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A.A MIGDAL

LOOP EQUATIONS

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LOOP EQUATIONS

The grounds, physical meaning and mathematical features of loop equations are discussed in detail for the Wilson averages in abelian and nonabelian gauge theories. A connection is established with planar diagrams. The law of areas is shown to serve as a self-consistent solution for large contours.

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ПЕТЛЕВЫЕ УРАВНЕНИЯ

Подробно обсуждаются обоснование, физический смысл и математические свойства петлевых уравнений для Вильсоновских средних в абелевых и неабелевых калибровочных теориях. Установлена связь с планарными диаграммами. Показано, что самосогласованным решением для больших контуров служит закон площадей.

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LOOP EQUATIONS

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Loop Equations (P)

This section is devoted to the loop equations. We start from the trivial loop equations corresponding to classical Y.M. eqc., discuss the rich variety of spurious solutions and the means to eliminate them. Then we discuss the abelian(Feynman) loop factor and the corresponding abelian loop equation. It gives us some insight into the problem of boundary conditions in loop space. Next we proceed to the nonabelian equations, discuss the rotating colour spin and vacuum pairs created at selfintersections of the loop. After that we derive basic loop equation at $N=\infty$, show how it gives the planar graphs, discuss it's physical meaning, and turn to the renormalization program. We discuss the peculiarities of the renormalization procedure in loop dynamics, and introduce the regularization of the loop equation. Finally we derive the area law as a self consistent solution at large C.

Loop Fields Corresponding to Classical Gauge Fields

The classical loop equations follow directly from the Y.M. equations

$$\nabla_\mu F_{\mu\nu} = 0 \tag{3.1}$$

The classical loop equation reads

$$\partial^\mu(x)\,\frac{\delta\Phi(c)}{\delta\sigma_{\mu\nu}} = 0 \tag{3.2}$$

Whenever the gauge field $A_\mu(x)$ satisfies the nonlinear Y.M.

eqc., the loop field satisfies the <u>linear</u> equation (3.2). Moreover note that the operator $L_\nu = \partial^\mu \frac{\delta}{\delta\sigma_{\mu\nu}(x)}$ satisfies the Leibnitz rule.

$$\begin{aligned} L_\nu AB &= (L_\nu A)B + A L_\nu B \\ &\quad + \frac{\delta A}{\delta\sigma_{\mu\nu}} \partial_\mu B + \frac{\delta B}{\delta\sigma_{\mu\nu}} \partial_\mu A \\ &= (L_\nu A)B + A(L_\nu B) \end{aligned} \tag{3.3}$$

Hence not only does any superposition of solutions of classical loop equation serve as another solution, functions of these solutions also satisfy (3.2).

$$L_\nu f(\phi_1, \phi_2) = L_\nu \phi_1 \frac{\partial f}{\partial \phi_1} + L_\nu \phi_2 \frac{\partial f}{\partial \phi_2} = 0 \tag{3.4}$$

Something went wrong. We lost some important piece of information concerning classical theory. The information is partly contained in the non linear Mandelstam relations (see appendix B). These relations between products of loop traces follow from the fact that these are traces of N^{th} rank unitary matrices. The more sophisticated relations in appendix C ensure that these matrices are unimodular. There is no theorem known which would guarantee that all the information is given by these relations. As usual, the missing information should be compensated by common sense. In perturbation theory, to which the classical equation serves as the zeroth approximation, we know that no classical solution should be added to the trivial one

$$\Phi(C)\Big|_{g_0=0} = \Phi(C = I) = 1 \tag{3.5}$$

This trivial solution corresponds to asymptotic freedom the amplitude of quark propagation along the small loop is the same as in the free quark theory. The formal perturbation theory also does not involve nontrivial classical solutions. This perturbation theory can be reproduced by direct iterations of the quantum loop equations starting from (3.5). Nontrivial solutions would arize were we to try to take into account the instanton corrections. Here we cannot offer any cure to the well known instability of instantons. Our plans concerning nonperturbative QCD are much more radical. We hope to work out an explicit solution of the $N=\infty$ problem and even the $1/N$ corrections. From this viewpoint the ambiguity in the solutions of the classical equation is in fact irrelevant. The loop equation at $N=\infty$ is nonlinear, and we have to find the proper solution of this nonlinear equation. The solution has to be asymptotically free and have to satisfy other physical requirements such as unitarity of the meson amplitudes corresponding to this solution of the Wilson loop. In finding an acceptable solution we would automatically reduce the ambiguity corresponding to the possibility of adding classical solution to the zeroth approximation of perturbation theory. For example in the random matrix model there was a similar ambiguity which was removed by the asymptotic freedom condition like (3.5) together with the unitarity condition

$$|\langle\Phi(C)\rangle| < 1 \tag{3.6}$$

The latter condition exist in any theory and follows from the positivity of the functional integral in P.C.D. together with unitarity of the loop matrix inside the trace. If there is still some anbiguity then this will be a physical problem rather than a mathematical one. This would mean that Q.C.D. is not unique at the nonperturbative level. So far there is no point in discussing this possibility. The problem is to find at least one solution.

