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SUPERSYMMETRIC DIMENSIONAL REGULARIZATION

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Эд.Ш.ЕГОРЯН

СУПЕРСИММЕТРИЧНАЯ РАЗМЕРНАЯ РЕГУЛЯРИЗАЦИЯ

В работе предлагается модифицированная схема размерной регуляризации, которая сохраняет суперсимметрию. Обсуждается применение этой схемы к двум моделям, обладающим расширенной суперсимметрией.

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SUPERSYMMETRIC DIMENSIONAL REGULARIZATION

A modified scheme of dimensional regularization that preserves supersymmetry is suggested. The application of the scheme to two models with extended supersymmetry is discussed.

Yerevan Physics Institute

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SUPERSYMMETRIC DIMENSIONAL REGULARIZATION

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A dimensional regularization supersymmetric scheme applicable to theories without γ_5 has been recently suggested [1] . In the work presented here this scheme is generalized so that it becomes applicable to all supersymmetric theories. Its application to two models with extended supersymmetry [2] is discussed.

When deriving supersymmetric Slavnov-Word identities in a functional integral for a generating functional $Z(J)$ a supersymmetric field shift is done. This generates terms of the type $f^{abc}\bar{\alpha}\gamma_\mu \lambda^a \bar{\lambda}^b \gamma_\mu \lambda^c$ which may break up supersymmetry [3] . At a naive level (i.e. without regularization) in a four-dimensional space one can prove with the help of Firtz transformations that these terms are zero. To preserve the Slavnov-Word naive identities the regularization must be such that this term could give no contribution into the generating functional $Z(J)$. It is of great importance for this to preserve the γ_μ-matrix indices μ of this term four-dimensional. This actually provides the equality of boson and fermion degrees of freedom and, as a result of that, supersymmetry in the regularized theory, too.

Formulate the regularization scheme.

The generating functional Z is defined by the functional integral

$$Z = \int \mathcal{D}\varphi \mathcal{D}\lambda \exp i \int dx \left(\mathcal{L} + \mathfrak{J}\varphi + \bar{\xi}\lambda + \bar{\lambda}\xi \right) \tag{1}$$

where $\mathcal{L}$ is a Lagrangian, φ , λ are the set of boson and fermion fields, $\mathfrak{J}$, ξ are boson and fermion sources. We shall give meaning to each term of expansion Z in sources in momentum representation, i.e. to each mean term of the type

$$\langle \mathfrak{J}\varphi(P_1) \ldots \mathfrak{J}\varphi(P_n)\, \bar{\xi}\lambda(K_1) \ldots \bar{\xi}\lambda(K_n)\, \bar{\lambda}(q_1)\xi \ldots \bar{\lambda}(q_n)\xi \rangle \tag{2}$$

The mean term (2) is a set of Feynman diagrams, and by the Feynman rules some mathematical expression corresponds to it. These expressions contain contractions of the type

$$\gamma_\mu \hat{\ell}_1 \ldots \hat{\ell}_\kappa \gamma^\mu \quad , \quad \delta^\mu_\mu \tag{3}$$

1. Carry out the contractions of the type (3) in a 4-dimensional space.

2. Take traces also in four dimension. This will cause new contractions of the type (3) as well as products of antisymmetric tensors $\epsilon^{\mu\nu\alpha\beta}$ The antisymmetric tensors products we shall replace in accord with the well-known formula by the determinant of the matrix constructed by tensors $g^{\mu\nu}$. Then carry out more new contractions also in four dimension. Note that such a technique removes the problem of determining the n -dimensional tensor $\epsilon^{\mu\nu\alpha\beta}$ and the difficulties connected with it [3] .

