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NORMAL GAUGE IN SUPERGRAVITY

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NORMAL GAUGE IN SUPERGRAVITY

A method of the construction of the superfield normal gauge in the vicinity of fixed point is given. In order to construct gauge-invariant theoretical quantities the question on the normal gauge application to the supergravity theory is discussed.

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НОРМАЛЬНАЯ КАЛИБРОВКА В СУПЕРГРАВИТАЦИИ

В работе указан метод построения нормальной калибровки суперполя в окрестности фиксированной точки. Обсуждается вопрос о применении нормальной калибровки для построения калибровочно-инвариантных величин теории.

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NORMAL GAUGE IN SUPERGRAVITY

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1. In Refs.[1, 2] Ogievetsky and Sokatchev suggest a scheme of supergravity on the basis of the pure geometrical consideration. In this scheme a complex superspace $^{*}C^{(4,2)}$ having complex dimension (4,2) is considered. Coordinates in this superspace are $z^A = (x^\mu, \theta^\alpha)$, where $x^\mu (\mu=1, 2, 3, 4)$ are complex even coordinates and $\theta^\alpha (\alpha = 1, 2)$ are odd ones. They consider in $C^{(4,2)}$ a real surface Ω which has real dimension (4,4). It is natural to correspond with the surface Ω a real four-dimensional superfield which is defined on the real (4,4) dimensional superspace with coordinates $\xi^1, \xi^2, \xi^3, \xi^4, \eta^1, \eta^2, \bar{\eta}^1, \bar{\eta}^2$; here $\xi^1, \xi^2, \xi^3, \xi^4$ are real even coordinates, η^1, η^2 are odd complex ones, $\bar{\eta}^1, \bar{\eta}^2$ are complex-conjugated to η^1, η^2, respectively. This is due to the fact that the arbitrary real superfield $H^\mu = H^\mu(\xi, \eta, \bar{\eta})$ ($\mu = 1, 2, 3, 4$, H^μ takes real even values) defines in the $C^{(4,2)}$ a supersurface $\Omega = \Omega(H)$ if we take $x = \xi + iH$, $\theta = \eta$. We shall adopt that supersurface $\Omega(H)$ in the $C^{4,2}$ and real superfield $H^\mu(\xi, \eta, \bar{\eta})$ correspond to each other.

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* For the definition of superspace as well as other superobjects (i.e. supersurfaces, berezinian, etc.) we shall deal with in this paper see Refs.[3, 4,].

Let us consider the group $\mathcal{G}$ of analytical transformations of the superspace $C^{(4,2)}$ onto itself.

$$z' = z^{g} = z'(z) \qquad (g \in \mathcal{G}) \tag{1}$$

which satisfy the following condition

$$\mathrm{Ber}\,\frac{\partial z'}{\partial z} = 1 \tag{2}$$

i.e. the jacobian of the transformation (1) is equal to unity (the preservation of a supervolume).

The transformation (1) of the superspace C transforms a supersurface Ω in some supersurface $\Omega' = \Omega^{g}$. The superfield H corresponding to surface Ω will turn into $H' = H^{g}$ (where H^{g} is the superfield corresponding to the superspace Ω^{g}). The main physical object of the theory is superfield H. The transformations $H \to H^{g}$, $\Omega \to \Omega^{g}$ ($g \in \mathcal{G}$, $\Omega = \Omega(H)$) are the gauge transformations of the H and $\Omega(H)$, respectively. The field H^{g} and surface Ω^{g} are gauge-equivalent to field H and surface Ω, respectively.

In Refs.[2] and [5] in terms of the field H the theory of the supergravity is considered. In particular, the gauge-invariant action and the corresponding motion equations are constructed. Moreover, the equivalence of this theory to the usual theory of supergravity is shown *.

We can always turn to zero a series of derivatives of the superfield in a fixed point by suitable transformation of the field H, i.e. we can construct the normal gauge of the superfield H in the vicinity of a fixe point. The normal gauge simplifies significantly the study of the theory.

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* We note that in [7,8] the Ogievetsky-Sokatchev approach acquires a more clear geometrical interpretation. Our paper is based upon the referenced papers being their logical continuation.

The H superfield power series members in the vicinity of this point that do not vanish in normal gauge are the "bricks" the gauge-invariant objects (lagrangian, etc.) can be built of. In [6] normal gauge was constructed in the vicinity of a fixed point in the first four orders of the H superfield Taylor series expansion. It means that the action of the gauge transformations group $\mathcal{G}$ on the derivatives of the superfield H was analyzed up to the fourth order.

