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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

ЕФИ-56I(48)-82

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DIMENSIONAL REGULARIZATION OF THE SUPERSYMMETRIC YANG-MILLS MODEL

ԵՐԵՎԱՆ 1982 ЕРЕВАН

1. Introduction

There exists now only one method of explicitly supersymmetric regularization of simple supersymmetric theories - the regularization by means of high covariant derivatives [1]. The application of this method for calculational purposes is practically impossible due to the bulkyness of regularization terms. An attempt of dimensional regularization of supersymmetric theories was made in ref. [2]. This method holds well in one- and two-loop calculations [3,4,5]. However, it has turned out that there are certain contradictions in it which make its application in high loops problematic [6,7].

In the present work a slightly modified scheme of dimensional regularization is proposed which preserves the supersymmetry. It is applicable to supersymmetric Yang-Mills model as well as to vector-like models with a simple supersymmetry in Wess-Zumino gauges. The proof given is for the Yang-Mills model. It can be easily generalized in the case of vector-like models with matter fields having simple supersymmetry.

2. Definition of regularization

Consider a simple supersymmetric Yang-Mills model in the Wess-Zunino gauge with SU(2) gauge symmetry:

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{2} i \bar{\lambda}^a \gamma_\mu (D_\mu \lambda)^a \tag{2.1}$$

It is symmetrical with respect to supersymmetric transformations

$$\delta A^a_\mu = \frac{1}{2} \bar{\alpha} \gamma_\mu \lambda^a$$
$$\delta \lambda^a = \frac{1}{4} \sigma_{\mu\nu} F^{a\mu\nu} \alpha \tag{2.2}$$

The spinors in all upper formulae are Mayorana.

$$\lambda = C \bar{\lambda}^T$$
$$\alpha = C \bar{\alpha}^T \tag{2.3}$$

The quantum theory is defined by the generating functional

$$Z(J,\xi) = \int \mathcal{D}A_\mu \mathcal{D}c \mathcal{D}\lambda \exp i \int [\mathcal{L} + \frac{1}{2\beta}(\partial_\mu A^a_\mu)^2 + \\ + \overset{*}{c}{}^a \partial_\mu (D_\mu c)^a + J^a_\mu A^a_\mu + \bar{\xi}^a \lambda^a] d^4x \tag{2.4}$$

Without a concrete regularization the above formulae are senseless. Let's now describe the concrete scheme of regularization in the framework of perturbation theory. We attach meaning to each term of the expansion $Z(J,\xi)$ by the sources J_μ, ξ in the momentum representation, i.e., to each mean type:

$$\langle J^{a_1}_{\mu_1} A^{a_1}_{\mu_1}(p_1) \cdots J^{a_n}_{\mu_n} A^{a_n}_{\mu_n}(p_n) \bar{\xi}\lambda(k_1) \cdots \bar{\xi}\lambda(k_m) \rangle \tag{2.5}$$

According to Feynman rules this mean value is a mathematical expression given by a set of Feynman diagrams. There are contractions of the type

$$\gamma_\mu (\hat{e}_1 \cdots \hat{e}_k) \gamma^\mu \quad , \quad \delta_\mu{}^\mu \tag{2.6}$$

in these diagrams. They appear, for example, when two vertices like $\bar{\lambda}\gamma_\mu A^\mu \lambda$ are contracting by the propagator $\langle A_\mu A_\nu \rangle$

In our scheme we contract expressions (2.6) in four-dimension. After that we carry out all trace calculations. This can originate new contractions of the type (2.6). . Carry out these contractions in four-dimension as well. Then pass on to n-dimensional space, i.e., put all momenta, vector indices in n-dimensional space.