3. Now we pass on to the n-dimensional space, i.e. we consider all the momenta and indices ν of matrices γ_ν n -dimensional ones, and integrate over internal momenta in the n -th dimension. After the integration new contractions of type (3) may arise. They must be contracted already in the n -th dimension by the following formulae:

$$\{\gamma_\mu \gamma_\nu\} = 2g_{\mu\nu}$$
$$\delta^\mu_\mu = \gamma_\mu \gamma^\mu = n = 4 - 2\varepsilon \tag{4}$$

For example, after the procedures 1 and 2 there may arise the integral of the type:

$$\int \frac{\hat{\ell}\hat{K}\hat{\ell}}{\ell^2(\ell-K)^2} d^n\ell = \int \frac{\gamma_i \hat{K} \gamma_j \ell_i \ell_j}{\ell^2(\ell-K)^2} d^n\ell \tag{5}$$

The result of the integration contains a term of the type $\gamma_i \hat{K} \gamma^i$ and to solve (5) unambiguously one should contract over index i according to (4), i.e. in the n -th dimension.

4. After that all the external momenta, sources and γ -matrices we put in the four-dimensional space.

Thus we obtain an unambiguous answer for each mean term of the type (2) and hence $Z(\mathcal{J})$ within the framework of the perturbation theory is regularized.

Now we consider two models [2] with extended supersymmetry and prove that the above scheme provides their supersymmetric regularization. The Lagrangian of these models has the form

$$\mathcal{L}_D = -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \frac{i}{2} \bar{\lambda}^a \Gamma_\mu (D^\mu \lambda)^a \tag{6}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$
$$(D_\mu \lambda)^a = \partial_\mu \lambda^a - g f^{abc} A^b_\mu \lambda^c$$

index D indicates the space dimension (D = 6.10), in which Lagrangian (6) is considered. Γ_μ are Dirac matrices in the D-th dimension f^{abc} are structure constants of the gauge group. If at D = 6 one imposes on λ the Weyl condition

$$(1-\Gamma_7)\lambda = 0$$

where $\Gamma_7 = \Gamma_0 \ldots \Gamma_5$, then $\mathcal{L}_6$ is invariant under the following supertransformations

$$\delta A_\mu = \frac{1}{2}(\bar{\alpha}\,\Gamma_\mu\,\chi - \bar{\chi}\,\Gamma_\mu\,\alpha) \tag{7}$$

$$\delta\lambda = \frac{1}{2}\,\sigma_{\mu\nu}\,F^{\mu\nu}\alpha$$

At D = 10 the condition of Weyl and Mayorana is imposed on λ :

$$(1-\Gamma_{11})\,\lambda = 0$$

$$\lambda = C\,\bar{\lambda}^T ,$$

$$\Gamma_{11} = \Gamma_0 \cdots \Gamma_9$$

Then $\mathcal{L}_{10}$ is invariant under the transformations

$$\delta A_\mu = i\bar{\alpha}\,\Gamma_\mu\,\lambda \tag{8}$$

$$\delta\lambda = \frac{1}{2}\,\sigma_{\mu\nu}\,F^{\mu\nu}\alpha$$

The 4-dimensional theory is obtained from (6) after the dimensional reduction which consists in the assumption that all the fields depend on X_i (i = 0, ..., 3) only and are independent of the rest of coordinates X_α (α = 4, ...,D -1). The 4-dimensional Lagrangian is derived from (6) by the replacement:

$$\mu \to (i, \alpha)$$

$$\partial_\alpha = 0 \tag{9}$$

$$i = 0, \dots, 3; \quad \alpha = 4, \dots, D-1$$

It has the form:

$$\mathcal{L}_D = -\frac{1}{4} F^a_{ij} F^{aij} - \frac{1}{2}(D_i A_i^{a\alpha})^2 + \frac{i}{2} \bar{\lambda}^a \Gamma_i (D^i \lambda)^{a} + \tag{10}$$

$$+ \frac{1}{4}(f^{abc} A^{b\alpha} A^{c\beta})^2 + \frac{ig}{2} f^{abc} \bar{\lambda}^a \Gamma_\alpha A^{b\alpha} \lambda^c$$