We shall construct the normal gauge of the superfield H in the vicinity of a fixed point in the all orders of the H superfield Taylor series expansion. It means that the action of the gauge transformations group $\mathcal{G}$ on all derivatives of the field H will be analyzed. The gauge-invariant quantities that depend on the higher derivatives of the field H (for example, counterterms) can be built by means of this gauge.

Moreover, it turns out that the use of the normal gauge is helpful for solving the problems of the conformal supergravity. In papers to come we observe in detail the structure of the counterterms in supergravity and the question on the conformal supergravity lagrangian uniqueness using normal gauge.

2. It is more convenient for our purposes to formulate the scheme of the supergravity theory in the other way. (Further we shall call the above scheme the H- scheme of the supergravity). We extend the gauge group $\mathcal{G}$ defined by (1) and (2) up to the $\mathcal{A}$ group of the all analytical transformations of the superspace C onto itself. It means that we ignore the condition (2) in the $\mathcal{G}$ group definition. We adopt the group $\mathcal{A}$ to be a gauge group of a reformulated scheme of the supergravity. The extension of gauge group from $\mathcal{G}$ up to $\mathcal{A}$ we compensate by introducing the additional dynamical object, i.e. function $V(z)$ defined on the $C^{4,2}$ and taking

complex even values. The H superfield and new object $V(z)$ (below we call $V(z)$ volume form) make the pair (V,H) which is the dynamical object of our supergravity scheme. The transformation of the field V under the gauge transformation

$$z' = z^{g} = z'(z), \quad g \in \mathcal{A} \tag{3}$$

we define by equation

$$V^{g}(z) = V^{g}(z') = V(z(z'))\,\mathrm{Ber}\,\frac{\partial z}{\partial z'} \tag{4}$$

It is obvious that if $V(z) \equiv 1$ and $g \in \mathcal{G} < \mathcal{A}$, then $V^{g} \equiv 1$. Just like in the H- scheme under the transformation (3) supersurface Ω transforms into the gauge equivalent supersurface Ω^{g} and respectively H into the H^{g}. Thus we defined the gauge transformations of the pair (V,H) under the action of the gauge group $\mathcal{A}$. The pair $(V^{g}, H^{g}), g \in \mathcal{A}$ is gauge-equivalent to pair (V, H). In what follows we shall call our scheme of the supergravity (V, H)- scheme. If in the H- scheme H field was the main dynamical object all the physical quantities were expressed through, in the (V, H) scheme the part of the dynamical object plays pair (V,H). The physical objects of the theory are expressed through the pair (V, H) and are gauge-invariant with respect to the gauge group $\mathcal{A}$ transformations.

We shall demonstrate that the (V, H) scheme of the supergravity is equivalent to the usual H- scheme. Indeed, if we restrict ourselves in the (V, H) scheme to such pairs (V, H) so that $V \equiv 1$, then the $\mathcal{A}$ group action on these pairs reduce to the $\mathcal{G}$ group action on the field H. This is on the one hand. On the other hand, for any volume form $V(z)$ there exists such a gauge transformation $g \in \mathcal{A}$ that

$$V^{g} \equiv 1 \tag{5}$$

Therefore these schemes are equivalent.

The quantity of the action on the pair (V, H) must be equal to the quantity of the Ogievetsky-Sokatchev action (the action in the H - scheme) on the field H^{g}, where g is given by condition (5).

3. Let us now investigate $\mathcal{A}$ gauge group action on the pairs (V, H). We analyze the action of the gauge transformations on the values of the fields V and H and all their derivatives in a fixed point.

Note that translation $z' = z - z_0$ is the gauge transformation. So each field H observed in the vicinity of a fixed point may be transformed to such a field H' that $H'(\xi, \eta, \bar\eta)\big|_{\xi=\eta=\bar\eta=0} = 0$. Hence for our purposes it is enough to investigate such pairs (V, H) in the vicinity of the zero that $H(0,0,0)=0$; (it means that surface $\Omega(H)$ goes through a zero point). Accordingly we exclude from group $\mathcal{A}$ all the translations, i.e. we consider such a subgroup $\mathcal{A}_0$ that the transformations belonging to this subgroup do not shift the zero point.