Let's postulate the following operations for γ_μ -matrices with n-dimensional index μ :

$$\{\gamma_\mu \gamma_\nu\} = 2 g_{\mu\nu} \cdot \hat{1}$$
$$Sp\, \gamma_\mu = 0 \tag{2.7}$$
$$\gamma_\mu \gamma^\mu = n = 4 - 2\varepsilon$$

$\hat{1}$ is the four-dimensional unit matrix. Then the trace of the odd number γ -matrix is zero, and the trace of the even number is defined unambiguously by tensors $g^{\mu\nu}$.

Note that there are C^{-1} (charge conjugation matrix) in vertices $\bar{\lambda}\gamma_\mu A_\mu \lambda$

$$\bar{\lambda}\gamma_\mu A_\mu \lambda = -\overset{T}{\lambda} C^{-1} \gamma_\mu A_\mu \lambda$$

but in fermion cycles they are contracted by C from propagators $\langle \lambda \lambda \rangle$. On the lines joining two spinor sources ξ

will contract in such a way that there will remain only one matrix C in front of the source and there will be no necessity of defining the n-dimensional matrix C.

Due to the absence of γ^5-matrix in the theory there are no difficulties associated with antisymmetric tensor $\epsilon^{\mu\nu\alpha\beta}$ [6].

We shall now integrate over all internal momenta in n-dimensional space and carry out all new contractions of γ -matrices and momenta. After that we put all momenta, sources and vector indices in four-dimensional space. It is obvious that we thus obtain an unambiguous answer for each term of the expansion $Z(J,\xi)$.

3. Supersymmteric Ward identities

Within the framewrok of our regularization one can shift the fields. Let's shift (2.3). We shall obtain the following expression for $Z(J,\xi)$:

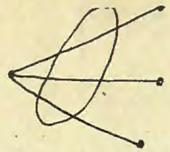
$$Z = \int \mathcal{D}A\, \mathcal{D}\lambda\, \mathcal{D}c\, \exp i \int d^n x \Big[\mathcal{L} + \frac{1}{2\beta}(\partial_\mu A^a_\mu)^2 + \overset{*}{c}{}^a \partial_\mu (D_\mu c)^a + J^a_\mu A^a_\mu + \bar{\xi}\lambda + \frac{g}{4}\epsilon^{abc}(\bar{\alpha}\gamma_\mu \lambda^a)(\bar{\lambda}^b \gamma_\mu \lambda^c) - \frac{1}{2\beta}(\partial_\mu \partial_\nu A^a_\nu)\bar{\alpha}\gamma_\mu \lambda^a + \epsilon^{abd}\overset{*}{c}{}^a c^d \partial_\mu (\tfrac{1}{2}\bar{\alpha}\gamma_\mu \lambda^b) + \frac{1}{2} J^a_\mu \bar{\alpha}\gamma_\mu \lambda^a + \bar{\xi}(\tfrac{1}{2}\sigma_{\mu\nu} F^{\mu\nu}\alpha) \Big] \tag{3.1}$$

Let's prove that the term

$$\epsilon^{abc}(\bar{\alpha}\gamma_\mu \lambda^a)(\bar{\lambda}^b \gamma_\mu \lambda^c) \tag{3.2}$$

makes no contribution in $Z(J,\xi)$.

A diagram in which the (3.2) can contribute has the form:



(3.3)

where the spinor lines outgoing from the vertex (3.3) come out and expire on spinor sources η (η is the spinor coefficient at the linear term λ in the expression (3.1)).

Each spinor in the vertex (3.3) is dressed in a certain coat of external momenta and γ -matrices. The general form or one of the diagram (3.3) components, after calculations by the scheme described in the previous section, is as follows:

$$\int \epsilon^{abc}\, \bar{\alpha}\gamma_\mu \xi^a_1 \overset{T}{\xi}{}^b_2 C^{-1}\gamma_\mu \xi^c_3 \, dp_{ext}$$

where ξ_i is the external spinor source (α or ξ) dressed in a coat of external momenta and γ -matrices; the integration is over external four-momenta.

