads to the dimensionless parameter of the type $\xi_N = -\xi_0 E$ which may be described as "unessential", since it doesn't alter the asymptotic limit (eq.(87)). When $E \rightarrow 0$, at the asymptotics $\xi_N \rightarrow 0$ and all quantities adopt the values they have had when there was no threshold. Thus the zero of β -function is stable with respect to the threshold, which is an example of Wilson universality [37]. The large values of the parameter ξ_N ($\xi_N > 0.1$) physically do not make any sense, since the experimental data show that $\xi_0 \approx 2$ and large ξ_N correspond..

to large E which are far from asymptotic values that we are interested in.

The dependence on κ^2 and the subcritical pomeron are an example of "nontrivial" parameters. For example, the dimensionless parameter $\rho = -4\delta/E$ is nontrivial, since the limit $E \rightarrow 0$ doesn't bring the critical exponents to the values they have had at $\delta = 0$, since the fixed point is unstable when the bare intercept changes. This should have been expected also for physical considerations: at $\delta_0 \neq 0$ a simple renormalized pomeron pole is the main singularity at asymptotic energies. At the same time there isn't any storage of singularities and there is no critical phenomenon, therefore, the asymptotic solution cannot be characterized by the approach to the fixed point.

In [11,12,22,24] a method of treating the nontrivial parameters was developed. It consisted in the construction of an integral representation for the propagator with a corresponding set of fixed dimensionless parameters (h and ρ were fixed, κ^2 and δ weren't). In the case of subcritical pomeron this has led to exponentially decreasing total cross sections (eq. (86)). When $\kappa^2 \neq 0$ this method has allowed to calculate the differential cross sections (eqs.(90)-(96)). The obtained differential cross sections agree with the results of [11,12,22], though they have been calculated in the one-loop approximation, as well as in ϵ -expansion (in [22]).

Consider in more detail the question of the asymptotics of total cross sections. As shown in [33], the correct consideration of energy dependence of jet-enhancement coefficients results in the fact that in the energy interval from

500 to $5 \cdot 10^4$ GeV the total cross sections increase as $\ln^2 s$, and this increase is due to the "dying out" of contributions of quasicikonal nonamplified branchings [35], without consideration of any amplified graphs. In this case the rate of the increase of total cross sections is slowed down, and at energies higher than 10^5 GeV the cross section increases as

$$\sigma_{tot} \propto A - B/\ln s \quad (97)$$

If we don't take into account the diagram amplification, such a behaviour will continue ad infinitum. Consider now what will happen if we include the amplified graphs. The first diagram of the type (18) occurs at energies higher than 10^3 GeV, the graphs of the type (40) start functioning at energies higher than $2 \cdot 10^4$ GeV, and at the further increase of energy diagrams of still higher orders are engaged. All this happens on the background of ordinary nonamplified contributions which give first cross sections increasing as $\ln^2 s$, and then - by eq.(97). According to fig. 3, the asymptotic behaviour of the contributions of amplified diagrams must begin with energies of the order 10^5 GeV, but at these energies the contribution of nonamplified graphs is still fairly large. Therefore, it will be very difficult to answer the question: due to what mechanism the increase of cross sections takes place? Is it due to nonamplified branchings or due to the asymptotic shape of the pomeron propagator? To answer this question we shall need measurements of total cross sections with 1% errors at quite fantastic energies of the order several million GeV. All that has been said at the end of the previous section about the beginning of

the transition region, has concerned only the contribution of amplified diagrams in the total pomeron propagator. For concrete predictions one should take into consideration all contributions of nonamplified and semi-amplified diagrams. The method of their consideration is developed in much detail in refs. [27,28,31-36].

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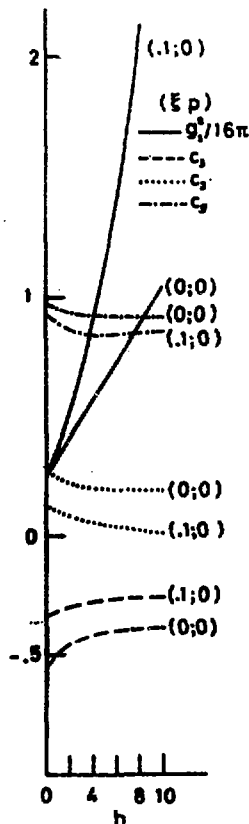


Fig. 1 Critical exponents C_3 , C_2 and C_1 , and $g_1^2/(16\pi)$ versus h , ρ and ξ_N in the two-loop approximation.

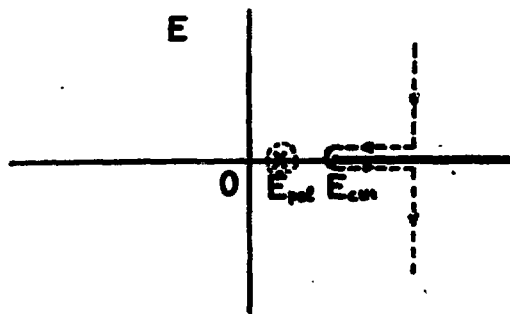


Fig. 2 Structure of singularities in E -plane at $\delta_0 \neq 0$ and $K^2 = \frac{2\gamma}{\gamma_0} = 0$

