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H.M.ASATRYAN, G.K.SAVVIDY

CONFIGURATION MANIFOLD OF YANG-MILLS CLASSICAL MECHANICS

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Г.М.АСАТРЯН, Г.К.САВВИДИ

КОНФИГУРАЦИОННОЕ МНОГООБРАЗИЕ КЛАССИЧЕСКОЙ МЕХАНИКИ ЯНГА-МИЛЛСА

Исследуется конфигурационное многообразие классической механики Янга-Миллса. Введены удобные переменные, соответствующие явному выделению "вращательных" степеней свободы пространственной и внутренней групп симметрии. Получены лагранжианы для невращательных степеней свободы.

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H.M.ASATRYAN, G.K.SAVVIDY

CONFIGURATION MANIFOLD OF YANG-MILLS CLASSICAL MECHANICS

Configuration manifold of the Yang-Mills classical mechanics is investigated. Convenient parameters are introduced corresponding to obvious separation of rotational degrees of freedom of space and internal symmetry groups. Effective Lagrangians for nonrotational degrees of freedom are obtained.

Yerevan Physics Institute

Yerevan 1982

Y E R E V A N   P H Y S I C S   I N S T I T U T E

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In [1-3] classical Yang-Mills equations are investigated in the case when the vector potential depends on the time only. The system described by such a potential $A_i^\alpha(t)$ in the gauge $A_0^\alpha = 0$ is reduced to a nonlinear mechanical system (Yang-Mills classical mechanics) with the Hamiltonian [1-3]

$$H_{YM} = \frac{1}{2}\dot{A}_i^\alpha \dot{A}_i^\alpha + \frac{g^2}{4} f^{\alpha\beta\gamma} f^{\alpha\delta\nu} A_i^\beta A_j^\gamma A_i^\delta A_j^\nu \qquad (1)$$

where $i, j, K = 1, 2, 3$, and $\alpha, \beta, \gamma = 1, \ldots N^2-1$ for the gauge group SU(N) ($f^{\alpha\beta\gamma}$ are the group structure constants). The Hamiltonian (1) is invariant with respect to global transformation group SU(N)⊗O(3), due to which are conserved the angular momenta [1,2]

$$m_i = \varepsilon_{ijK} A_j^\alpha \dot{A}_K^\alpha \qquad (2)$$

$$n^\alpha = f^{\alpha\beta\gamma} A_i^\beta \dot{A}_i^\gamma \qquad (3)$$

eq. (3) coinciding with coupling equations at the presence of a constant external density of colour charge.

As is shown in [1] in subsystems of (1) with m_i and n^α being zero, there occurs a strong local unstability of trajectories leading to the nonintegrability and stochasticity of the Yang-Mills system.

We believe that the investigation of the Yang-Mills classical mechanics may shed light on the problem of confinement [4-6] and vacuum structure in QCD [7] therefore, it is very important to consider the general situation when both m_i and n^α differ from zero.

The configuration manifold of the Yang-Mills system (1) is the direct product $E_{2N^2-5} \otimes R_{SU(N)} \otimes R_{O(3)}$ where E is the Euclidean space, R is the space of the adjoined representation of the appropriate group. In this paper convenient parameters are introduced which obviously separate degrees of freedom of appropriate spaces.

First consider the group SU(2). One may always present the real matrix $A_{\alpha i} \equiv A_i^\alpha$ (i , α = 1,1,3) in the form

$$A = O_1 E O_2^T \tag{4}$$

where

$$E = \begin{pmatrix} x(t) & & \\ & y(t) & \\ & & z(t) \end{pmatrix}, \tag{5}$$

O_1 and O_2 are orthogonal matrices which may be written via time-dependent Euler parameters. The Hamiltonian (1) in the representation (4) has the form

$$H_{YM} = \frac{1}{2} S_p(\dot{E}\dot{E}) + \frac{g^2}{4}\left[S_p^2(EE) - S_p E^4\right] + T_{YM} \tag{6a}$$

$$T_{YM} = \frac{1}{2}\left\{S_p(\dot{O}_1 E^2 O_1^T) + S_p(E \dot{O}_2^T O_2 E) + 2S_p(\dot{O}_1 E O_2^T \dot{O}_2 E O_1^T)\right\} \tag{6b}$$