So, *the classical loop dynamics is quite complicated and* implicit, but presumably it is irrelevant as well as the classical colour dynamics.

Abelian Loop Equation

Let us turn to quantum loop dynamics and first recall the abelian loop factor considered first by Feynman thirty years ago. He introduced the abelian loop factor as the amplitude for a charged Klein-Gordon particle to propagate along a closed world line. In loop dynamics we have the same physical picture in mind, but generalized Feynman's ideas to Dirac particles interacting with nonabelian gauge fields. In the nonabelian case such a simple expression for the amplitude can no longer be found, but the functional equation for this amplitude can always be written down.

We start by describing the loop equation in the abelian case, where the exact solution is known in advance. This will provide us with experience necessary to proceed with the nonabelian loop equations. The abelian loop factor

$$\Phi(c) = \exp\left(\int_C A_\mu dx^\mu \right) \tag{3.7}$$

reduces to the source term in the action

$$S(C, a) = \int d4x \left(\frac{1}{2e_0^2} f_{\mu\nu}^2 + j_\mu(x) a_\mu(x) \right) \tag{3.8}$$

with the well known current created by a point-like relativistic particle with the world line C

$$j_\mu(x) = \int_C dy_\mu \, \delta^d(x-y) \tag{3.9}$$

Note that a_μ is purely imaginary in our notations. This removes all the factors of i from the Euclidean gauge theory. The Gaussian functional integral over a_μ in any gauge yields the same as elimination of a_μ from the Maxwell equations (classical equations for this action)

$$\frac{\partial f_{\mu\nu}}{\partial x_\mu} = e_0^2 J_\nu \tag{3.10}$$

The solution decreasing at Euclidean infinity

$$a_\nu = - e_0^2 \int dy_\nu \, D(x-y) \tag{3.11}$$

$$D(x-y) = \int \frac{d^d k}{(2\pi)^d} \frac{e^{ik(x-y)}}{k^2} = \pi^{-d/2} \Gamma\left(\frac{d}{2} - 1\right) (x-y)^{2-d} \tag{3.12}$$

yields the following classical action

$$S(c) = -\frac{e_0^2}{2} \int_C dx_\mu \int_C dy_\mu \, D(x-y) \tag{3.13}$$

The abelian loop factor is just the exponential of this classical action

$$< \Phi(c) > = exp(S(c)) \tag{3.14}$$

Note that the Euclidean action is negative, in agreement with inequality (3.6). It is most easily seen in the momentum representation

$$S(c) = -\frac{e_0^2}{2} \int \frac{d^d K}{(2\pi)^d} \frac{1}{K^2} \left| \int_c dx_\mu e^{iKx} \right|^2 \tag{3.15}$$

This expression diverges at $d = 4$ due to large momenta, or due to small distances $(x-y)$ in (3.13).

It can be regularized, say, by inserting the factor $K^{-2\varepsilon}$ The regularization can be removed after performing the path integrals relating observables to the loop factor. This path integrals produce additional divergences, which lead in particular to the charge renormalization. The charge e_0 in the above relations is the bare charge, depending on the physical charge e_K and on the cutoff ε . The latter dependence will be compensated by the divergences coming from summation over paths. This point of view will be developed below in nonabelian theory. Let us proceed with the abelian loop equation. As is well known, the classical equations of motion are valid for expectation values in quantum theory. In our case

$$< \left(\frac{\partial f_{\mu\nu}}{\partial x_\mu} - e_0^2 j_\nu\right) \Phi(c) > = 0 \tag{3.16}$$

