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DIAGRAM SUMMATION METHOD BY MEANS OF LEGENDRE TRANSFORMATION BY A SCALAR PARAMETER AND ITS APPLICATION TO AN ANHARMONIC OSCILLATOR

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МЕТОД СУММИРОВАНИЯ ДИАГРАММ С ПОМОЩЬЮ ПРЕОБРАЗОВАНИЯ ЛЕЖАНДРА ПО СКАЛЯРНОМУ ПАРАМЕТРУ И ЕГО ПРИМЕНЕНИЕ К АНГАРМОНИЧЕСКОМУ ОСЦИЛЛЯТОРУ

Проведено суммирование бесконечного ряда диаграмм. Обсуждается случай размерности $d = 1$ (ангармонический осциллятор). Показано, что предложенный метод суммирования позволяет на один порядок увеличить область применения по сравнению с обычной теорией возмущения.

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The summation of an infinite number of diagrams is carried out. The case of the dimension d = 1 (anharmonic oscillator) is discussed. It is shown that the proposed summation method allows to enlarge by an order the application region as compared to a standard perturbation theory.

Yerevan Physics Institute

Yerevan 1982

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1. Introduction

The progress of quantum electrodynamics (QED) is determined first of all by the fact that its interaction constant is small and perturbation theory may be applied here. In quantum chromodynamics (QCD) the weak-coupling expansion can be used only at small distances (asymptotic freedom) and the perturbation theory is applicable only in this region. At large distances the interaction between quarks can't be described by a weak-coupling expansion. Therefore, there arises a need in developing a calculation technique different from the standard perturbation theory.

There are many ways of improving the weak-coupling expansion for the interaction constants $\lambda \sim 1$ [1,2]. In this paper we propose a method of Feynman diagrams summation allowing to enlarge by an order the application region of the weak-coupling expansion. The essential feature of the method is that for diagram summation a finite number of diagrams by the number of lines [3] should be calculated in each order.

In sections 2 and 3 the diagrams summation up to the eighth order by the number of lines is carried out on the basis of the results obtained in [4] . In sec. 3 the obtained formulae are applied to an anharmonic oscillator. The obtained expressions for the ground-state energy $\mathcal{E}_o(\lambda)$ (3.7) and (3.12) in the region of constants $0 \leq \lambda \leq 1$ are in a satisfactory agreement with the exact results of [2] . The agreement is particularly good $\sim 10\%$ at $\lambda \lesssim 0.3$, whereas the weak-coupling expansion is applicable in the region $\lambda \lesssim 0.01$.

2. Summation method

Write the generating functional $Z(J, K, M, L)$ for the $g\varphi^4$-theory with the sources J, K, M, L [4] :

$$\exp\left\{\frac{i}{\hbar} Z(J,K,M,L)\right\} = N^{-1}\int D\varphi \cdot \exp\left\{\frac{i}{\hbar}\left[S(\varphi) + J(x)\varphi(x) + \right.\right.$$

$$\left.\left. + \frac{1}{2}K(x,y)\varphi(x)\varphi(y) + \frac{1}{3!}M(x,y,z)\varphi(x)\varphi(y)\varphi(z) + L\cdot S(\varphi)\right]\right\} \qquad (2.1)$$

where $S(\varphi)$ is the classical action of $g\varphi^4$ -theory,

$$S(\varphi) = \frac{1}{2} i D^{-1}(x,y)\, \varphi(x)\,\varphi(y) + \frac{1}{4!} g\varphi^4(x) \qquad (2.2)$$

Here and later, if there are no special reservations, we integrate over repeating arguments. N^{-1} is the normalization constant, and the operator iD^{-1} is defined as

$$iD^{-1}(x, y) = -(\Box + m^2)\,\delta^4(x-y)$$

Let us pass from the independent variables J, K, M, L to new independent variables $\varphi(x) \equiv Z_{J(x)}$; $i\,G(x,y) \equiv Z_{J(x)J(y)}$;

$i^2 H(x,y,z) \equiv Z_{J(x)J(y)J(z)}$; $\quad S = Z_L$

by means of the fourth order Legendre transformation [3] :

$$\Gamma(\varphi, G, H, S) = Z - J\varphi - \frac{1}{2}K\varphi\varphi - \frac{\hbar}{2}KG - \frac{1}{3!}M\varphi\varphi\varphi - \tag{2.3}$$
$$- \frac{\hbar}{3!}MG\varphi - \frac{\hbar^2}{3!}MH - L\cdot S$$

Here and later, if there is no special need, the integration variables are omitted.

