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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

ЕФИ-567(54)-82

D.B.SAHAKYAN

DEFINITION OF STRING TENSION IN GAUGE THEORIES ON A LATTICE AND NONPLANAR CONTOURS

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An off-axis string analog is considered on a lattice in the Euclidean approach to gauge fields. The string tension may be calculated either by means of Monte Carlo calculations or by the method of high-temperature expansion. For a three-dimensional Ising model a Coulomb term in the string potential is calculated in the first orders.

Yerevan Physics Institute

Yerevan 1982

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ОПРЕДЕЛЕНИЕ НАТЯЖЕНИЯ СТРУНЫ В КАЛИБРОВОЧНЫХ ТЕОРИЯХ НА РЕШЕТКЕ И НЕПЛАНАРНЫЕ КОНТУРЫ

В евклидовом подходе к калибровочным полям на решетке рассмотрен аналог внеосевой струны. Натяжение струны можно вычислить или с помощью Монте-Карло расчетов, или методом высокотемпературного разложения. Для трехмерной модели Изинга вычислен в первых порядках кулоновский член в потенциале струны.

Ереванский физический институт

Ереван 1982

YEREVAN PHYSICS INSTITUTE

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It is known that the phase transition of roughning deteriorates the accuracy of the string tension definition in gauge theories on a lattice.

In ref. 1 the dynamics of the string between a quark and an antiquark off the axis is considered. The energy of the system was calculated in the Hamiltonian SU(3) gauge theory by the method of high-temperature expansion in the first orders of perturbation theory. The string was found to be in the "roughning" phase in the whole range $0 < g^2 < \infty$

In the present paper we treat an off-axis string analog in the Euclidean approach to gauge fields on a lattice.

Consider the following process: at the time moment a quark and an antiquark are produced at a distance $\vec{R}$ from each other and after the time t they vanish. If $\vec{R}$ does not lie on one axis of the lattice, then there is a large number of contours C_i with a minimum area (the area of the surface that may be pulled on a contour) describing the quark and antiquark trajectories.

By analogy with 2 the physical amplitude of the process is

$$A = \sum W(C_i) \tag{1}$$

where $W(C)$ is the Wilson order parameter. The potential energies of the system $V(\vec{R})$ and the string tension α are, respectively

$$V(\vec{R}) = -\lim_{t\to\infty} \frac{\ln A}{t} \tag{2}$$

$$\alpha = \lim_{R\to\infty} \frac{V(R)}{R} \tag{3}$$

In principle, $W(C)$, defined for any nonplanar contour, may as well serve as a source of the definition of α in the limit of large contours. But since just A is the physical amplitude, α, defined from A should be more regular (as a function of g^2). There is a hope that this is true for both Monte Carlo calculations and at high-temperature expansion.

Besides, considering the quantity A, one may define the Coulomb term in the string potential energy.

In ref. 3 at most general assumptions the following expression is found for the boson string:

$$V_{-1}(R) = -\frac{(d-2)}{2} \frac{\pi}{12} \cdot \frac{1}{R} \tag{4}$$

In accord with Polyakov's hypothesis the three-dimensional Ising model near the phase transition point should lead to the fermion string. An accurate calculation of $V_{-1}(R)$ for this model would allow to check the type of the string (bosonic or fermionic).

Let us show how the high-temperature expansion is constructed in the Euclidean approach to gauge theories on a lattice and calculate to the second order the $V_{-1}(R)$ for a three-dimensional Ising gauge model. Denote the i-th minimum path between a quark and and antiquark in d-1 dimensional space by $|i\rangle$. Then by the definition of

$$A = \sum_i \langle i|T|i\rangle \qquad (5)$$

is the T transition matrix spur in a linear space pulled on N vectors $|i\rangle$.