Note that it is not the Maxwell equations in the vacuum, but rather the Maxwell equations with the current created by our loop. The formal proof of equation (3.16) from the functional integral can also be given. It follows from the translation identity

$$0 = \int \mathcal{D} a_\mu \frac{\delta}{\delta a_\mu(x)} \phi(c) \exp\left(\int dx \frac{1}{2e_0^2} f_{\mu\nu}^2\right)$$

$$\frac{\delta}{\delta a_\mu(x} \phi(c)$$

$$= \frac{\delta}{\delta a_\mu(x)} \exp \oint a_\mu(y) d\ell_\mu$$

$$= \phi(c) \frac{\delta}{\delta a_\mu(x)} \oint a_\mu d\ell_\mu$$

$$= \phi(c) \oint \delta^d(x-y) d\ell_\mu(y)$$

$$= \phi(c) J_\mu(x)$$

(3.17)

$$\frac{\delta}{\delta a_\mu} \int d^4x \frac{1}{\partial e_0^2} f_{\alpha\beta}^2$$

$$\int d4x' \frac{1}{\nabla e_0^2} f \quad \frac{\delta}{\delta a_\mu(x')} f_{\alpha\beta}$$

$$= \frac{\delta}{\delta a_\mu(x')} \frac{\partial A_\alpha}{\partial x^\beta} - \frac{\partial A_\beta}{\partial x^\alpha}$$. Either α or $\beta = \mu$ (not both)

$$= \frac{\partial}{\partial x^\beta} \delta(x-x') \delta_{\mu\alpha}$$

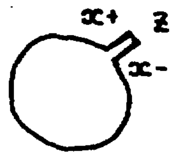
$$\int d4x' \frac{1}{e_0^2} \frac{\partial}{\partial x'^\beta} f_{\alpha\beta} \delta_{\mu\alpha}$$

$$\int \mathcal{D} a_\mu \left(\frac{1}{e_0^2} \frac{\partial}{\partial x'^\beta} f \quad \beta - J_\mu\right) \phi(c) \exp S$$

$$=$$

$$= \left\langle \left(\frac{1}{e_0^2} \frac{\partial}{\partial x'_\beta} f_{\mu\beta} - j_\nu \right) \phi(c) \right\rangle .$$

$$\partial_\mu(x) \oint_c dy_\nu \, x y_\mu$$



$$\oint_x = \oint_{x+\to x} dy_\nu \, y_\mu + \left(\int_{\Gamma_{x_- x}} + \int_{\Gamma_{z x+}} \right) dy_\nu \, y_\mu$$

$$y_\mu = x_\mu + O\left((y_\mu - x_\mu)^2 \right)$$

$$x_\mu \left(\int_{\Gamma_{x_- z}} + \int_{\Gamma_{z x-}} \right) dy_\nu = 0 .$$

This is an integral of the total derivative of a function which vanishes at the (infinite) end points. The integral is therefore zero. The variation of the exponential yields the L.H.S. of Maxwell's equations, and variation of the loop factor produces the current. It will come about in the same way in nonabelian theory, though the loop factor will not reduce to the source term in the action. The abelian equation (3.16) can be immediately translated to loop space using the methods of the previous section

$$\partial^{\mu}(x)\frac{\delta}{\delta\sigma^{\mu\nu}(x)}<\phi\,(c)> = e_0^2 <\phi(c)>\left(\int_c dy_\nu \delta^d(x-y)\right) \quad (3.1?)$$

Or, using the Leibnitz rule

$$\partial^{\mu}(x)\frac{\delta}{\delta\sigma^{\mu\nu}(x)}\ln <\phi(c)> = e_0^2\int_c dy_\nu \delta^d(x-y) \qquad (3.1?)$$

We already know the solution (3.13) to this equation, and it is trivial to verify it by the methods of the previous section. The nontrivial problem is now to eliminate the rich variety of classical solutions which could be added to $\ln <\phi(C)>$

In the old quantization the Euclidean boundary conditions for the Green functions served this purpose. What substitute these boundary conditions in loop dynamics?

Elimination of Spurious Solutions

To get some insight to this problem, let us take a specifi example.Consider the following solution of the loop equation (3.19)

$$\ln \langle \phi(c) \rangle = S(c) + \tilde{S}(c) \tag{3.20}$$

$$\tilde{S}(c) = 1 - \sqrt{1 + M^4 \sigma_{\mu\nu}^2(c)} \tag{3.21}$$

The additional term $\tilde{S}(c)$ in the loop action satisfies the classical equation

$$\partial^\mu \frac{\delta \tilde{S}}{\delta \sigma_{\mu\nu}} = -\partial^\mu \left(\frac{M^4 \sigma_{\mu\nu}(c)}{\sqrt{1 + M^4 \sigma_{\mu\nu}^2(c)}} \right) = 0 \tag{3.22}$$

This is trivial, since $\sigma_{\mu\nu}(c)$ (see (2.25)) is a Stokes type functional without marked point. At first glance the term $\tilde{S}(c)$ is acceptable. Say, for the big rectangular loop $-\tilde{S}$ grows like the area inside . So, is it the confining solution of abelian theory? A closer examination reveals that it is not. This term can be expanded in a series of multiple loop integrals, (2.15) as any Stokes type functional. The coefficient func - tions in $\langle \phi \rangle$ will correspond to many photon Green functions. In particular the two point term will get the following contri- bution from $\tilde{S}$.

