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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

ЕФИ-569(56)-82

H.M.ASATRYAN

FERMION MASS RELATIONS AND THE t-QUARK MASS

ԵՐԵՎԱՆ 1982 ЕРЕВАН

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Г.М.АСАТРЯН

МАССОВЫЕ СООТНОШЕНИЯ ДЛЯ ФЕРМИОНОВ И МАССА t - КВАРКА

Рассмотрены массовые соотношения для фермионов в моделях великого объединения. В предположении линейной связи между массовыми матрицами заряженных лептонов и кварков с зарядами +2/3 и -I/3 получено ограничение на массу t - кварка.

Ереванский физический институт
Ереван 1982

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Fermion mass relations are considered in grand unified models. In the assumption of linear coupling between the mass matrices of charged leptons and quarks with charges +2/3 and -1/3 a restriction is obtained on the t-quark mass $m(t) \leqslant 24.5$ GeV.

Yerevan Physics Institute
Yerevan 1982

ЕФИ-569(56)-82

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Yerevan 1982

Recently there appeared a number of works dedicated to the estimation of the t-quark mass [1-]. Searches are carried out for bound $\bar{t}t$ states in e^+e^--annihilation.

In the present paper the fermion mass relations in grand unified models are investigated which allows to obtain a restriction on the t-quark mass.

Let $\mathcal{M}(Q)$ ($Q = 2/3, -1/3, -1$) be 3x3 mass matrices of the quarks(u, c, t) with the charge 2/3, (d, s, b) with the charge -1/3 and of the leptons (e, μ, τ) with the charge -1. In the general case one may present $\mathcal{M}(Q)$ in the form[4]

$$\mathcal{M}(Q) = \sum_{a} F^{a}(Q)\, G^{a}, \qquad (1)$$

where $F^{a}(Q)$ are the numbers and G^{a} are the 3x3 matrices independent of Q. Glashow [4] paid attention on the possibility when there are only two terms in the sum (1). Such a situation occurs, for instance, in the SO(10) grand unified model where a single complex $\underline{10}$ and $\underline{126}$ of Higgs boson generate fermion masses as well as in the model E_6 where fermions become massive by coupling to $\underline{27}$ and $\underline{251}$ representations of Higgs fields.

We then obtain from (1):

$$\mathcal{M}(-1) = \alpha\,\mathcal{M}(2/3) + \beta\,\mathcal{M}(-1/3) \tag{2}$$

Assume that the mass matrices $\mathcal{M}(Q)$ are symmetric (it is so for the above examples). Then one may rewrite the relation (2) for diagonalized mass matrices:

$$A\,\mathcal{M}^{\circ}(-1)\,A^{T} = \alpha\,\mathcal{M}^{\circ}(2/3) + \beta\,K^{+}\mathcal{M}^{\circ}(-1/3)\,K^{*}, \tag{3}$$

where $\mathcal{M}^{\circ}(Q)$ are the diagonal (though generally complex) matrices, $A, \mathcal{K}$ are the unitary matrices. The matrix $\mathcal{K}$ describes the mixing in weak left currents [5]. The matrix $\mathcal{K}$ is expressed, with an accuracy up to phase redetermination, through three mixing angles $\Theta_1, \Theta_2, \Theta_3$ and one phase δ connected with the CP-violation [5] (the angle Θ_1 coincides with the Cabibbo angle Θ_c). If mixing angles $\Theta_1, \Theta_2, \Theta_3$ are neglected ($\sin\Theta_c = 0.23$, for restrictions on the values Θ_2, Θ_3 see 6,7), the matrix $\mathcal{K}$ becomes diagonal. We then obtain from (3) the relation for particles masses [1]

$$\det\begin{pmatrix} u e^{i\varphi_u} & c e^{i\varphi_c} & t e^{i\varphi_t} \\ d e^{i\varphi_d} & s e^{i\varphi_s} & b e^{i\varphi_b} \\ e e^{i\varphi_e} & \mu e^{i\varphi_\mu} & \tau e^{i\varphi_\tau} \end{pmatrix} = 0, \tag{4}$$