Varying $\Gamma(\varphi, G, H, S)$ by the variables φ, G, H and S we obtain:

$$\Gamma_\varphi = -J - K\varphi - \frac{1}{2}M\varphi\varphi - \frac{\hbar}{3!}MG;$$
$$\Gamma_H = -\frac{\hbar^2}{3!}M; \quad \Gamma_G = -\frac{\hbar}{2}K - \frac{\hbar}{3!}M\varphi; \quad \Gamma_S = -L \tag{2.4}$$

The complete set of equations determining the generalized effective potential Γ (2.3) has been written in [4] and has the form

$$S - S(\varphi) - \frac{i\hbar}{2}\mathcal{D}^{-1}G - \frac{g\hbar}{4}G\varphi^2 -$$
$$- \frac{g\hbar^2}{8}G^2 - \frac{g\hbar^2}{3!}H\varphi = \frac{g}{4!}\left(\frac{\hbar}{i}\right)^3 Z_{JJJJ}, \tag{2.5}$$

$$(1-\Gamma_S)\left\{i\mathcal{D}^{-1}(x,y)\varphi(y) + \frac{g}{3!}\left[\varphi^3(x) + \frac{3\hbar}{i}\,i\,G(x,x)\varphi(x) + \right.\right.$$
$$\left.\left. + \left(\frac{\hbar}{i}\right)^2 i^2 H(x,x,x)\right]\right\} = \Gamma_{\varphi(x)} \tag{2.6}$$

$$\delta(x-y)=\frac{2i}{\hbar}\Gamma_{G(x,z)}G(z,y)+\frac{3i}{\hbar}\Gamma_{H}(x,z,t)\,H(z,t,y)+$$

$$+iQ_{\varphi(x)\varphi(z)}G(z,y)+iQ_{\varphi(x)}G(z,t)H(z,t,y)-$$

$$-Q_{\varphi(x)}H(z,t,u)\,Z_{J(z)J(t)J(u)J(y)} \qquad (2.7)$$

$$0=\frac{3i}{\hbar}\Gamma_{H(x,y,t)}G(t,z)+iQ_{G(x,y)\varphi(t)}G(t,z)+$$

$$+iQ_{G(x,y)G(t,u)}H(t,u,z)-Q_{G(x,y)H(t,u,v)}Z_{J(t)J(u)J(v)J(z)}, \qquad (2.8)$$

$$0=iQ_{H(x,y,z)\varphi(u)}G(u,t)+iQ_{H(x,y,z)G(u,v)}H(u,v,t)-$$

$$-Q_{H(x,y,z)H(u,v,\omega)}Z_{J(u)J(v)J(\omega)J(t)} \qquad (2.9)$$

where $$Q_{AB}=\Gamma_{AS}\Gamma^{-1}_{SS}\Gamma_{SB}-\Gamma_{AB} \qquad (2.10)$$

In the set of eqs. (2.5) - (2.9) the quantity Z_{JJJJ} is left, which should be expressed through derivatives of Γ by means of eq. (2.9) and substitute in eqs. (2.5) - (2.8). The set of equations (2.5) - (2.9) completely determines the functional Γ

From the Schwinger linear equation (2.6) follows that the general solution can be presented in the form $\Gamma=S+F$ where F depends on the invariant variable

$$\tilde{S}=S-S(\varphi)-\frac{i\hbar}{2}\mathcal{D}^{-1}G-\frac{g\hbar}{4}G\varphi^2-\frac{g\hbar^2}{8}G^2-\frac{g\hbar^2}{3!}H\varphi \qquad (2.11)$$

and the variables G and H (see [4]). Then eqs. (2.5), (2.7) - (2.9) will be written as

$$\tilde{S} = \frac{g}{4!}\left(\frac{\hbar}{i}\right)^3 Z_{JJJJ} \qquad (2.12)$$

$$1 = \frac{2i}{\hbar} F_G G + \frac{3i}{\hbar} F_H H - \frac{g\hbar^2}{3!} F' Z_{JJJJJ} \, , \qquad (2.13)$$

$$0 = \frac{3i}{\hbar} F_H G + \frac{ig\hbar^2}{4} F' H + i Q_{GG} H - Q_{GH} Z_{JJJJ} \qquad (2.14)$$

$$0 = \frac{ig\hbar^2}{3!} F' G + i Q_{GH} H - Q_{HH} Z_{JJJJ} \qquad (2.15)$$

where $Q_{\tilde{A}\tilde{B}} = F'_{\tilde{A}} F^{n-1} F'_{\tilde{B}} - \Gamma_{\tilde{A}\tilde{B}}$ (2.16)

and F' denotes the differentiation with respect to $\tilde{S}$ (2.11).