Bearing in mind the restrictions on the pairs (V, H) and gauge transformations defined above we expand them in a Taylor power series.

$$z^{g} = z'(z) = \sum P_{A_2 \ldots A_n} z^{A_2} \ldots z^{A_n}, \quad g \in \mathcal{A}_0 \tag{6}$$

here the power series begins from the first order members in z.

$$V(z) = V_0 + V_\alpha \theta^\alpha + V_\mu x^\mu + \ldots \tag{7a}$$

$$H^\mu(\xi, \theta, \bar\theta) = a^\mu(\xi) + [b^\mu_\alpha(\xi)\theta^\alpha + c.c.] + \Gamma^\mu_{\alpha\dot\beta}(\xi)\theta^\alpha\bar\theta^{\dot\beta} + \tag{7b}$$
$$+ [D^\mu_\alpha(\xi)\theta^\alpha\bar\theta\bar\theta + c.c.] + [K^\mu(\xi).\theta\theta + c.c.] + A^\mu(\xi)\theta\theta.\bar\theta\bar\theta.$$

(Here and what follows we take $x = \xi + iH$, $\theta = \eta$)

Here $a(\xi)$, $b(\xi)$, $\Gamma(\xi)$, $D(\xi)$, $K(\xi)$ and $A_\mu(\xi)$ are the formal powers in ξ, $\alpha(0)=0$, c.c. is complex conjugation, $\theta\theta=\theta^\alpha\theta_\alpha$, where $\theta_\alpha=\epsilon_{\alpha\beta}\theta^\beta$ ($\epsilon_{\alpha\beta}=-\epsilon_{\beta\alpha}$; $\epsilon_{12}=1$, $\epsilon^{12}=-1$, all other are zeros). We use spinor indices $(\alpha\dot\beta)$ on the equal footing, just as vector indices μ. So $H^{\alpha\dot\beta}$ corresponds to H^μ; $H^\mu=\sigma^\mu_{\alpha\dot\beta}H^{\alpha\dot\beta}$, σ^μ are Pauli vertices. For example, if we rewrite (7b) in spinor indices, we get

$$H^{\alpha\dot\beta}(\xi)=a^{\alpha\dot\beta}(\xi)+[b^{\alpha\dot\beta}_{\pi}(\xi)\theta^{\pi}+h.c.]+\dots$$

where h.c. is the operation of the hermitian conjugation.

The objects we shall deal with below are the formal power series. We shall analyze the action of the power series (6) corresponding to $\mathcal{A}_0$ gauge group transformations on the power series (7a), (7b) corresponding to the pair (V,H). We shall not deal with the problems concerning the convergence of these formal power series. Our considerations are purely algebraic.

Let us denote by $\mathcal{H}$ the space of the formal power series (7a), (7b). It is natural to single out in $\mathcal{A}_0$ subsets in the following way.

1) The Lorentz subgroup $\mathcal{L}$ that is defined as

$\ell\in\mathcal{L}$ (elements of $\mathcal{L}$ we denote by ℓ) if

$$\mathfrak{Z}^{\ell}(\mathcal{Z})=\begin{cases}x'^\mu=t^\mu_\nu\,x^\nu\\ \theta'=g^\alpha_\beta\,\theta^\beta\end{cases}$$

here

$$\mathrm{Det}\,g^\alpha_\beta=1,\quad t^\mu_\nu=g^\alpha_\beta\,\bar g^{\dot\pi}_{\dot\rho}\,\sigma^\mu_{\alpha\dot\pi}\,\sigma^{\beta\dot\rho}_\nu$$

g^α_β are even Grassman numbers *.

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* Strictly speaking $\mathcal{L}$ subgroup is not isomorphic to the Lorentz group, but as far as the difference between them is inessential for us we shall treat it as a Lorentz one.

2) The dilatation subgroup $\mathcal{D}$ that is defined as $d \in \mathcal{D}$ (elements of $\mathcal{D}$ we denote by d) if

$$z^{d}(z) = \begin{cases} x'^{\mu} = |\varepsilon|^{2} x^{\mu} \\ \theta'^{\alpha} = \varepsilon \theta^{\alpha} \end{cases} \tag{8}$$

here ε are even Grassman numbers.

3) The subset $\mathcal{K}$ so that

$$\kappa \in \mathcal{K} \text{ if } z^{\kappa} = z^{\kappa}(z) = \begin{cases} x' = x'(x, \theta) \\ \theta' = \theta'(x, \theta) \end{cases} \tag{9}$$

$$\text{is such that } \left.\frac{\partial \theta'(x,\theta)}{\partial \theta}\right|_{x=\theta=0} = 1$$

It is easy to obtain that every element $g \in \mathcal{A}_{0}$ is unequally expressed as

$$g = d \ell \kappa$$

where $d \in \mathcal{D}$ $\ell \in \mathcal{L}$, $\kappa \in \mathcal{K}$ $(\mathcal{D} < \mathcal{A}_{0}, \mathcal{L} < \mathcal{A}_{0}, \mathcal{K} \subset \mathcal{A}_{0})$.