Let's introduce antisymmetrical matrices Ω and ω

$$\omega = O_1^T \dot{O}_1 = - \dot{O}_1^T O_1$$
$$\Omega = O_2^T \dot{O}_2 = - \dot{O}_2^T O_2 \qquad (7)$$

allowing to transform (6b) into

$$T_{YM} = -\frac{1}{2}\left\{ Sp(\omega^2 E^2) + Sp(\Omega^2 E^2) - 2 Sp(\omega E \Omega E) \right\} . \qquad (8)$$

and then calculate the trace

$$H_{YM} = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + T_{YM} + \qquad (9a)$$
$$+ \frac{g^2}{2}(x^2y^2 + y^2z^2 + z^2x^2)$$

$$T_{YM} = \frac{1}{2}\sum_{a=1}^{3}\left\{ I_a(\omega_a^2 + \Omega_a^2) - 2 J_a \omega_a \Omega_a \right\} \qquad (9b)$$

where

$$\omega_\alpha = \frac{1}{2}\varepsilon_{\alpha\beta\gamma}\omega_{\beta\gamma} , \quad \Omega_i = \frac{1}{2}\varepsilon_{ijk}\Omega_{jk} \qquad (10)$$

and

$$I_1 = y^2 + z^2 , \quad I_2 = x^2 + z^2 , \quad I_3 = x^2 + y^2$$
$$J_1 = 2yz , \quad J_2 = 2xz , \quad J_3 = 2xy \qquad (11)$$

In [1] , when investigating the system (9a) with $\omega_a = \Omega_a = 0$ $(T_{YM} = 0)$, the authors have used the analogy with the point mechanics with the coordinates x, y, z (E_3-space). Now, when $T_{YM} \neq 0$ the following analogy with the solid body mechanics is relevant. In some sence in (9b) we have a "gauge" body with time-dependent "inertia" moments (11) which rotates in the usual and internal spaces. Continuing this analogy, we see that (9b) is written in the "moving" coordinate system connected with the "body". Projections of conserved angular momenta m_i, n^α (2), (3) on this moving system may be obtained by substituting (4) in (2) and (3)

$$n^\alpha = O_1^{\alpha\beta} N^\beta , \qquad m_i = O_2\, ij\, M_j \tag{12}$$

$$N^\beta = I_\beta \omega_\beta - \mathcal{J}_\beta \Omega_\beta , \quad M_K = I_K \cdot \Omega_K - \mathcal{J}_K \omega_K \tag{13}$$

Differentiating (12) with respect to the time ($\dot{n}^\alpha = m_i = 0$) we shall obtain the equation for angular momenta in the moving coordinate system:

$$\frac{d\vec{N}}{dt} - [\vec{N}\vec{\omega}] = 0, \quad \frac{d\vec{M}}{dt} - [\vec{M}\vec{\Omega}] = 0 \tag{14}$$

These equations are similar to the Euler equations of the solid body classical mechanics.

Thus, in the expressions (9), (11), (13) and (14) is accomplished an obvious separation of variables corresponding to spatial and rotational degrees of freedom.

One may generalize the scheme described above to an arbitrary group SU(N). In that case, as for the group SU(2), the potential A_i^α is presented in the form (4) where $O_1^{\alpha\beta}$ ($\alpha, \beta = 1, \ldots$ $N^2 - 1$) is the real matrix of the adjoined representation of the group SU(N) and depends on $N^2 - 1$ parameters, and the matrix E has the form

$$E = \begin{pmatrix} x & & & \\ & y & & \\ & & z & \\ & & & 0 \end{pmatrix} \tag{15}$$