As has been mentioned in ref. [4] , the functional F , depends. in fact, on its variables in the combinations $\tilde{S}^2/G^4$ and H^2/G^3

$$F(\tilde{S},G,H) = \frac{\hbar}{2i} \operatorname{tr} \ln G + \Theta\left(\frac{\tilde{S}^2}{G^4} , \frac{H^2}{G^3}\right) \qquad (2.17)$$

(see eqs. (2.12) and (2.13)) and, hence, it is quite enough to single out from the set (2.12) - (2.15) an equation of the type $F_G = \dots$, where higher derivatives of F are on the right [3] . Multiplying (2.13) by G and (2.14) by H , subtracting them from each other, and then substituting from (2.15) and (2.12), we obtain the desired equation:

$$2F_G = \hbar G^{-1} H Q_{GG} H G^{-1} - \frac{3\hbar}{2} G^{-1} H Q_{\underline{HG}} \underline{H G^{-1} H} G^{-1} +$$

$$+ 2i\hbar G^{-1} \mathcal{Z}_{JJJJ} Q_{HG} H G^{-1} -$$

$$- \hbar G^{-1} \mathcal{Z}_{JJJJ} Q_{HH} \mathcal{Z}_{JJJJ} G^{-1} -$$

$$- \frac{3i\hbar}{2} G^{-1} H Q_{\underline{HH}} \underline{\mathcal{Z}_{JJJJ} G^{-1} H} G^{-1} \qquad (2.18)$$

The sign ⌊______⌋ denotes the integration (summation) over argument. In all other cases arguments integrate (sum) in that succession in which operators are written.

From eqs. (2.14) and (2.15) the $\mathcal{Z}_{JJJJ}$ is

$$\mathcal{Z}_{JJJJ} = -i \frac{4!\tilde{S}}{g\hbar^3} \cdot \frac{Q^{-1}G}{tr(Q^{-1}G)} - i\,tr(Q^{-1} Q_{HG} H) \left[\frac{Q^{-1}G}{tr(Q^{-1}G)} - \frac{Q^{-1} Q_{HG} H}{tr(Q^{-1} Q_{HG} H)} \right] \qquad (2.19)$$

where $Q^{-1}\,\bullet$ is defined as

$$Q_{H(x,p,q)H(u,v,\omega)} Q^{-1}_{(u,v,\omega|s,t,z)} = \frac{1}{6} \left[\delta(x-s)\delta(p-t)\delta(q-z) + \ldots \right] \qquad (2.20)$$

and

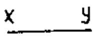
$$tr(Q^{-1}G) \equiv \int Q^{-1}_{(x,x,x|y,y,y)} G_{(x,y)}\, d^4_x\, d^4_y$$

$$tr(Q^{-1}Q_{HG}H) \equiv \int Q^{-1}_{(x,x,x/u,v,t)}\, Q_{H}(u,v,t)\, G(p,q)\, H_{(p,q,x)}\, d^4(x,u,v,t,p,q)$$

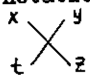
Eq. (2.18) is solved by the iteration method proposed in [3] . Its idea consists in the expansion of F in series of G^{-1} Let us write the expression for F up to the fifth order by G^{-1} inclusively:

$$F=\frac{\hbar}{2i}tr\,\ell_n G+\frac{i\hbar^2}{12}\ominus-\frac{i\hbar}{48}\left(\frac{4!\tilde{S}}{g\hbar^2}\right)^2\frac{1}{\ominus}-\frac{3i\tilde{S}}{g}\frac{\bigcirc\!\!\!\!\!\!\triangle}{\ominus} \qquad (2.21)$$

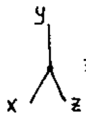
where the following notations are introduced:



$$\equiv G(x,y)$$



$$\equiv \delta(x-y)\,\delta(x-z)\,\delta(x-t)$$



$$\equiv H(\alpha,\beta,\gamma)\,G^{-1}(\alpha,x)\,G^{-1}(\beta,y)\,G^{-1}(\gamma,z)$$