Note that subset $\mathcal{K}$ is not subgroup in $\mathcal{A}_{0}$

It is easy to check the action of the groups $\mathcal{L}$ and $\mathcal{D}$ on the space $\mathcal{H}$ and on the subset $\mathcal{K} \subset \mathcal{A}_{0}$. Really, the coefficients of the formal power series (7a), (7b) behave as the Lorentz group tensors under the action of the gauge transformations belonging to group $\mathcal{L}$.

For example, if

$$H^{\alpha\dot{\beta}} = \theta^{\alpha} \bar{\theta}^{\dot{\beta}} + [S^{\alpha\dot{\beta}}_{\mu\rho} \xi^{\mu} \theta^{\rho} + h.c.]$$

then $H^{\alpha\dot{\beta}}$ behaves as spin-tensor of the (1.1) valence , $S^{\alpha\dot{\beta}}_{\mu\rho}$ is vector-like over the index μ , spin-like over the index α and so on. It should be also noted that the coefficients of the formal power series (9) corresponding to the gauge transformations from the subset $\mathcal{K}$ behave as the tensors under the transformation $\kappa \to \ell \kappa \ell^{-1}$ $(\ell \in \mathcal{L}, \kappa \in \mathcal{K})$

Now let us check the $\mathcal{D}$ group action on the space $\mathcal{H}$ and subset $\mathcal{K}$. For example, under the dilatation transformation (8) monomial $F^{\mu}_{\nu\alpha\dot{\beta}}\xi^{\nu}\theta^{\alpha}\bar{\theta}^{\dot{\beta}}$, belonging to $\mathcal{H}$ transforms into the $\frac{1}{|\varepsilon|^4}F^{\mu}_{\nu\alpha\dot{\beta}}\xi^{\nu}\theta^{\alpha}\bar{\theta}^{\dot{\beta}}$. We adopt that the weight of this monomial is equal to 4. In the following we adopt that the element $F(\xi,\theta,\bar{\theta})$ belonging to $\mathcal{H}$ has the weight p if

$$F^{d} = \frac{1}{|\varepsilon|^{p}} F$$

where d is the dilatation transformation defined by (8). Analogously we consider the action of the dilatation group $\mathcal{D}$ on the group $\mathcal{A}_0$ that includes $\mathcal{K}$ subset.

If
$$z^{g} = \sum P_{B_1\dots B_n} z^{B_1}\dots z^{B_n}$$

then
$$z^{gd} = \sum P'_{B_1\dots B_n} z^{B_1}\dots z^{B_n}$$

where d is defined by (8), too. It is obvious that modules of every coefficients $P'_{B_1\dots B_n}$ differ from the ones of the coefficients P by the multiplier $|\varepsilon|$ in some power p. We shall take p as the weight of the corresponding monomial.

The question we deal with is the investigation of the $\mathcal{A}_0$ group action on the space $\mathcal{H}$. Now the problem of the $\mathcal{K}$ set action on the space $\mathcal{H}$ is to be solved.

We formulate our main statement as a theorem. We shall impose on the fields from $\mathcal{H}$ (i.e. on the values of these fields and their all derivatives in the zero point) the gauge conditions (so that they would be invariant with respect to Lorentz group $\mathcal{L}$) extracting from $\mathcal{H}$ some subspace $\mathcal{H}_0$. We define subspace $\mathcal{H}_0$ as a set of the pairs (V, H) such that the following restrictions on the pair (V, H) add to the restrictions (7a), (7b):

$$V_0 = 1, \qquad V_\alpha = 0, \qquad \mathrm{Re}\, V_\mu = 0 \tag{10}$$

$$a(\xi) \equiv b(\xi) \equiv K(\xi) \equiv 0 \tag{11}$$

$$\Gamma^{\mu}_{\mu}(\xi) \equiv 0, \ \underset{(\nu,\, p_1, \dots p_k)}{\mathrm{Symm}}\ \Gamma^{\mu}_{\nu\, p_1 \dots p_k} = 0, \ \Gamma^{\mu}_{\nu}(\xi) \equiv \Gamma^{\nu}_{\mu}(\xi) \tag{12}$$

where $\Gamma^{\mu}_{\alpha\dot\beta}(\xi) = \sum \Gamma^{\mu}_{\alpha\dot\beta\, p_1 \dots p_k}\, \xi^{p_1} \dots \xi^{p_k}$

$$\mathcal{D}^{\mu}_{\alpha}(0) = 0, \quad \underset{(\mu,\, p_1, \dots, p_k)}{\mathrm{Symm}}\ \mathcal{D}^{\pi\dot\rho}_{\alpha\, p_1 \dots p_k}\, \sigma^{\alpha}_{\dot\rho\mu} = 0, \ \mathcal{D}^{\alpha\dot\rho}_{\alpha}(\xi) \equiv 0 \tag{13}$$