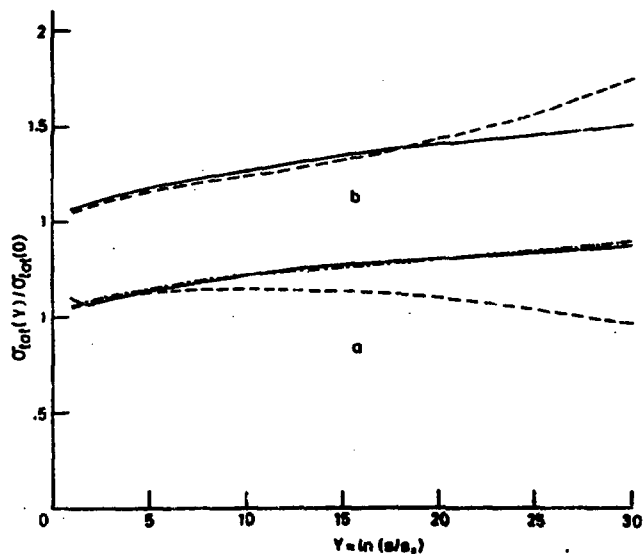


Fig. 3 Comparison of total cross sections obtained from perturbation theory and from the scale solution (81).
 a) In the one-loop approximation: solid line - asymptotic solution, dashed line - perturbation theory up to terms $\sim \gamma_0^2$, dashed-dot line - perturbation theory up to terms γ_0^4 . b) In the two-loop approximation: solid line - asymptotic solution, dashed line perturbation theory up to terms $\sim \gamma_0^6$.

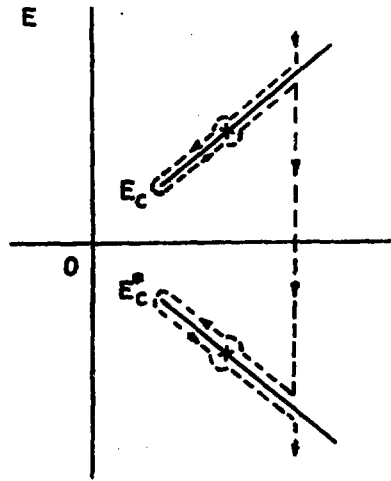


Fig. 4 Singularities at asymptotic pomeron propagator in complex E -plane at $k^2 > 0$. The cross shows the position of the pole corresponding to h_0 . The dashed line shows the integration outline.

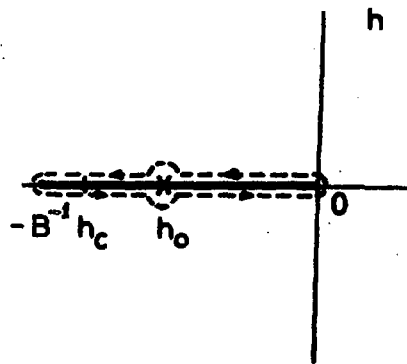


Fig. 5 Singularities of the pomeron asymptotic propagator in complex h -plane. The cross indicates the position of the pole h_0 , and the point h_c is the image of branch points in E -plane.

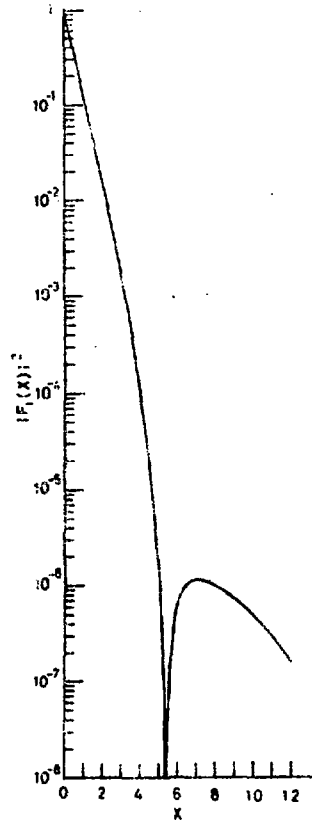


Fig. 6 Square of asymptotic scale function $F_1^2(x)$ from eq. (91).

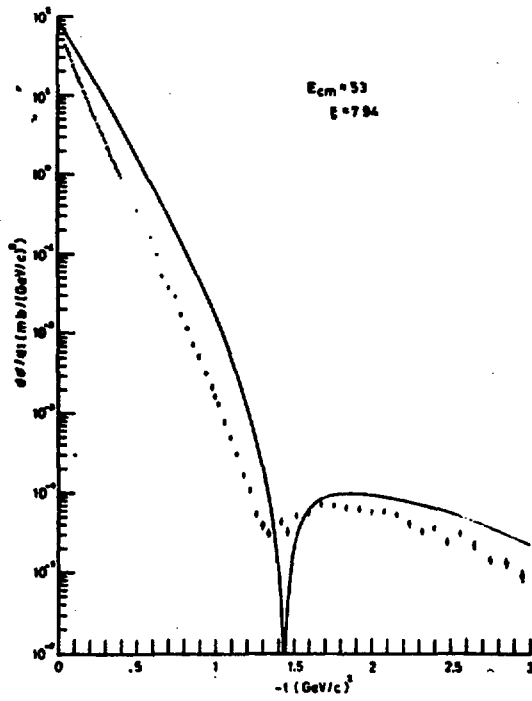


Fig. 7 Comparison between the ISR data at $E_{cm} = 53$ GeV and the asymptotic solution of eq.(96).

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