However, in accord with our regularization scheme, we put this expression (already integrated over all internal momenta) in a four-dimensional space, and, therefore, ξ_i may be considered as a Mayorana (α and ξ are Mayorana, and the multiplication by four-dimensional γ -matrices and by the four-dimensional momenta leads to the Mayorana spinors ξ_i).

It is obvious that in (3.3), along with this component, its partners are present as well:

$$\int dp_{ext}\,\epsilon^{abc}\,[\,\bar\alpha\gamma_\mu\,\xi_1^a\,\bar\xi_2^b\,\gamma_\mu\,\xi_3^c\; + \;\bar\alpha\gamma_\mu\,\xi_2^a\,\bar\xi_1^b\,\gamma_\mu\,\xi_3^c\; + \;\bar\alpha\gamma_\mu\,\xi_3^a\,\bar\xi_2^b\,\gamma_\mu\,\xi_1^c\,]\,. \tag{3.4}$$

In (3.4) everything is already four-dimensional. Let's prove that the sum of the first two is equal to the third with a minus sign.

We can already use the Fiertz expansion because in (3.4) everything is four-dimensional

$$\epsilon^{abc}(\,\bar\alpha\gamma_\mu\,\xi_1^a\,\bar\xi_2^b\gamma_\mu\,\xi_3^c + \bar\alpha\gamma_\mu\,\xi_2^a\,\bar\xi_1^b\gamma_\mu\,\xi_3^c\,) =$$

$$= -\frac{1}{4}\,\epsilon^{abc}(\,\bar\xi_2^b\,\Gamma^i\,\xi_1^a + \bar\xi_1^b\,\Gamma^i\,\xi_2^a\,)\,\bar\alpha\gamma_\mu\,\Gamma^i\gamma_\mu\,\xi_3^c$$

$$= -\frac{1}{4}\,\epsilon^{abc}(\,\bar\xi_2^b\,\Gamma^i\,\xi_1^a - \bar\xi_2^b\,C\,\Gamma^{iT}C^{-1}\,\xi_1^a\,)\,\bar\alpha\gamma_\mu\,\Gamma^i\gamma_\mu\,\xi_3^c$$

$$= \epsilon^{abc}\,\bar\xi_2^b\gamma_\nu\,\xi_1^a\,\bar\alpha\gamma_\nu\,\xi_3^c = -\,\epsilon^{abc}\,\bar\alpha\gamma_\nu\,\xi_3^a\,\bar\xi_2^b\gamma_\nu\,\xi_1^c$$

In these calculations Γ^i is the expansion basis:

$$\Gamma^i = (\,1,\, i\gamma_5\,,\gamma_\mu\,,\, i\gamma_\mu\gamma_5\,,\,\sigma_{\mu\nu}\,)$$

There is also one possibility when (3.2) can make a contribution in (3.1). In this case two spinors λ from (3.2) close on each other and the third one, outgoing from the vertex (3.2), finishes on external spinor source. The resulting expression, after calculations by the scheme described in the previous section, is a sum of expressions of the following

type:

$$\int dp_{ext}\;\epsilon^{abc}(\,2\bar\alpha\gamma_\mu\,\langle\lambda^a\bar\lambda^b\rangle\gamma_\mu\,\xi^c + \bar\alpha\gamma_\mu\,\xi^a\langle\bar\lambda^b\gamma_\mu\,\lambda^c\rangle\,),$$

$$\langle\,\lambda^a\,\bar\lambda^b\,\rangle = P^{ab} \tag{3.5}$$

$$\langle\,\bar\lambda^b\gamma_\mu\,\lambda^c\,\rangle = -\,Sp\,(\,P^{cb}\gamma_\mu\,)$$

