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TO THE QUESTION ON YANG-MILLS CLASSICAL MECHANICS STOCHASTICITY

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1. The study of classical solutions of nonlinear field equations is of great interest. This is connected with the hopes that classical solutions act an important role when constructing both the ground state (vacuum) and the spectrum of this theory . Y.M. theory is the most interesting nonlinear theory pretending on the description of physical reality.

When studying the solutions of the Y.M. classical equations in the four-dimensional Euclidean space the instanton solutions [1] were found out that played an important role in understanding vacuum structure in nonabelian theories.

Monopole solutions [2] were found out for the Y.M. fields in the Higgs condensate. All these solutions possess a topological charge and are stable due to it.

Recently an interest has arisen to a new class of solutions of Y.M. equations, i.e. to nonlinear plane waves. This class of problems has an obvious mechanical analog. Their investigation reduces to that of mechanical Hamiltonian systems. Coleman was the first [3] who studied the wave solutions in nonabelian theories. He imposed the equality condition of the magnitude of the Poynting vector and the energy density. This assumption is typical of linear plane waves and therefore only linear waves were obtained in Ref.[3].

The authors of Ref.[4] denied this assumption. They obtained a solution in the form of a massive nonlinear wave. In this work there occurred difficulties connected with the description of the interaction of different color degrees of freedom. Only the colorless solutions [4] were described analytically.

The question of the interaction of two color degrees of freedom in a particular case reduces to the study of a set of the two second-order differential equations:

$$\ddot{x} + xy^2 = 0 \qquad (1)$$
$$\ddot{y} + yx^2 = 0$$

together with the first motion integral

$$2\mathcal{E} = \dot{x}^2 + \dot{y}^2 + x^2y^2$$

In Ref.[5] they carried out a computer experiment in order to study the set (1) solutions. A series of periodical solutions was found out. They also studied the question on the stability of the set (1) solutions and there was found out the instability of the solutions with respect to small perturbations,which in fact is the indication of a stochasticity of these solutions. A more rigid proof of the set (1) stochasticity was obtained (again by computing) in Ref.[6] .

2. Consider classical equations of Y.M. fields for SU(2) group. We are interested in wave solutions for which $A_\mu(x) = A_\mu(\xi)$, where $\xi = K_\mu x^\mu$, K_μ is a time-like vector $(K^2 > 0)$. Then one may turn to the reference frame, where $K_\mu = (K_0 ; \vec{0})$ and the potential will depend on time only. As distinct from Refs.[4, 5] we shall work in axial gauge $A_3 = 0$. In such a statement the problem possesses an addi-

tional symmetry relative to the transformations

$$\hat{A}_{1;2} \to S^{-1}\hat{A}_{1;2}S$$
$$\hat{A}_0 \to S^{-1}\hat{A}_0 S + S^{-1}\dot{S}$$

where $\hat{A} = A^a T_a$ (T_a are generators of SU(2) group), and S depends on time only.

Making use of this symmetry we can cancel A_0 .

Then the motion equations are as follows:

$$\ddot{\vec{A}}_1 + \vec{A}_1(\vec{A}_2\vec{A}_2) - \vec{A}_2(\vec{A}_1 \cdot \vec{A}_2) = 0 \qquad (2a)$$

$$\ddot{\vec{A}}_2 + \vec{A}_2(\vec{A}_1 \cdot \vec{A}_1) - \vec{A}_1(\vec{A}_1 \cdot \vec{A}_2) = 0 \qquad (2b)$$

$$[\vec{A}_1 \times \dot{\vec{A}}_1] + [\vec{A}_2 \times \dot{\vec{A}}_2] = 0 \qquad (2c)$$

The set (2) has the integrals analogous to the mechanical energy $\mathcal{E}$ and moment μ which are of the form:

$$\mathcal{E} = \frac{1}{2}[(\dot{\vec{A}}_1)^2 + (\dot{\vec{A}}_2)^2] + \frac{1}{2}[(\vec{A}_1)^2(\vec{A}_2)^2 - (\vec{A}_1 \cdot \vec{A}_2)^2] \qquad (3a)$$

$$\mu = (\vec{A}_1 \cdot \dot{\vec{A}}_2 - \dot{\vec{A}}_1 \cdot \vec{A}_2) \qquad (3b)$$

In Ref.[5] a particular case of the set (2) (when $A_1^2 = A_1^3 = A_2^1 = A_2^3 = 0$) was actually analyzed. In this case Eqs.(2c) are satisfied automatically, and Eqs.(2a, 2b) coincide with the set (1).

It should be noted that in this case the moment integral $\mu = 0$. In this work the set (2) transforms into the set of two differential equations (with no additional simplifications) which can be computer-studied.

A sufficient condition for the stochasticity of the set (2) solutions is obtained.