If we conclude, proceeding from the theory symmetry properties, that in the ground state $\varphi_{vac}=H_{vac}=0$, then we can obtain another equation determining F' :

$$F'=-\frac{1}{\hbar}\left(\frac{4!}{g\hbar^2}\right)^2\frac{\tilde{S}}{4tr(GQ^{-1})} \qquad (2.22)$$

In the approximation (2.21), allowing for $\varphi_{vac}=H_{vac}=0$,

$$tr(GQ^{-1})=\frac{6i}{\hbar^2}\ominus(1+x) \qquad (2.23)$$

where $x=\frac{36}{g\hbar^2}\frac{\bigcirc\!\!\!\!\!\!\triangle}{\ominus^2}\tilde{S}$ and $\tilde{S}=S-\frac{i\hbar}{2}\mathcal{D}^{-1}G-\frac{g\hbar^2}{8}\infty$

and
$$F=\frac{i\hbar}{54}\frac{\ominus^3}{\bigcirc\!\!\!\!\!\!\triangle^2}\int\frac{x\,dx}{1+x} \qquad (2.24)$$

The integral (2.24) sums up an infinite number of diagrams of the type  and  . Such a summation is in many respects alike to the Hartree approximation where an infinite series of "bubble" diagrams is summed up, or to the random phase approximation ("rings" are summed up). In our case the introduction of the scalar parameter $\widetilde{S}$ (2.11) has allowed to sum up in a similar way an infinite series of diagrams of a definite type by means of (2.22). This is the basic idea of the diagram summation proposed in this paper.

The effective potential Γ after the integration of (2.24) will have the form

$$\Gamma = S + F = \frac{i\hbar}{2} \mathcal{D}^{-1} G + \frac{\hbar}{2i} \operatorname{tr} \ln G + \frac{g\hbar^2}{8} \bigcirc\!\!\bigcirc + \frac{g\hbar^2}{36} \frac{\ominus^2}{\bigtriangleup} x + \frac{i\hbar}{54} \frac{\ominus^3}{\bigtriangleup^2} \left\{ \varkappa + \ln(1+\varkappa) \right\} \tag{2.25}$$

In the ground state the quantities G_{vac} and S_{vac} are determined from eqs. $\Gamma_S = 0$ ($\Gamma_x = 0$), $\Gamma_G = 0$ (2.4), and the energy difference between the vacuum states of the full ($g \neq 0$) and free ($g = 0$) theories is determined by the formula [3]

$$-\mathcal{E}(g) \int dt = \Gamma \Big/_{\substack{S = S_{vac} \\ G = G_{vac}}} - \frac{i\hbar}{2} \operatorname{tr} \hat{1} - \frac{\hbar}{2i} \operatorname{tr} \ln \mathcal{D} \tag{2.26}$$

3. Anharmonic oscillator

Let us now discuss the case when the space dimension is $d = 1$ (anharmonic oscillator). The formula (2.25) derived in sec. 2 allows to calculate the ground-state energy (2.26) as a coupling constant function (in the approximation $G = \mathcal{D}$) which can be compared with the exact results presented in [2].

In the weak-coupling expansion the ground-state energy $\mathcal{E}(\pi)$, as a function of the interaction constant λ , agrees with the exact value $\mathcal{E}_{exact}$ to within 0.06% at $\lambda \sim 0.01$ [5]. With the increase of the interaction constant λ the accuracy gets worse with the increase of the approximation order (see fig. 1). For example, at $\lambda = 0.1$ in the third order by λ the accuracy is 1.86%, in the fourth order - 2.45% etc (the minus sign means that the point $\mathcal{E}^4$ (0.1) is lower than $\mathcal{E}^{exact}(0.1)$) *.

The proposed summation of diagrams allows to penetrate into the region of stronger constants.

Let us write the Lagrangian for the anharmonic oscillator in the following form:

$$\mathcal{L} = \frac{m\dot{x}^2}{2} - \frac{m\omega^2 x^2}{2} + \frac{g x^4}{4!} \qquad (3.1)$$

The green function $\mathcal{D}(t, t')$, corresponding to the line $\underline{t \qquad t'}$ in the formula (2.25) diagrams, is

$$\mathcal{D}(t,t') = \frac{1}{2m\omega}\left[\theta(t-t')\,e^{-i\omega(t-t')} + \theta(-t+t')\,e^{i\omega(t-t')}\right] \qquad (3.2)$$

* At $\lambda = 0.2$ these accuracies are 18.1% and -45.98%, respectively.