Let eigenvectors and matrix eigenvalues be $|i\rangle$ and $e^{-E_i t}$. Then

$$V(\vec{R}) = -\lim_{t\to\infty} \frac{\ln\left(\sum_\kappa e^{-E_\kappa t}\right)}{t} = -\lim \frac{\ln\langle 1|T|1\rangle}{t} = E_1 \qquad (6)$$

where $|1\rangle$ is the state with the maximum $e^{-E_1 t}$. We wish to obtain the expansion $V(R)$ as a series in terms of powers of $\beta^n\ (\beta = 1/g^2)$. From (6) we have the condition $(E_2 - E_1)t \to \infty$. Since $E_2 - E_1 \sim \beta$, we obtain

$$\begin{aligned} \beta t &\to \infty \\ \beta &\to 0 \end{aligned} \qquad (7)$$

Let's take instead of the exact eigenvector $|1\rangle$ the solution of the appropriate secular equation $|1'\rangle$. We can then obtain $V(R)$ to the terms β^2 inclusively. We obtain

$$\langle 1'|T|1'\rangle = \beta\sqrt{2}\,R\,t_e - |C_1|\beta t\left[e^{c_2\beta + c_3\beta^2 + c_4\beta^2 t + o(\beta^3 t^2)}\right], \qquad (8)$$

$$\lim_{\substack{\beta t\to\infty \\ \beta\to 0}} \frac{\ln\langle 1'|T|1'\rangle}{t} = \ln\beta\sqrt{2}\,R - |C_1|\beta + C_4\beta^2$$

Here C_1, C_2, C_3, C_4 are the functions of R . If we had considered $|X\rangle'$ for one simple contour instead of $|1'\rangle$, which is the linear combination of simple contours, we should not be able to collect the terms $\langle X|T|x\rangle$ in the form of an exponent. There would be no condition overlap region $\begin{matrix}\beta t \to \infty \\ \beta \to 0\end{matrix}$ where we could have calculated $\frac{\ln\langle x|T|x\rangle}{t}$. It would be prevented by large terms of the type $\beta^n t^n$.

The calculations were carried out for $\vec{R}$ directed at an angle $\pi/4$ with respect to the axis X . We have calculated $V(\vec{R})$ for $\vec{R}=(M,M)$ in the points (3.3), (4.4), (5.5). The following quantities enter the perturbation theory series for

$$P_i(M) = n_i/R \tag{9}$$

where n_i is the number of surfaces of definite type (with the same angular coefficient) pulled on contours describing quark and antiquark trajectories.

The quantities $P_i(M)$ were approximated by the formulae

$$P_i(M) = C_1 + \frac{C_2}{M} + \frac{C_3}{M^2} \tag{10}$$

The quantities C_1, C_2, C_3 rapidly become constants (with an accuracy ~ 0.01) with the increase of M . They have been calculated on a computer.

Approximating the potential $V(\vec{R})$ by the expression $-\alpha R + b + \gamma/R$ we have found for α and γ

$$\begin{aligned} \alpha &= -1.414 \ln u - 0{,}9014 + 0{,}213 u^2 \\ \gamma &= -0{,}181 u + 0.003 u^2 \end{aligned} \tag{11}$$

where $C_1 = th\beta$

High orders of perturbation theory may be calculated by Rayleigh-Schrodinger formulae.

In conclusion the author expresses his thanks to S.G.Matinyan, A.A.Migdal, A.M.Polyakov and A.G.Sedrakyan for valuable discussions.

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The manuscript was received 27 May 1982.

Д.Б.СААКЯН

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(на английском языке, перевод Л.Н.Багдасаряна)

Ереванский физический институт

Редактор Л.П.Мукаян

Тех. редактор А.С.Абрамян

Заказ 280	ВФ-03877	Тираж 299
Препринт ЕФИ		Формат издания 60х84/16
Подписано к печати 15/VII-82г.		0,5 уч.-изд.л. Ц.7 к.

Издано отделом научно-технической информации

Ереванского физического института, Ереван 36, Маркаряна 2