3. It follows from Eqs.(2c) that $\dot{\vec{A}}_1$ and $\dot{\vec{A}}_2$ are in a plane determined by vectors $\vec{A}_1$ and $\vec{A}_2$. Let us introduce in this plane two orthogonal unit vectors $\vec{n}_1$ and $\vec{n}_2$:

$$\vec{n}_1 \cdot \vec{n}_1 = \vec{n}_2 \cdot \vec{n}_2 = 1; \qquad \vec{n}_1 \cdot \vec{n}_2 = 0$$

Expand now vectors $\vec{A}_1$ and $\vec{A}_2$ and their first derivatives in this basis:

$$\vec{A}_1 = \alpha_1 \vec{n}_1 + \beta_1 \vec{n}_2 \qquad \vec{A}_2 = \alpha_2 \vec{n}_1 + \beta_2 \vec{n}_2 \qquad (4a)$$

$$\dot{\vec{A}}_1 = \gamma_1 \vec{n}_1 + \delta_1 \vec{n}_2 \qquad \dot{\vec{A}}_2 = \gamma_2 \vec{n}_1 + \delta_2 \vec{n}_2 \qquad (4b)$$

Eqs.(3c) and integrals (3a, b) take the form

$$\alpha_1 \delta_1 + \alpha_2 \delta_2 = \beta_1 \gamma_1 + \beta_2 \gamma_2 \qquad (5a)$$

$$\mu = \alpha_2 \gamma_1 + \beta_2 \delta_1 - \alpha_1 \gamma_2 - \beta_1 \delta_2 \qquad (5b)$$

$$\mathcal{E} = \frac{1}{2}\left(\gamma_1^2 + \gamma_2^2 + \delta_1^2 + \delta_2^2\right) + \frac{1}{2}\left(\alpha_1 \beta_2 - \alpha_2 \beta_1\right)^2 \qquad (5c)$$

The self-consistency conditions of expansions (4a) and (4b) give the following equations

$$\gamma_1 = \dot{\alpha}_1 \qquad \gamma_2 = \dot{\alpha}_2 \qquad \delta_1 = \dot{\beta}_1 \qquad \delta_2 = \dot{\beta}_2 \qquad (6)$$

From the self-consistency conditions of expansions (4a, 4b) with Eqs.(

we obtain a set which using (6) can be written in the form:

$$\ddot{\alpha}_1 + \beta_2(\alpha_1\beta_2 - \alpha_2\beta_1) = 0$$

$$\ddot{\alpha}_2 - \beta_1(\alpha_1\beta_2 - \alpha_2\beta_1) = 0 \qquad ($$

$$\ddot{\beta}_1 - \alpha_2(\alpha_1\beta_2 - \alpha_2\beta_1) = 0$$

$$\ddot{\beta}_2 + \alpha_1(\alpha_1\beta_2 - \alpha_2\beta_1) = 0$$

The coupling condition (5a) and integrals (5b, 5c) take the form:

$$\alpha_1\dot{\beta}_1 + \alpha_2\dot{\beta}_2 = \beta_1\dot{\alpha}_1 + \beta_2\dot{\alpha}_2 \qquad (8$$

$$\mu = \alpha_2\dot{\alpha}_1 + \beta_2\dot{\beta}_1 - \alpha_1\dot{\alpha}_2 - \beta_1\dot{\beta}_2 \qquad (8'$$

$$\mathcal{E} = \frac{1}{2}\left(\dot{\alpha}_1^2 + \dot{\alpha}_2^2 + \dot{\beta}_1^2 + \dot{\beta}_2^2\right) + \frac{1}{2}\left(\alpha_1\beta_2 - \alpha_2\beta_1\right)^2 \qquad (8$$

The set (7) can be obtained from the variation of the following Lagrangia

$$\mathcal{L} = \frac{1}{2}\left(\dot{\alpha}_1^2 + \dot{\alpha}_2^2 + \dot{\beta}_1^2 + \dot{\beta}_2^2\right) - \frac{1}{2}\left(\alpha_1\beta_2 - \alpha_2\beta_1\right)^2 \qquad (9$$

Let us now pass on to the coordinates which diagonalize the potential ene

$$\alpha_1 = x_1 + x_2 \qquad \alpha_2 = x_3 + x_4$$

$$\beta_2 = x_1 - x_2 \qquad \beta_1 = x_3 - x_4$$

The Lagrangian will take the form:

$$\mathcal{L} = \frac{1}{2}\left(\dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2 + \dot{x}_4^2\right) - \frac{1}{4}\left(x_1^2 + x_4^2 - x_2^2 - x_3^2\right)^2$$