Using (3.2) it is not difficult to calculate all the types of graphs entering the expression for the ground-state energy (2.26)

$$\infty = \frac{\int dt}{(2m\omega)^2}\ ;\quad \ominus = \frac{\int dt}{(2i\omega)(2m\omega)^4}\ ;\quad \bigcirc\!\!\!\!\!\triangle = \frac{3}{2}\,\frac{\int dt}{(2i\omega)^2(2m\omega)^6} \tag{3.3}$$

From the stationary condition $\Gamma_x = 0$ we have

$$x_{vac} = -\frac{3g\hbar}{2i}\cdot\frac{\bigcirc\!\!\!\!\!\triangle}{\ominus}\cdot\frac{1}{1+\frac{3g\hbar}{2i}\cdot\frac{\bigcirc\!\!\!\!\!\triangle}{\ominus}} \tag{3.4}$$

or, taking into account (3.3)

$$x_{vac} = -\frac{27\lambda}{4}\cdot\frac{1}{1+\frac{27\lambda}{4}} \tag{3.5}$$

where a convenient dimensionless parameter is introduced

$$\lambda = -\frac{g\hbar}{4!\,m^2\omega^3} \tag{3.6}$$

The energy difference between the full ($\lambda \neq 0$) and free ($\lambda = 0$) theories (2.26) (see (2.25) and (3.5)) is given by the formula

$$\frac{\tilde{\mathcal{E}}^{(3)}(\lambda)}{\hbar\omega} = \frac{23}{36}\lambda + \frac{4}{243}\ln\left(1+\frac{27}{4}\lambda\right) \tag{3.7}$$

As is seen from fig. 1, though we have used at calculations diagrams not higher than the third order by λ, the accuracy we have obtained is much better in the large region ($\lambda \leq 1$) as compared with the third order of perturbation theory (see curves 2 and 4 in fig. 1). The table with the results of the comparison of our summation and the third order of the standard perturbation theory with exact values [2] is given below.

The values $\tilde{\mathcal{E}}^{(3)} = \frac{1}{2} + \frac{\tilde{\mathcal{E}}^{(3)}(\lambda)}{\hbar\omega}$ presented in Table 1, indicate that the formula (3.7) may be applied in a wider region over λ than the standard perturbation theory.

We have summed also the diagrams of the fourth order over λ. For that purpose we have had to calculate F (2.21) up to the seventh order over G^{-1}. In this approximation the relation (2.24) is replaced by

$F = \frac{i\hbar}{54}$ ( $)^3$ / ( $)^2$ $\int \frac{\varkappa \, d\varkappa}{1+\varkappa+A\varkappa^2}$ (3.8)

where

$A = -1 + \frac{1}{3}$  / ( $)^2$ $[$  $+ 4$  $]$ (3.9)

Here is the expression for diagrams of the fourth order, contained in (3.9) for an anharmonic oscillator (3.1):

 $= \frac{5}{2} \frac{\int dt}{(2i\omega)^3 (2m\omega)^8}$;  $= \frac{7}{3} \frac{\int dt}{(2i\omega)^3 (2m\omega)^8}$ (3.10)

$$\varkappa_{vac} = -\frac{81}{122}\left\{1 + \frac{4}{27\lambda} - \sqrt{\left(1+\frac{4}{27\lambda}\right)^2 - \frac{244}{81}}\right\} \quad (3.11)$$

$$\frac{\widetilde{\mathcal{E}}^{(4)}(\lambda)}{\hbar\omega} = \frac{3}{4}\lambda + \frac{1}{9}\lambda\cdot x_{vac} + \frac{2}{183}\left\{ \ln\left(-\frac{4}{27\lambda}\cdot X_{vac}\right) - \right.$$

$$\left. - \frac{18}{\sqrt{163}}\, \operatorname{arctg}\left(\frac{\sqrt{163}}{9}\cdot\frac{x_{vac}}{x_{vac}+2}\right)\right\} \qquad (3.12)$$

It is seen from (3.11) that the stationary point in this order exists for $\lambda \lesssim 0.201$ only. However, in any odd order over λ a polynom of the odd power over x will be present in denominator (3.8), and, hence, x_{vac} will exist in any values of λ.