In (3.5) spinors in $\langle\;\rangle$ close on each other; the factor 2 at the first term is a consequence of two possible closings of λ^a on λ^b or λ^c ; P^{ab} is a matrix 4×4 depending on external sources, momenta and it satisfies equations

$$\int dp_{ext}\;\epsilon^{abc}(\,Sp\,P^{bc}\,\Gamma^i\,)\,\bar\alpha\gamma_\mu\,\Gamma^i\gamma_\mu\,\xi^a = 0 \tag{3.6}$$

$$\text{at}\quad \Gamma^i = 1,\,\gamma^5,\,\gamma^\mu\gamma^5,\,\sigma^{\mu\nu}$$

(3.6) is a consequence of the condition $\bar\lambda = -\overset{T}{\lambda}\,C^{-1}$. P^{ab} as a matrix 4×4, can be expanded on Fiertz basis Γ^i

$$P^{ab}_{\alpha\beta} = \frac{1}{4}\,\Gamma^i_{\alpha\beta}\;Sp\;P\,\Gamma^i \tag{3.7}$$

By means of (3.7), (3.6), taking into account that μ in (3.5) is four dimensional in our scheme, one can show that (3.5) is zero. Thus, the vertex (3.2) doesn't contribute in (3.1) and one obtains the following supersymmetric identities:

$$\int \mathcal{D}A_\mu\,\mathcal{D}\lambda\,\mathcal{D}c\,[\,-\frac{1}{2\beta}(\partial_\mu\partial_\nu A_\nu^a)\bar\alpha\gamma_\mu\,\lambda^a + \epsilon^{abd}\bar c^a c^d\,\partial_\mu\,(\frac{1}{2}\bar\alpha\gamma_\mu\,\lambda^b\,)\; +$$

$$+\frac{1}{2} J^{a}_{\mu} \bar{\alpha} \gamma_{\mu} \lambda^{a} + \bar{\xi}\left(\frac{1}{2}\sigma_{\mu\nu} F^{\mu\nu} \alpha\right)\Big] \exp i \int d^{n}x \Big[\mathcal{L} + \frac{1}{2\beta}(\partial_{\mu} A^{a}_{\mu})^{2} +$$

$$+ \overset{*}{C}{}^{a} \partial_{\mu}(D_{\mu}C)^{a} + J^{a}_{\mu} A^{a}_{\mu} + \bar{\xi}\lambda \Big] = 0 \qquad (3.8)$$

Conclusion

Thus our regularization scheme consists in the following:

Carry out all (2.6)-type contractions in four-dimensional space.

2. Carry out all trace calculations. This can originate new contractions of type (2.6). Carry out these contractions in four-dimension as well.

3. After that put all in n-dimension and carry out all integrations over internal momenta and, according to (2.7), all originated by integrations contractions of γ -matrices and momenta in n-dimensional space.

4. After that put all the external momenta, sources and γ -matrices in four-dimensional space.

In this scheme supersymmetric identities (3.8) are fulfilled and the relation

$$Z_2 = Z_3$$

should follow from them, where Z_2, Z_3 are the renormalization constants of the fields A_{μ}, λ , respectively. The relation of the remaining renormalization constants are defined by the Slavnov gauge identities [8,9].

I am greatly indebted to A.A.Slavnov, A.A.Vladimirov for helpful discussions as well as to the staff of the laboratory headed by S.G.Matinyan (EPI).

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The manuscript was received 28 April 1982

Эд.Ш.ЕГОРЯН

РАЗМЕРНАЯ РЕГУЛЯРИЗАЦИЯ СУПЕРСИММЕТРИЧНОЙ МОДЕЛИ ЯНГА-МИЛЛСА

(на английском языке, перевод Л.Н.Багдасаряна)

Ереванский физический институт

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Заказ 213 ВФ- 03809 Тираж 299

Препринт ЕФИ Формат издания 60х84/16

Подписано к печати 25/У-82 0.9 уч-изд.л.Ц. 14 к.

Издано Отделом научно-технической информации Ереванского физического института, Ереван 36, Маркаряна 2