$$-\frac{1}{2} M^4 \sigma_{\mu\nu}^2(c) = -\frac{1}{8} M^4 \int_c dx_\mu (x_\nu - y_\nu) \int_c dy_\mu (y_\nu - x_\nu)$$
$$= \frac{1}{8} M^4 \int_c dx_\mu \int_c dy_\mu (x - y)^2$$

This two point function does not decrease at infinity. We ru- led out such functions in solving the Maxwell equations. But maybe we did it wrong? Are there any inherent paradoxes in loop space with the term (3.21)? Consider the case of two separated loops C_1 and C_2 connected by the wires Γ_{12}, Γ_{12}^{-1}. The tensor area for the total loop $C_1\,\Gamma_{12}\,C_2\,\Gamma_{21}$ reduces to the sum of areas for C_1 and C_2

$$\sigma_{\mu\nu}(C_1\,\Gamma_{12}\,C_2\,\Gamma_{12}^{-1}) = \sigma_{\mu\nu}(\Gamma_1) + \sigma_{\mu\nu}(C_2) \qquad (3.23)$$

The contributions from the wires cancel, as one may easily check. Now we come to the paradox, since for arbitrary separation between C_1 and C_2 there will remain the correlation in (3.21). The implied decoupling

$$< \phi(C_1\,\Gamma_{12}\,C_2\,\Gamma_{21}) > = < \phi(C_1)\,\phi(C_2) > \qquad (3.24)$$

corresponds to the Mandelstam relation in the abelian case. So we impose the Mandelstam relation together with cluster decomposition (i.e. unitarity) to eliminate the spurious classical solution. Analysing such examples we get the feeling that the loop equations can be saved in loop space, without involving any information about colour dynamics. This makes loop dynamics a genuine alternative to the standard quantization of gauge theories. The question is whether any tractable scheme will emerge from loop dynamics, beyond perturbation theory. Remember the S matrix approach, which also arose as an alternative to local field theory, led nowhere basically because of the technical difficulties. Hopefully it will not happen to loop dynamics.

Rotating Colour Spin

The basic difference between the propagation of charged particles in the abelian and nonabelian theory reduces to local rotations of the colour spin in the latter case. The colour of any particle, however heavy, cannot avoid rotations induced by vacuum fluctuations of the gauge field. This is taken into account by the nonabelian loop factor i.e. the Wilson loop.

$$W(c) = \frac{1}{N} < \mathrm{Tr}\, \mathcal{U}\ (C_{xx}) > \tag{3.25}$$

The colour spin matrices $(\tau_a)_{ik}$ in the loop product enter through the gauge fields

$$A_\mu = i \sum \tau_a A_\mu^a \tag{3.26}$$

They are ordered along the world line C. The colour spin at a given point of the trajectory depends on the whole trajectory rather than on the local gluon field. The local avarage colour spin

$$\frac{1}{W(c)} < \frac{1}{N} \mathrm{tr}\, \tau_a\, \mathcal{U}(C_{xx}) > = 0 \tag{3.27}$$

vanishes due to gauge invariance, but there are correlations described by the functions

$$\rho_{ab}(C_{12}, C_{21}) = \frac{1}{W(c)} \frac{1}{2} < \tau_a\, \mathcal{U}(C_{12})\, \tau_b\, \mathcal{U}(C_{21}) > \tag{3.28}$$

If x_1 coincides with x_2 in co-ordinate space then this function may be finite. From gauge invariance it is diagonal.

$$\rho_{ab} = \delta_{ab}\, \rho(C_{12}, C_{21}) \tag{3.29}$$

The correlation function ρ, describes the average "scalar product" of colour spins at the two points x_1, x_2. If x and x_2 coincide at the loop i.e. if $t_1 = t_2$ then $\rho = 1$ in our normalization.

For self-intersecting loops there may be nontrivial cor - relations between colour spins (Fig.2).

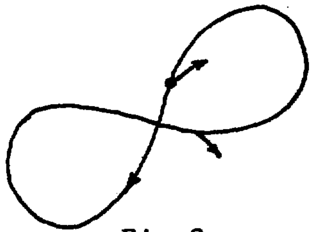


Fig.2

From the point of view of Minkowski space the self intersecting loop describes pairs created from the vacuum. So, this correlation function measures the angles between colour spins of particles and antiparticles.