In fig. 2 the dependence $\widetilde{\mathcal{E}}^{(4)} = \frac{1}{2} + \frac{\widetilde{\mathcal{E}}^{(4)}(\lambda)}{\hbar\omega}$ (3.2) is presented. Note that at $\lambda = 1$ $\frac{\Delta\widetilde{\mathcal{E}}^{(4)}}{\mathcal{E}_{exact}}$ 100% $\simeq$ 2.3% and at $\lambda = 0.2$ the accuracy is 6.1% whereas the corresponding accuracies in the fourth order of the standard perturbation theory are -2.5% and -46.0%.

In the standard perturbation theory an oscillation of curves occurs at transition from order to order, i.e., curves are now above, now below the exact curve. And each successive order gives a worse accuracy than the previous already at $\lambda \simeq 0.1$.

It is seen from fig. 2 that the summation proposed in this paper allows to get rid of curves oscillations and improves the accuracy at transitions to higher orders. The detailed proof of these two facts for higher orders will be published elsewhere. It should be also noted that

$$\widetilde{\mathcal{E}}^{n}(\lambda) \sim a\lambda + b\ln\lambda \qquad \text{when } \lambda \to \infty$$

Such a summation method may be applied in non-linear field theories and in problems of statistical physics.

In conclusion the authors wish to thank S.G.Matinyan and O.M.Khudaverdyan for useful discussions.

Table 1

$\mathcal{E}(\lambda)$ \ λ	0.1	0.2	0.3	0.4	0.5	0.6	1
$\mathcal{E}_{exact}$	0.5591	0.6024	0.6380	0.6688	0.6962	0.7210	0.8038
$\mathcal{E}^{(3)}_{w.c.exp.}$	0.5696	0.7115	1.0507	1.712	2.8203	4.5005	19.4375
$\frac{\Delta\mathcal{E}^{(3)}}{\mathcal{E}_{exact}}\cdot 100\%$	1.9%	18.1%	64.7%	156.0%	305.1%	524.2%	2318.3%
$\tilde{\mathcal{E}}^{(3)}$	0.5724	0.6418	0.7099	0.7771	0.8437	0.9100	1.1726
$\frac{\Delta\tilde{\mathcal{E}}^{(3)}}{\mathcal{E}_{exact}}\cdot 100\%$	2.4%	6.5%	11.2%	16.2%	21.2%	26.2%	45.9%

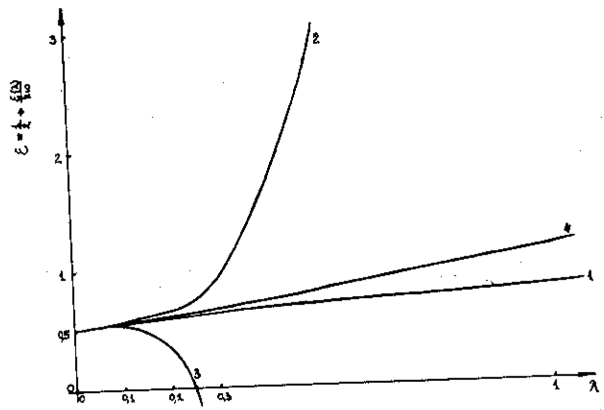


Fig. 1 Graphs of the dependence of ground-state energy on 1 is the exact curve [2] ; 2 and 3 are the third and fourth orders of the standard perturbation theory, respectively; 4 is the curve corresponding to the formula (3.7).

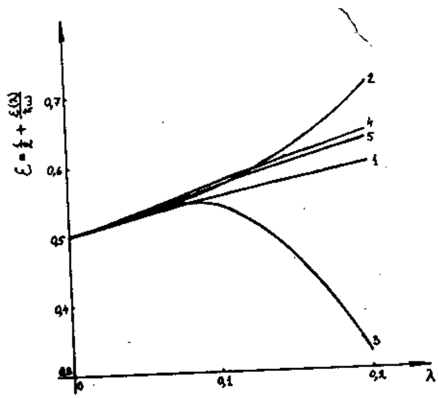


Fig. 2 Curves 1,2,3 and 4 correspond to the same curves as in fig. 1; curve 5 corresponds to the formula (3.12).

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МЕТОД СУММИРОВАНИЯ ДИАГРАММ С ПОМОЩЬЮ ПРЕОБРАЗОВАНИЯ ЛЕЖАНДРА ПО СКАЛЯРНОМУ ПАРАМЕТРУ И ЕГО ПРИМЕНЕНИЕ К АНГАРМОНИЧЕСКОМУ ОСЦИЛЛЯТОРУ

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