where $\mathcal{D}^{\pi\dot\rho}_{\alpha}(\xi) = \mathcal{D}^{\mu}_{\alpha}(\xi)\,\sigma^{\pi\dot\rho}_{\mu} = \sum \mathcal{D}^{\pi\dot\rho}_{\alpha\, p_1 \dots p_k}\, \xi^{p_1} \dots \xi^{p_k}$

$$A^{\mu}(0) = 0, \qquad \underset{(\mu,\, p_1, \dots, p_k)}{\mathrm{Symm}}\ A^{\mu}_{p_1 \dots p_k} = 0; \tag{14}$$

where

$$A^{\mu}(\xi) = \sum A^{\mu}_{p_1 \dots p_k}\, \xi^{p_1} \dots \xi^{p_k}$$

Here Symm means symmetrization over the corresponding indices.

It can be noted that the conditions (12), (13), (14) can be rewritten in a more elegant and usual for physicists form.

$$\Gamma^{\mu}_{\mu}(\xi)\equiv 0,\ \Gamma^{\mu}_{\nu}(\xi)\,\xi^{\nu}\equiv 0,\ \Gamma^{\mu}_{\nu}(\xi)\equiv\Gamma^{\nu}_{\mu}(\xi) \quad (12a)$$

$$\mathcal{D}^{\alpha\dot\rho}_{\alpha}(\xi)\,\sigma^{\alpha}_{\dot\rho\nu}\,\xi^{\nu}\equiv 0,\ \mathcal{D}^{\alpha\dot\rho}_{\alpha}(\xi)\equiv 0 \quad (13a)$$

$$A_{\mu}(\xi)\,\xi^{\mu}=0 \quad (14a)$$

The conditions (10)-(14) we denote as the conditions of the normal gauge. If the conditions of the normal gauge hold for a pair (V, H), we call this pair a normal pair. It is easy to note that the normal gauge conditions are Lorentz-invariant (i.e. they are invariant with respect to the group $\mathcal{L}$).

Theorem. Any pair (V, H) can be reduced to the normal form by means of gauge transformation which is determined unequally to a precision up to the Lorentz group transformation.

More formally this statement can be formulated as follows.

Let pair (V, H) be an arbitrary element from $\mathcal{H}$. Then , there exists such an element $g\in\mathcal{A}_{o}$ that $(V^{g}, H^{g})\in\mathcal{H}_{o}$. If $(V^{g_1}, H^{g_1})\in\mathcal{H}_{o}$ and $(V^{g_2}, H^{g_2})\in\mathcal{H}_{o}$, then $g_1 = \ell g_2$, where $\ell\in\mathcal{L}$.

Note that the proof of the Theorem is constructive.

Before this Theorem being proved let us discuss its consequences. The theorem actually states that the study of the $\mathcal{A}$ group action on $\mathcal{H}$ reduces to the study of the $\mathcal{L}$ group action on $\mathcal{H}_{o}$. The gauge-invariant functionals defined on $\mathcal{H}$ are in the one-one correspondence with the ones defined on $\mathcal{H}_{o}$, being invariant with respect to $\mathcal{L}$ group.

Now it is obvious that for the gauge-invariant physical quantity construction the Lorentz-invariant functional $F(V, H)$ defined on $\mathcal{H}_{o}$ must be fixed $(F(V^{\ell}, H^{\ell})=F(V, H),\ \ell\in\mathcal{L})$. Then by the suitable gauge transformation any pair (V, H) we reduce to the normal form $(V^{g}, H^{g})\in\mathcal{H}_{o}$ and consider the functional

$$\mathcal{J}(V,H) = F(V^g, H^g)$$

Functional $\mathcal{J}(V,H)$ is gauge-invariant and unequally determined.

Analogously, studying the gauge-invariant motion equations reduces to studying the Lorentz-invariant equations if we turn to normal gauge.

Let us prove the Theorem. Note that our proof stands close to the one of the Theorem 2.2 in Ref.[9]. In that paper (2n-1)-dimensional real surfaces lying in the complex space C^n (with complex dimension n) are considered. The action of the space C^n analytic transformations group on these surfaces was studied.

To prove our Theorem it is convenient to extract in the space $\mathcal{H}$ subspace $\mathcal{H}_1$ ($\mathcal{H}_0 < \mathcal{H}_1 < \mathcal{H}$) such that a pair (V,H) from $\mathcal{H}$ belongs to $\mathcal{H}_1$ if

$$V_0 = 1, \quad H^\mu(\xi,\theta,\bar\theta) = \sigma^\mu_{\alpha\dot\beta}\theta^\alpha\bar\theta^{\dot\beta} + \text{terms with weight} \geqslant 3$$

Respectively in subset $\mathcal{K}$ that was defined by (9)) we extract subgroup R in $\mathcal{A}_0$ defined by the condition

$$\tau \in R \quad \text{if} \quad z^\tau(z) = \begin{cases} x' = x + \text{terms with weight} \geqslant 3 \\ \theta' = \theta + \text{terms with weight} \geqslant 2. \end{cases}$$

Then the statement of the Theorem reduces to the following Lemma.