Now we are in a position to write down the nonabelian equation of motion. Instead of the abelian identity (3.17) we employ the corresponding matrix identity

$$0 = \int \mathcal{D}A \frac{\delta}{\delta A_{a\nu}(x)} \left(i\tau_a U(C_{xx}) \right)^j_i \exp\left(\int d^4x \frac{1}{4g_0^2} \mathrm{tr} F_{\mu\nu}^2 \right) \qquad (3.30)$$

Varying the exponential we find the L.H.S. of the Yang Mills equations and varying the factor in front we find

$$\left\langle \frac{\delta}{\delta A_{a\nu}} \left(i\tau_a U(C_{xx}) \right\rangle^j_i = -\int_{C_{xx}} dy_\nu\, \delta^4(x-y) \left\langle \left(\tau_a U(C_{xy}) \tau_a U(C_{yx}) \right)^j_i \right\rangle \qquad (3.31)$$

$$= \frac{2(N^2-1)}{N} \int_{C_{xx}} dy_\nu\, \delta^d(x-y)\, \rho(C_{xy}, C_{yx})$$

Collecting the terms we arrive at the following matrix equation of motion

$$\langle (\nabla_\mu F_{\mu\nu} - g_0^2 \bar{j}_\nu (C_{xx})) \, U(C_{xx}) \rangle = 0 \tag{3.32}$$

here

$$\bar{j}_\nu(x) = \frac{N^2-1}{N} \int_{C_{xx}} dy_\nu \, \delta^d(x-y) \, \rho(C_{xy}, C_{yx}) \tag{3.33}$$

serves as an effective gauge invariant current created at the point x by a particle with world line C_{xx}. This current depends in a nontrivial way on the form of the loop. In the abelian case, when the correlation function was absent, the current could be regarded as a function of the point x in space depending in addition on some closed loop C. In notations of the previous section, this was a Stokes type functional without marked point. The point x could be freely shifted aside from C, and one should not care about the wires in the abelian case. In the case at hand, this is a Stokes type functional with a marked point. The point x enters not only as an argument of the δ function but also as an endpoint of the paths C_{xy}, C_{yx} in the correlation function. This correlation function represents a Stokes type functional with two marked points. The significance of these mathematical properties will become clear below.

Let us discuss the physical properties of our current. In the absence of self-intersections it does not differ from the abelian current. So we have to include self-intersecting loops to distinguish between Q.E.D. and Q.C.D.. The self-intersecting loops, as it was mentioned above, correspond to vacuum pairs. At multiple self-intersections the current contains some number of terms corresponding to the solutions of the equations

$$C_\mu(t_i) = X_\mu \equiv C(t_o) \quad i = 1, \dots n \tag{3.34}$$

In each term we may take out, which yields

$$\dot{j}_\nu(C_{xx}) = \frac{N^2-1}{N} \sum_{i=0}^{n} \rho(C_{oi}, C \quad) \, \dot{j}_\nu^{(i)}(x) \tag{3.35}$$

with abelian currents

$$\dot{j}_\nu^{i}(x) = \int_{L_i} dy_\nu \, \delta^d(x-y) \tag{3.36}$$

L_i being the contour consisting of two rays- one coming to x with the same direction as C_{oi} and another coming out of x in the same direction as C_{io}.

$$L_i \quad \left\{ \begin{array}{lll} x_\mu = C_\mu(t_i+0)\tau & & \tau > 0 \\ x_\mu = \dot{C}_\mu(t_i-0)\tau & & \tau < 0 \end{array} \right\} \tag{3.37}$$

Naturally these currents should be understood as distributions, like in the abelian case. The problem of regularization will be discussed at length below. At the moment note, that expansion (3.35) is a counterpart of the expansion of the current in creation and annihilation operators in the second quantized version of the theory. In the path integral representation pair creation is taken into account implicitly by letting the world lines intersect and turn backwards an arbitrary number of times. The same is true in loop dynamics. By incorporant self-intersecting loops we take into account part of the vacuum quark pairs. Another part corresponds to disconnected loops and enter with the factor N^{-1}, as we discuss later. Naturally, the physical quarks have ordinary spin and flavour as well as colour, which was disregarded here.

However this can be accounted for afterwards in loop dynamics. The amplitude for the physical quark to propagate along C equals to the Wilson loop factor times the standard amplitude in Dirac flavour space.

The Loop Equation at $N = \infty$

We come to the most important part of loop dynamics - the closed equation for the Wilson loop at $N = \infty$. This equation follows from the factorization property, which in turn can be proven nonperturbatively from the more general loop equations, as in the random matrix model. For pedagogical reasons we proceed in a more natural way. First we assume factorization as a working hypothesis, and later we consider the generalization to finite N, which confirms our hypothesis.