Making use of the fact that the kinetic part of the Lagrangian is invariant with respect to O(4) transformations group, and the potential part is invariant with respect to O(2) x O(2) group we introduce the parametrization

$$x_1 = r\sin\varphi \qquad x_2 = R\sin\theta$$

$$x_4 = r\cos\varphi \qquad x_3 = R\cos\theta$$

The coupling condition (8a) and the moment integral (8b) will take a simple form:

$$R^2\dot{\theta} - r^2\dot{\varphi} = 0$$

$$R^2\dot{\theta} + r^2\dot{\varphi} = \mu/2$$

After excluding $\dot{\theta}$ and $\dot{\varphi}$ the energy and Lagrangian will be

$$\mathcal{E} = \frac{1}{2}\left[\dot{r}^2 + \dot{R}^2 + \frac{\mu^2}{16}\left(\frac{1}{r^2} + \frac{1}{R^2}\right)\right] + \frac{1}{4}\left(r^2 - R^2\right)^2 \qquad (11a)$$

$$\mathcal{L} = \frac{1}{2}\left(\dot{r}^2 + \dot{R}^2\right) - \frac{\mu^2}{32}\left(\frac{1}{r^2} + \frac{1}{R^2}\right) - \frac{1}{4}\left(r^2 - R^2\right)^2 \qquad (11b)$$

Varying Lagrangian (11b) we get a set:

$$\ddot{r} = \frac{\mu^2}{16 r^3} - r\left(r^2 - R^2\right) \qquad (12a)$$

$$\ddot{R} = \frac{\mu^2}{16 R^3} + R\left(r^2 - R^2\right)$$

It should be noted that at $\mu = 0$ the set (12) changes into the set (1) (after denoting $x = r - R$; $y = r + R$) which was studied in Refs. [5, 6] . Making use of the results of Refs.[5, 6] one can claim: the moment canceling condition $\mu = 0$ is a sufficient condition of the set (2)

stochasticity.

The scale transformations

$$r \to (\mu/4)^{1/3} r\,; \quad R \to (\mu/4)^{1/3} R\,; \quad t \to (\mu/4)^{-1/3} t$$

bring the energy (11a) and Lagrangian (11b) to the form:

$$\mathcal{E} = \left(\frac{\mu}{4}\right)^{4/3}\left\{\frac{1}{2}\left(\dot{r}^2 + \dot{R}^2\right) + \frac{1}{2}\left(\frac{1}{r^2} + \frac{1}{R^2}\right) + \frac{1}{4}\left(r^2 - R^2\right)^2\right\} \qquad (13a)$$

$$\mathcal{L} = \frac{1}{2}\left(\dot{r}^2 + \dot{R}^2\right) - \frac{1}{2}\left(\frac{1}{r^2} + \frac{1}{R^2}\right) - \frac{1}{4}\left(r^2 - R^2\right)^2 \qquad (13b)$$

Motion equations are as follows:

$$\ddot{r} = \frac{1}{r^3} - r\left(r^2 - R^2\right)$$

$$\ddot{R} = \frac{1}{R^3} + R\left(r^2 - R^2\right) \qquad (14)$$

The question on stochasticity of the (14) set with the energy integral (13a) will be studied in a computer experiment.

4. In conclusion we shall discuss briefly the possibility of introducing vacuum charges.

To restore the stability of the set (1) solutions there was introduced the Higgs condensate in Ref.[7] . It seems more natural to introduce vacuum charge in order to restore the solutions stability. Since there are every reason now to assume that the Yang-Mills vacuum, besides gauge fields, includes fermion fields, too.

In the reference frame (where $K_\mu = (K_0;\ 0\ )$) we introduce vacuum current in the form of $J^a_\mu = (\rho^a;\ 0)$

From the continuity equation $\nabla^{ab}_\mu J^b_\mu$ it follows that $\dot{\rho}^a = 0$.

lot losing community we can choose ρ^a in the form:

$$\rho^a = \rho\, \epsilon^{abc} n_1^b n_2^c$$

Then integrals (10) will take the form:

$$r^2\dot{\varphi} + R^2\dot{\theta} = \mu/2$$

$$r^2\dot{\varphi} - R^2\dot{\theta} = \rho/2$$

The finite Lagrangian (after the scale transformations $r \to \left(\frac{\mu+\rho}{4}\right)^{1/3} r \ldots$) take the form of

$$\mathcal{L} = \frac{1}{2}\left(\dot{r}^2 + \dot{R}^2\right) - \frac{1}{2}\left(\frac{1}{r^2} + \frac{\lambda^2}{R^2}\right) - \frac{1}{4}\left(r^2 - R^2\right)^2$$

where

$$\lambda = \left(\frac{\mu-\rho}{\mu+\rho}\right)$$

If the computer experiment shows that the set (14) solutions (which corresponds to $\lambda = 1$ case) are stochastic, then it will be interesting to investigate at what value of $\lambda_{\kappa p}$ the stochasticity vanishes. The computing results will be published afterwards.

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