<u>Lemma</u>. For an arbitrary pair $(V,H) \in \mathcal{H}_1$ there exists one and only one element $\tau \in R$ such that

$$(V^\tau, H^\tau) \in \mathcal{H}_0$$

First, we demonstrate that the Theorem follows from this Lemma, then the Lemma will be proved.

Let (V, H) be an arbitrary pair in $\mathcal{H}$. It can be shown that there exists such a transformation $p_0 \in \mathcal{A}_0$ that

$$Z^{p_0} = \begin{cases} x'^{\mu} = A^{\mu}_{\nu} x^{\nu} + B^{\mu}_{\alpha} \theta^{\alpha} + L^{\mu} \theta\theta \\ \theta'^{\alpha} = \varepsilon \theta^{\alpha} \end{cases} \tag{15}$$

and $$(V^{p_0}, H^{p_0}) \in \mathcal{H}_0 \tag{16}$$

From the Lemma we obtain that there exists such a $\tau_0 \in \mathcal{R}$ that $(V^{p_0 \tau_0}, H^{p_0 \tau_0}) \in \mathcal{H}_0$. Thus we have reduced the pair (V, H) to the normal form (V^{g}, H^{g}), where $g = p_0 \tau_0$

The conditions of the normal gauge are Lorentz-invariant, so if $(V^{g}, H^{g}) \in \mathcal{H}_0$, then $(V^{g\ell}, H^{g\ell}) \in \mathcal{H}_0$, too. $(\ell \in \mathcal{L})$.

The fact to be proved is that every gauge transformation transforming the pair (V, H) to a normal form differs from the above transformation $g = p_0 \tau_0$ by the Lorentz transformation. Let $(V^{g'}, H^{g'}) \in \mathcal{H}_0$ ((V, H) is the above-stated pair reduced to the normal form by the transformation $g = p_0 \tau_0$). We prove that $g' = g\,\ell$ (where $\ell \in \mathcal{L}$). It is easy to check that every element $g \in \mathcal{A}_0$ can be expressed in the form

$$g' = p' \tau' \ell'$$

here p' is the (15) type transformation, $\tau \in \mathcal{R}, \ell \in \mathcal{L}$, $(V^{p'}, H^{p'}) \in \mathcal{H}_1$ because $\mathcal{H}_1$ is the invariant space with respect to $\mathcal{L}$ and $\mathcal{R}$ groups action. The analysis of Eq.(15) convinces us that the transformation p is unequally determined for a fixed pair (V, H) by the condition (16), so $p' = p_0$. Since $\mathcal{H}_0$ subspace is Lorentz-invariant, the transformation τ' transforms the pair (V^{p_0}, H^{p_0}) to $\mathcal{H}_0$

$((V^{P_0\tau'}, H^{P_0\tau}) \in \mathcal{H}_0)$ as well as the transformation $\tau' \ell'$. Using the Lemma we get that $\tau' = \tau_0$, too. The theorem is proved.

4. In this section we will prove the Lemma. Let pair (V, H) belong to $\mathcal{H}_1$, then

$$V(z) = V(x, \theta) = 1 + V_\alpha \theta^\alpha + V_\mu x^\mu + \dots \tag{17}$$

$$H^\mu(\xi, \theta, \bar\theta) = \sigma^\mu_{\alpha\dot\beta}\, \theta^\alpha \bar\theta^{\dot\beta} + \sum_{p \geq 3} F^\mu_p(\xi, \theta, \bar\theta), \tag{18}$$

here p is the weight of the $F^\mu_p(\xi, \theta, \bar\theta)$

We consider the arbitrary transformation $Z^\tau(z)$, where $\tau \in R$. .

$$Z^\tau(z) = \begin{cases} \tilde{x}^\mu = x^\mu + G^\mu(x, \theta) = x^\mu + \sum\limits_{p \geq 3} G^\mu_p(x, \theta) \\ \tilde\theta^\alpha = \theta^\alpha + f^\alpha(x, \theta) = \theta^\alpha + \sum\limits_{p \geq 2} f^\alpha_p(x, \theta) \end{cases} \tag{19}$$