In order to apply the factorization we use in the correlation function the Fiertz identity

$$\frac{1}{2}(\tau_a)^{ij}(\tau\ )^{k\ell} = \delta_{jk}\delta_{i\ell} - \frac{S}{N}\delta_{ij}\delta_{k\ell} \tag{3.38}$$

Here

$$S = 0 \quad \text{for } U(N) \text{ and } 1 \text{ for } SU(N) \tag{3.39}$$

For the $U(N)$ group this identity follows from completeness of the basic τ_a for the $N \times N$ hermitian matrices. For $SU(N)$ the term with $(\tau_a)^{ij} \doteq \sqrt{2/N}\,\delta_{ij}$ should be substituted with the Fiertz identity we find

$$\rho^{aa}(C_{xy}, C_{yx}) = N^2 \frac{\langle \phi(C_{xy})\,\phi(C_{yx})\rangle}{\langle \phi(C)\rangle} \tag{3.40}$$

The last term is negligible at $N \to \infty$, and the first simplifies due to factorization

$$\langle \phi(C_{xy})\,\phi(C_{yx})\rangle = \langle \phi(C_{xy})\rangle\langle \phi(C_{yx})\rangle + O\left(\frac{1}{N^2}\right) \tag{3.41}$$

Note that still the correlation remains nontrivial

$$\rho = \frac{1}{N^2-1}\,\rho_{aa}(C_{xy}, C_{yx}) \to \frac{W(C_{xy})\,W(C_{yx})}{W(C_{xx})} \tag{3.42}$$

unlike the abelian case. In fact the limit $N \to \infty$ is the extreme opposite to the abelian limit $N \to 1$ in $U(N)$ theory. The equation of motion can always be written as a loop equation

$$\partial^{\mu}(x)\frac{\delta}{\delta\sigma_{\mu\nu}(x)} W(C) = g_0^2\, W(C)\, j_\nu(C_{xx}) \tag{3.43}$$

At $N = \infty$ the general expression for the loop current simplifies

$$\partial^{ai}\frac{\delta}{\delta\sigma_{\mu\nu}(x}\, W(C) = \lambda \int_{C_{xx}} dy_\nu\, \delta^d(x-y)\, W(C_{xy})\, W \tag{3.44}$$

with

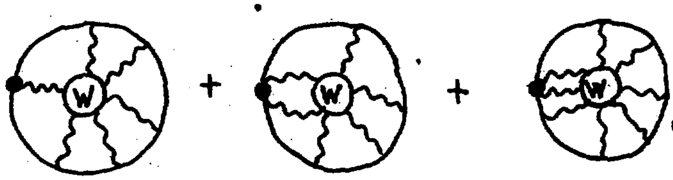
$$\lambda = N g_0^2 = \text{constant} \tag{3.45}$$

This equation was first derived in 1978 within the framework of lattice gauge theory. The physical meaning of this equation was completely unclear at the beginning because of lattice artefacts. It took the whole year to reformulate this equation in continuous terms, and establish the connection with gluon exchange graphs. Only after that could we proceed further with the area law and with the string solution.

Planar Graphs

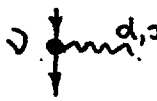
As is well known at $N = \infty$ we are left with planar graphs in Q.C.D.. So the loop equation (3.44) sums the planar graphs. Let us explicitly demonstrate it. With the graph notation of section II the L.H.S. of the loop equation can be represented as a sum of three terms

$$\partial^{\mu}_{x}\frac{\delta W(C)}{\delta\sigma_{\mu\nu}(x)} =$$

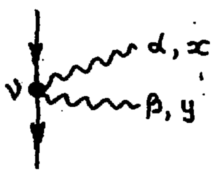


(3.46)

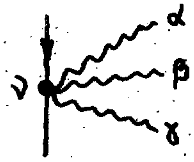
The three vertices here arise after applying the path derivative ∂^{α} to the previous vertices (2.43), (2.45) namely



$$= \frac{\partial^2}{\partial x^2}\delta_{\nu\alpha} - \frac{\partial}{\partial x_\nu}\frac{\partial}{\partial x_\alpha} \qquad (3.47)$$



$$= \left(\frac{\partial}{\partial x_\alpha}+2\frac{\partial}{\partial y_\alpha}\right)\delta_{\nu\beta} - \left(\frac{\partial}{\partial y_\beta}+2\frac{\partial}{\partial x_\beta}\right)\delta_{\nu\alpha} + \delta_{\alpha\beta}\left(\frac{\partial}{\partial x_\nu}-\frac{\partial}{\partial y_\nu}\right) \qquad (3.48)$$



$$= \delta_{\alpha\beta}\,\delta_{\nu\gamma} + \delta_{\beta\gamma}\,\delta_{\nu\alpha} - 2\,\delta_{\alpha\beta}\,\delta_{\nu\beta} \tag{3.49}$$