Supersymmetric Lagrangian transformations (10) are derived from (7) and (9) with the help of the same replacement (9). Further on, using the explicit representation of matrices Γ solving also the additional conditions imposed on λ the Lagrangian (10) can be rewritten in the terms of four-component spinors and γ_μ matrices [2]. After that the theory is regularized in accord with the above scheme. But one may act differently. One may operate directly by Lagrangian (10) (i.e. in supermultiplet representation) and reformulate the regularization scheme as applied to the supermultiplet representation of the theory. The regularization scheme will differ from the above-described one only by the fact that instead of the former four-dimensional contractions the following ones will arise:

$$\Gamma_\mu \hat{\ell}_1 \dots \hat{\ell}_\kappa \Gamma^\mu, \quad \delta_\mu^\mu, \tag{11}$$

$$\mu = i, \alpha$$

instead of the four-dimensional antisymmetric tensors D-dimensional ones arise and their products are replaced as before by the determinant from the matrix formed of tensors g^{ij}, $g^{\alpha\beta}$. All the contractions of (11) type are carried out by the formulae:

$$
\begin{aligned}
&\{\Gamma_\mu \Gamma_\nu\} = 2g_{\mu\nu} \\
&\Gamma_i \Gamma^i = \delta_i^{\,i} = 4 \\
&\delta_\alpha^\alpha = \Gamma_\alpha \Gamma^\alpha = D-4 \\
&\mu = i, \alpha\ ; \ \nu = j, \beta \\
&i, j = 0, \ldots, 3;\ \alpha, \beta = 4, \ldots, D-1
\end{aligned}
\tag{12}
$$

Then, according to item 3, all the integrals over internal momenta are to be taken. The occurred after the integration contractions are carried out already in accord with the algebra

$$
\begin{aligned}
&\{\Gamma_\mu \Gamma_\nu\} = 2g_{\mu\nu} \\
&\Gamma_i \Gamma^i = \delta_i^{\,i} = n = 4-2\varepsilon \\
&\Gamma_\alpha \Gamma^\alpha = \delta_\alpha^\alpha = D-4
\end{aligned}
$$

After that in all the formulae the n-dimensional indices, momenta we consider 4-dimensional ones again (item 4) and finish with that the procedure of the theory regularization in the supermultiplet representation.

Now we shall prove the regularization superinvariance. After the supershift in the functional integral for Z

$$
\begin{aligned}
&A_\mu \to A_\mu + \delta A_\mu \\
&\lambda \to \lambda + \delta\lambda
\end{aligned}
$$

where δA_μ , $\delta\lambda$ are determined by formulae (7), (8), in the integrand exponential the term will arise (both for $D = 6$ and $D = 10$)

$$
\int f^{abc} \bar{\alpha}\, \Gamma_\mu \lambda^a(x)\, \bar{\lambda}^b(x)\, \Gamma_\mu \lambda^c(x)\, dx \tag{14}
$$

Prove that within our regularization (14) does not contribute in Z .

Owing to the fact that in our scheme μ remains actually D -dimensional we avoid the difficulty mentioned in Ref.[4] .

The proof is common both for $D = 6$ and $D = 10$.

Let us introduce the quantity $\Pi^{abc}_{\alpha\beta\gamma}(J)$:

$$\Pi^{abc}_{\alpha\beta\gamma}(J) = \int d^nK_1 d^nK_2 d^nK_3 \delta(K_1+K_2+K_3) < \lambda^a_\alpha(K_1)\bar{\lambda}^b_\beta(K_2)\lambda^c_\gamma(K_3)> \tag{15}$$

This quantity is calculated unambiguously within the framework of the above regularization scheme. In (15) J is the generalized designation of all the external sources. Π^{abc} , evidently satisfies the relations:

$$\begin{aligned}
&\Pi^{abc}_{\alpha\beta\gamma} = -\Pi^{cba}_{\gamma\beta\alpha} \\
&\Pi^{abc}_{\alpha\beta\gamma} = -C_{\alpha\alpha'}C^{-1}_{\beta\beta'}\,\Pi^{bac}_{\beta'\alpha'\gamma} \\
&\tfrac{1}{2}(1+\Gamma_{D+1})_{\alpha\delta}\,\Pi^{abc}_{\delta\beta\gamma} = \Pi^{abc}_{\alpha\beta\gamma} \\
&\tfrac{1}{2}(1-\Gamma_{D+1})_{\beta\delta}\,\Pi^{abc}_{\alpha\delta\gamma} = \Pi^{abc}_{\alpha\beta\gamma}
\end{aligned} \tag{16}$$

where the chirality of fields λ is taken into account.