It may be shown that the obvious separation of variables into spatial and rotational degrees of freedom is possible only in this case. The kinetic energy (9b) will now be rewritten as

$$T_{YM} = \frac{1}{2}\left\{\sum_{i=1}^{3} I_i \Omega_i^2 + \sum_{\alpha=1}^{N^2-1} K_\alpha \omega_\alpha^2 - 2\sum_{a=1}^{3} J_a \Omega_a \omega_a\right\} \quad (16)$$

where

$$\delta_{\alpha\mu} K_\alpha = f_{\alpha\beta\gamma} f_{\mu\nu\gamma} E^{\beta i} E^{\nu i} \quad (17)$$

The first three components of K_α (17) coincide with the three components of I_K , the remaining components of the momentum (13) being defined as

$$N_a = K_a \omega_a \left(a = 4, \ldots N^2 - 1\right). \quad (18)$$

Instead of the first equation in (14) we obtain

$$\frac{dN_\alpha}{dt} + f_{\alpha\beta\gamma}\omega_\beta N_\gamma = 0 \quad (19)$$

To investigate the statistical properties of the Yang-Mills classical mechanics [1,2], it is desirable to have an effective Hamiltonian written via variables X, y, z , therefore, one should integrate eq.(14) and exclude $\vec{\omega}$ and $\vec{\Omega}$ from (9b). Two integrals of eq.(14) are known

$$N^2 = N^\beta N^\beta, \qquad M^2 = M_i M_i . \quad (20)$$

The difficulty of a complete exclusion of variables $\vec{\omega}$ and $\vec{\Omega}$ from (9b) lies in the fact that the inertia moments I, J (11) depend on time, therefore, the kinetic energy (9b) is not conserved:

$$\frac{dT_{YM}}{dt} = -\frac{1}{2}\sum_{a=1}^{3}\left\{\dot{I}_a\left(\Omega_a^2 + \omega_a^2\right) - 2\dot{J}_a \Omega_a \omega_a\right\} \quad (21)$$

For vacuum fields (i.e. when the density of the quark charge

$N^{\alpha} = 0$) it follows from (13) that

$$\omega_a = \frac{J_a}{I_a} \, \Omega_a , \qquad M_K = \frac{I_K^2 - J_K^2}{I_K} \, \Omega_K \tag{22}$$

and the kinetic energy (9b) is

$$T_{YM} = \sum_{K=1}^{3} \frac{I_K^2 - J_K^2}{2 I_K} \, \Omega_K^2 \tag{23}$$

The angular velocities $\vec{\omega}$ and $\vec{\Omega}$ may be excluded from (9) if $\vec{M}$ is colinear to $\vec{N}$ and their direction coincides with one of the main axes.

Consider two cases. Let $N_2 = N_3 = 0$, then from (13) and (14) we have

$$\begin{aligned} &\dot{N}_1 = 0 , \quad \dot{M}_1 = 0 , \quad M_2 = M_3 = 0 , \\ &\omega_2 = \omega_2 = \Omega_2 = \Omega_3 = 0 , \end{aligned} \tag{24}$$

hence, the angular velocities ω_1, Ω_1 may be expressed via conserving moments N_1, M_1 , and the effective Hamiltonian, depending on x, y, z only, may be obtained:

$$T_{YM} = \frac{1}{2} \, \frac{(M_1^2 + N_1^2)(y^2 + z^2) + 4 M_1 N_1 y z}{(y^2 - z^2)^2} , \tag{25}$$

at $y = z$ it follows from (13) that $M_1 = -N_1$ and $T_{YM} = \frac{M_1^2}{4 y^2}$. As for the case $\vec{N} = 0$, using (23) and taking $z = y$ one may obtain

$$H_{YM} = \frac{1}{2} \left(\dot{x}^2 + 2 \dot{y}^2 \right) + \frac{1}{2} \frac{M^2 (x^2 + y^2)}{(x^2 - y^2)^2} + \frac{g^2}{2} \left(2 x^2 y^2 + y^4 \right) \tag{26}$$

The investigation of stochastic properties of these systems will be carried out in a following paper.

Thus, we succeed in obtaining a natural separation of variables in the Hamiltonian (1) into spatial (x, y, z) and "rotational" (O_1, O_2) variables with a natural analogy with the "solid body's" mechanics, whose inertia moments are time-dependent.

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Г.М.АСАТРЯН, Г.К.САВВИДИ

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