$G^\mu_p(x, \theta)$ and $f^\alpha_p(x, \theta)$ have the weight p. Let under the action of the transformation (19) the field (18) changes into the field

$$\tilde{H}^\mu(\tilde\xi, \tilde\theta, \bar{\tilde\theta}) = \sigma^\mu_{\alpha\dot\beta}\, \tilde\theta^\alpha \bar{\tilde\theta}^{\dot\beta} + \sum_{p \geq 3} \tilde{F}^\mu_p(\tilde\xi, \tilde\theta, \bar{\tilde\theta}), \tag{20}$$

It is obvious that

$$\tilde{H}^\mu(\tilde\xi, \tilde\theta, \bar{\tilde\theta}) = \operatorname{Im} \tilde{x}^\mu = \operatorname{Im}(x^\mu + G^\mu(x, \theta)) = H^\mu(\xi, \theta, \bar\theta) + \operatorname{Im} G^\mu(x, \theta) =$$
$$= \sigma^\mu_{\alpha\dot\beta}(\theta^\alpha + f^\alpha(x, \theta))(\bar\theta^{\dot\beta} + \bar{f}^{\dot\beta}(x, \theta)) + \sum \tilde{F}^\mu_p(\xi + \operatorname{Re} G(x, \theta), \theta + f(x, \theta), \bar\theta +$$
$$+ \bar{f}(x, \theta)), \qquad (x = \xi + iH, \quad \tilde{x} = \tilde\xi + i\tilde{H}).$$

So (18) and (20) are related by the equations:

$$\tilde{F}^\mu_p(\xi, \theta, \bar\theta) = F^\mu_p(\xi, \theta, \bar\theta) + [\operatorname{Im} G^\mu_p(x, \theta) - \tag{21}$$
$$-2 \operatorname{Re}(\sigma^\mu_{\alpha\dot\beta}\, \theta^\alpha \bar{f}^{\dot\beta}_{p-1}(x, \theta))]\Big|_{(x^\mu = \xi^\mu + i\sigma^\mu_{\alpha\dot\beta}\theta^\alpha\bar\theta^{\dot\beta})} + \dots$$

Here dots denote the members depending on G_t, f_{t-1}, F_t, with $t < P$. Eqs.(21) are the recurrent system of the algebraic equations. If we solve the first K equations, then we shall solve the $(K+1)$-th equation. Thus step by step we can calculate all the $\widetilde{F}_p$.

We denote by T the space of the power series in ξ, θ, $\bar{\theta}$ such that $t \in T$ if superfield

$$H = \theta\bar{\theta} + t$$

satisfies condition (18). Respectively, we denote by T_0 the subspace of T so that superfield

$$H = \theta\bar{\theta} + t, \quad \text{where} \quad t \in T_0 < T$$

satisfies conditions (11), (12), (13), (14) (or the equivalent ones (11), (12a), (13a), (14a). On the space of power series $\begin{pmatrix} G^{\mu}(x,\theta) \\ f^{\alpha}(x,\theta) \end{pmatrix}$ (see (19)) that will be denoted by $\mathcal{F}$ we define linear operator L.

$$L\begin{pmatrix} G^{\mu}(x,\theta) \\ f^{\alpha}(x,\theta) \end{pmatrix} = \left[\operatorname{Im} G^{\mu}(x,\theta) - 2\operatorname{Re} \sigma^{\mu}_{\alpha\dot{\beta}}\, \theta^{\alpha} \bar{f}^{\dot{\beta}}(x,\theta) \right] \Big|_{x = \xi + i\theta\bar{\theta}}$$

Operator L takes the values in the space T. Eqs.(21) could be rewritten as

$$\widetilde{F}_p = F_p - L\begin{pmatrix} G \\ f \end{pmatrix} + \ldots \tag{22}$$

Here dots denote the terms depending on the monoms G_t, f_{t-1}, F_t with weight $t < P$. Let us write out in detail the action of the operator L on $\mathcal{F}$. We denote by

$$G^{\mu}(x,\theta) = G^{\mu}_{0}(x) + G^{\mu}_{1\alpha}(x)\,\theta^{\alpha} + G^{\mu}_{2}(x)\cdot\theta\theta,$$

$$f^{\alpha}(x,\theta) = f_0^{\alpha}(x) + f_{1\beta}^{\alpha}(x)\,\theta^{\beta} + f_2^{\alpha}(x).\theta\theta\ .$$