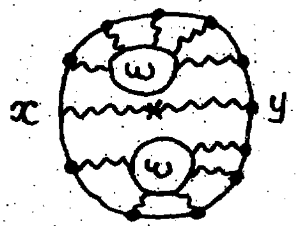
These three vertices correspond to the three terms in the Yang-Mills equations

$$\nabla_\mu F_{\mu\nu} = \partial_\mu \partial_\mu A_\nu + \left(\partial_\mu [A_\mu, A_\nu] + [A_\mu, \partial_\mu A_\nu]\right) + [A_\mu, [A_\mu, A_\nu]] \tag{3.50}$$

Let us stress once more that we derived them from the geometric definitions rather than from (3.50). Agreement with (3.50) indicates the consistency of the geometric language. As for the R.H.S. of the loop equation it can be represented as the following graph

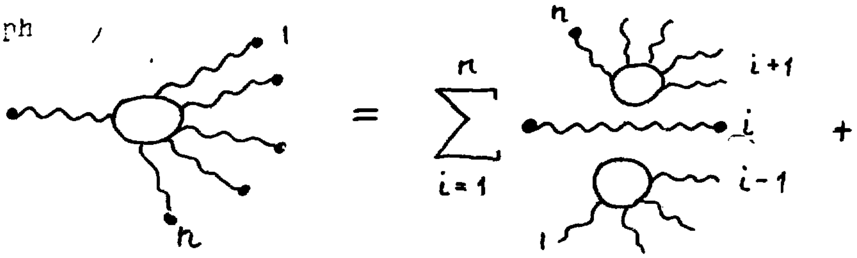
$$\lambda \int dy_\nu\ \delta^d(x-y)\, W(C_{xy})\, W(C_{yx})$$

$$= \sum \tag{3.51}$$



Here the line with the cross stands for $\lambda\,\delta_{\nu\alpha}\,\delta^d(x-y)$, the vertex without a dot denotes the unintegrated point x, ν and the rest of the notations are the same as in section II. This graph is planar since the points at the two parts C_{xy}, C_{yx} of the loop C are ordered and do not cross the point y. If the reader is not satisfied with this loose definition of planar graphs he is addressed to the original paper by t'Hooft, where the precise definition was given and various fundamental topological observations were made. Let us return to the loop equation.

Note that the first term in (3.46) contains exactly the same graph as in (3.51). This is the disconnected one-gluon graph ,



$$+ \quad \text{connected graphs} \quad + \quad \text{nonplanar graphs} \qquad (3.52)$$

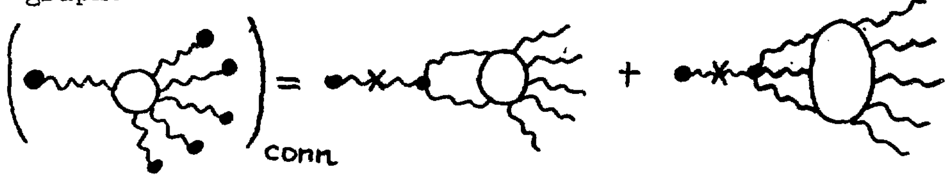
In the Feynman gauge the first term in the vertex (3.47) just produces the δ - function

$$\frac{\partial^2}{\partial x^2} \lambda \mathcal{D}(x-y)\delta_{\nu\alpha} = \lambda\, \delta^d(x-y)\delta_{\nu\alpha} \qquad (3.53)$$

Therefore the rest of the terms in (3.46) should cancel among themselves. As is shown in our papers with Makeenko, this concellation followed from the Dyson equations and Ward identities. The ghost loops are necessary to cancel the longitudinal terms.

$$-\frac{\partial}{\partial x_\nu} \frac{\partial}{\partial x_\alpha} \lambda \mathcal{D}(x-y)$$

in the connected and disconnected graphs. The second and third terms in (3.46) cancel with self-energy parts in the connected graphs



+(ghost loops) + longitudinal terms.