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[1] Here we have excluded nonphysical solutions of (3) of the type $\tau e^{i\varphi_\tau} = \alpha u e^{i\varphi_u} + \beta d e^{i\varphi_d}$, $\mu e^{i\varphi_\mu} = \alpha c e^{i\varphi_c} + \beta s e^{i\varphi_s}$, $e e^{i\varphi_e} = \alpha t e^{i\varphi_t} + \beta b e^{i\varphi_b}$, when the mass of heavy τ -lepton, for example, is connected with masses of light u and d -quarks.

where particle names are their masses which enter (4) with arbitrary phases (in ref. [4] these phases are taken equal to zero). From (4) we obtain

$$t \leq \frac{\tau(cd + us) + b(u\mu + ec)}{d\mu - es} \qquad (5)$$

Current masses of quarks, included in (5) are functions of momentum transfer. Following [4] we define the observed mass of the heavy quark q in the point equal to the mass of the bound state $\bar{q}q$ (masses of light u, d, S - quarks are defined at 1 GeV transfer momentum). Then, taking the following values for the current masses of the first five quarks - $m(u) = 4.2$ MeV, $m(d) = 7.5$ MeV, $m(S) = 150$ MeV, $m(c) = 1.2$ GeV, $m(b) = 4.4$ GeV [1], we shall obtain the restriction for the t-quark mass:

$$\begin{aligned} m(t) &\leq 24.0 \text{ GeV for } \Lambda = 0.1 \text{ GeV} \\ m(t) &\leq 22.5 \text{ GeV for } \Lambda = 0.3 \text{ GeV}. \end{aligned} \qquad (6)$$

where Λ is the well known scale parameter of QCD.

How can this result change the account of mixing angles ? To answer this question we have done the following. Assume that fermion mass matrices have the form

$$\mathcal{M}(Q) = \begin{pmatrix} 0 & a(Q) & 0 \\ a(Q) & 0 & b(Q) \\ 0 & b(Q) & c(Q) \end{pmatrix}, \qquad (7)$$

which leads to reasonable values for mixing angles [1]. For simplicity we shall consider the mass matrices (7) to be real, i.e., neglect the CP-violation. Then the matrix (7) elements are expressed through particles masses as follows [1]:

$$a^2(2/3) \approx uc \qquad b^2(2/3) \approx ct \tag{8}$$

$$c^2(2/3) \approx (t-c)^2$$

and similarly for $Q = -1/3, -1$. For the Cabibbo angle we obtain [1,8]

$$\sin\theta_c \approx \sqrt{\frac{d}{s}} \pm \sqrt{\frac{u}{c}} \tag{9}$$

(the signs + and - correspond to the values $\delta = 0$ and $\delta = \pi$ of the CP-violation phase δ). From (2), (7), (8) with consideration of (9) [8] we obtain the restriction on the t-quark mass ($\delta = \pi$ corresponds to the maximum value of $m(t)$):

$$\begin{aligned} m(t) &\leq 24.5 \text{ GeV for } \Lambda = 0.1 \text{ GeV} \\ m(t) &\leq 24.3 \text{ GeV for } \Lambda = 0.3 \text{ GeV}, \end{aligned} \tag{10}$$

which doesn't differ much from the restriction (6). The result obtained is valid at consideration of CP-violation as well: if $\sin\delta$ is small then the restriction on $m(t)$ in the first order over $\sin\delta$ won't change.

Thus we may expect that the bound $\bar{t}t$ state will have a mass less than 50 GeV.

In conclusion the author expresses his thanks to S.G.Matinyan and A.G.Sedrakyan for helpful discussions.

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The manuscript was received 27 April 1982.

Г.М.АСАТРЯН

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(на английском языке, перевод Л.Н.Багдасаряна)

Ереванский физический институт

Редактор Л.П.Мукаян

Тех.редактор А.С.Абрамян

Заказ 254 ВФ-05300 Тираж 299

Препринт ЕФИ Формат издания 60х84/I6
Подписано к печати 24/УI-82 0.6 уч.-изд.л. Ц. 8 к.

Издано Отделом научно-технической информации
Ереванского физического института, Ереван 36, Маркаряна 2