The contribution of (14) to Z is the sum of expressions of the type:

$$A \equiv (\bar{\alpha}\Gamma_\mu)_\alpha (\Gamma_\mu)_{\beta\gamma}\, f^{abc}\, \Pi^{abc}_{\alpha\beta\gamma} \tag{17}$$

In our scheme μ remains D -dimensional (and not $D-2\varepsilon$), and α , β , γ take $2^{D/2}$ values, therefore we can use Firtz transformation:

$$\Pi^{abc}_{\alpha\beta\gamma} = \sigma^i_{\alpha\beta}\left(\Pi^{abc}_{\alpha'\beta'\gamma'}\ \sigma^i_{\beta'\alpha'}\right)/\sigma^i\sigma_i \tag{18}$$

where σ_i is the Firtz basis in D dimension. Using (12) we rewrite A in the form:

$$A = (\bar{\alpha}\Gamma_\mu \sigma^i \Gamma^\mu)_\gamma f^{abc} (\Pi^{abc}_{\alpha'\beta'\gamma}\ \sigma^i_{\beta'\alpha'}) / \sigma^i \sigma_i \tag{19}$$

From Eqs.(16) we obtain

$$\Pi^{abc}_{\alpha'\beta'\gamma}\ \sigma^i_{\beta'\alpha'} = \Pi^{abc}_{\alpha\beta\gamma} \frac{1}{2}(1-\Gamma)_{\beta\beta'}\ \sigma^i_{\beta\beta'}\ \frac{1}{2}(1+\Gamma)_{\alpha'\alpha}$$

whence it follows that σ^i can be composed of only odd number of matrices Γ. σ^i is antisymmetric combination of Γ matrices and i denotes the number of indices in σ^i. For example,

$$\sigma^3 = \Gamma^{[\mu}\Gamma^\nu\Gamma^{\lambda]}$$

Thus in (19) i takes only odd values: $i = 1, \ldots D/2, \ldots D-1$ ($D = 6, 10$).

Taking into account the formulae

$$\Gamma_\mu \sigma^i \Gamma^\mu = (-1)^i (D-2i)\sigma^i \tag{20}$$

$$\sigma^{\mu_1\ldots\mu_K} = a\,\epsilon^{\mu_1\ldots\mu_K\mu_{K+1}\ldots\mu_D}\ \sigma_{\mu_{K+1}\ldots\mu_D}\Gamma_{D+1}$$

where a is some constant, one can readily prove that for $D = 6$ the contribution in (19) from σ^5 is equal to that from σ^1, and the contribution from σ^3 is zero. For $D = 10$ the contribution from σ^9 is equal to that from σ^1, the contribution from σ^7 to that of σ^3, and the contribution from σ^5 is zero. Taking into account all the mentioned above and making use of the second formula of (16) we obtain:

$$A = -\frac{1}{2} A \qquad \text{for} \quad D = 10$$

$$A = - A \qquad \text{for} \quad D = 6$$

Thus $A = 0$ in the models $\mathcal{L}_6$ and $\mathcal{L}_{10}$ and hence we have proved that after a supersymmetric shift of fields in the functional integral for the generating functional $Z(J)$ we obtain $Z(J,\alpha)$ coinciding in form with $Z(J,\alpha)$ received by a shift from nonregularized functional integral. This means that the naive supersymmetric identities of Slavnov [5] hold at a regularized level, and the above-described regularization scheme preserves supersymmetry.

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СУПЕРСИММЕТРИЧНАЯ РАЗМЕРНАЯ РЕГУЛЯРИЗАЦИЯ

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