Detailed calculations yield

$$L\begin{pmatrix} G^{\mu}(x,\theta) \\ f^{\alpha}(x,\theta) \end{pmatrix} = a^{\mu}(\xi) + \Gamma^{\mu}_{\alpha\dot\beta}(\xi)\,\theta^{\alpha}\bar\theta^{\dot\beta} + [\,b^{\mu}_{\alpha}(\xi)\,\theta^{\alpha} + \mathrm{c.c.}\,] +$$

$$+[\,D^{\mu}_{\alpha}(\xi)\,\theta^{\alpha}.\bar\theta\bar\theta + \mathrm{c.c.}\,] + A^{\mu}(\xi).\theta\theta.\bar\theta\bar\theta + [\,K^{\mu}(\xi).\theta\theta + \mathrm{c.c.}\,]$$

where

$$a^{\mu}(\xi) = \mathrm{Im}\ G_0^{\mu}(\xi)$$

$$\Gamma^{\mu}_{\alpha\dot\beta}(\xi) = \mathrm{Re}\,\frac{\partial G_0^{\mu}(\xi)}{\partial\xi^{\alpha\dot\beta}} - \left(f^{\pi}_{1\alpha}(\xi)\,\delta^{\dot\rho}_{\dot\beta} + \bar f^{\dot\rho}_{1\dot\beta}(\xi)\,\delta^{\pi}_{\alpha} \right)\sigma^{\mu}_{\pi\dot\rho}\ ,$$

$$A^{\mu}(\xi) = \frac{1}{8}\,\mathrm{Im}\,\frac{\partial^2 G_0^{\mu}(\xi)}{\partial\xi^{\alpha\dot\beta}\,\partial\xi^{\pi\dot\rho}}\,\epsilon^{\alpha\pi}\epsilon^{\dot\beta\dot\rho} -$$

$$-\frac{1}{4}\,i\left(\frac{\partial \bar f^{\dot\beta}_{1\dot\rho}(\xi)}{\partial\xi^{\dot\lambda}_{\alpha}}\,\epsilon^{\dot\rho\dot\lambda} - \frac{\partial f^{\alpha}_{1\rho}(\xi)}{\partial\xi^{\lambda}_{\dot\beta}}\,\epsilon^{\rho\lambda} \right)\sigma^{\mu}_{\alpha\dot\beta}\ , \qquad (23)$$

$$b^{\mu}_{\alpha}(\xi) = G^{\mu}_{1\alpha}(\xi) + \delta^{\pi}_{\alpha}\,\bar f^{\dot\rho}_0(\xi)\,\sigma^{\mu}_{\pi\dot\rho}\ ,$$

$$\bar D^{\mu}_{\dot\rho}(\xi) = -\frac{1}{2}\,i\,\frac{\partial G^{\mu}_{1\pi}(\xi)}{\partial\xi^{\beta\dot\rho}}\,\epsilon^{\pi\beta} + \left(\frac{1}{2}\,i\,\frac{\partial \bar f_0^{\dot\beta}(\xi)}{\partial\xi^{\alpha\dot\rho}_{\alpha}} - f_2^{\alpha}(\xi)\,\delta^{\dot\beta}_{\dot\rho} \right)\sigma^{\mu}_{\alpha\dot\beta}\ ,$$

$$K^{\mu}(\xi) = G_2^{\mu}(\xi).$$

Not difficult, though a cumbersome analysis of equations (23) shows that subspace T_0 is complementary to the image of the operator L

$$T = T_0 \oplus L\,\mathcal{F}. \tag{24}$$

It is obvious from the recurrent eqs.(22) consideration that for any p there is such an element $y \in \mathcal{F}$ that $\widetilde{F}_p \in T_0$. (So using relation (24) we get that for any $x \in T$ there is such $y \in \mathcal{F}$ that $x - Ly \in T_0$).

Therefore any pair $(V, H) \in \mathcal{H}$ can be reduced under the action of the corresponding transformation $\tau \in R$ to such a pair (V^{τ}, H^{τ}) that field H^{τ} satisfies the conditions (11), (12), (13), (14).

However a kernel of the operator L differs from zero *. So if in one of Eqs.(22) we change element y by such an element y' that $L(y'-y)=0$ then we shall construct the other transformation $\tau' \in R$ that reduces pair $(V, H) \in \mathcal{H}_1$ to the pair $(V^{\tau'}, H^{\tau'})$ such that $H^{\tau'}$ satisfies the conditions (11), (12), (13), (14). Although, generally speaking, $H^{\tau'}$ does not coincide with H^{τ}. Thus the procedure suggested above is ambiguous.

Recall, however, that in order the pair (V, H) be in a normal form, not only conditions (11), (12), (13), (14), but also conditions (10) are to be held. It can be demonstrated considering the action of the transformations (19) by means of (4) on the volume form (17) that conditions (10) unequally fix the elements from the L operator kernel. The Lemma is proved.

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* Note that the corresponding to the L operator kernel subgroup of the transformations is tightly connected with the group of superconformal transformations.

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