Gluon Exchange and the Loop Equation

We do not dwell on details, they can be found in the original papers. Our aim here is to surpass perturbation theory, the above companion with planar graphs was only given for methodical purposes. Many people, including the author, have a kind of affection to Feynman graphs, and always try to translate complicated dynamical equations into graphical language. Then we feel safer and start understanding what is going on. In the case of the loop equation we find out that the special variation of the loop on the L.H.S. emits only one gluon. All other gluons, usually emitted when the charged particle changes it's trajectory, completely interfere each other. However, this single gluon does not propagate. The effective gluon field is zero everywhere but in the infinitesimal vicinity of the varied point. The gluon propagator $1/K^2$ is cancelled by the vertex $\sim K^2$. Hence only quark in the infinitesimal vicinity can absorb this gluon which happen when the loop intersect itself at this point. The total amplitude equals the product of all quark amplitude $W(C_{xy}), W(C_{yx})$ before and after gluon absorption. This factorization is explained as follows. There are N colour components of quarks and N^2 components of gluons. When the quark number i units the gluon number (ij) it goes in most cases to a different quark $j \neq i$. In further emission it will become j', j'' and it is unlikely that it will be itself again at $N \to \infty$. Note that the interaction of gluons goes with a wider phase space-gluon (i, j) goes to gluon (i, K) plus gluon (K, j) with any $K = 1 \dots = 1 \dots N$. The correlation between different quark loops corresponds to identification of emitted and absorbed gluons, which is unlikely at $N \to \infty$. Only one of the N^2 gluons will be the one we need this gives the estimates $1/N^2$ for the correlation. This explanation shows that factorization has nothing to do with classical gauge fields. Some types of quantum factorization have more phase space than other types, that is all. In the functional integral it corresponds to the fact that most of the relevant field configurations are charged i.e. they correspond to off diagonal components of the gauge field. These charged components produce an entropy $\sim N^2$ which changes the effective action, like the energy is changed to the free energy in a statistical system. The large N limit in this sense is analogous to the statistical limit. The number of colour degrees of freedom goes to infinity. We avoided hopeless problem of the calculation of the entropy in loop dynamics by deriving a direct equation of motion in loop space. Now we see, that it sums planar graphs.

Renormalization Program in Loop Dynamics

Renormalization is the most delicate part of gauge theories, since the regularization has to preserve gauge invariance. Such important phenomena as quantum anomalies, e.g. would be missed would we not insist on exact gauge invariance at finite cutoff.

In loop dynamics gauge invariance corresponds to closed loops. As long as the regularized loop equation involves closed loop factors gauge invariance is preserved. In the old language it corresponds to the insertion of path factors $\mathcal{U}(\Gamma_{xy})$ between the split points in the regularized currents. The form of the wires Γ_{xy} connecting split points is arbitrary. This produces the ambiguity of regularization procedure.

The renormalizability of Q.C.D. guarantees that in any order in the effective coupling constant the ambiguities in the regularization procedure will disappear. The problem has to be reexamined if we intend to go beyond perturbation theory. The universality of renormalized perturbation theory is replaced by the existence of universality classes, or phases, of regularized gauge theories. For example, with lattice regularization, there are generally two phases depending on the struc - ture of the lattice action. One is the vertex monopole phase with strong fluctuations at the lattice scale and another is the gluon phase we need. In the gluon phase one may vary the bare coupling constant together with the lattice spacing without affecting the physical quantities, such as meson amplitudes (up to powerlike corrections at vanishing lattice spacing). This is a nonperturbative analogue of the renormalization group law. What are the implications of these ideas to the Wilson loop factor? This factor depends upon the bare coupling g_0 and C in the regularized theory, it also depends on the cut-off ε. The meson amplitudes are certain path integrals involving the Wilson loop factors as well as other factors depend ing upon the quark parametre such as spin, flavour, etc. These path integrals produce divergences of their own. There are, therefore, two sources of divergences: 1) the ε - dependence of the path integrals relating observables to the Wilson loops. In the simplest case of $\langle\bar{q}\, q\rangle$ expectation values

$$m_o\langle\bar{q}\,q\rangle = m_o \sum_{C_{xx}} I\left(C_{xx}\right) W\left(C_{xx}\right)$$

The sum over paths C_{xx} and the explicit expression for the weight $I(C_{xx})$ (Dirac amplitude) will be derived later. By varying the bare coupling $g_o(\varepsilon)$ and the bare mass $m_o(\varepsilon)$ according to the renormalization group law with the β function of the pure gauge theory we make keep the $m_o\bar{q}\,q$ expectation value finite at $\varepsilon = 0$ in (3.53). Both types of divergence mentioned above will be eliminated. By varying the bare coupling $g_o(\varepsilon)$ in $W(C_{xx})$ we eliminate part of the singularities, but some dependence of $W(C_{xx})$ on ε will remain. It will be eliminated later on, when the path integration is performed. This path integration will produce the factor $Z_\mu^{-1}(\varepsilon)$ which will cancel with the factor $Z_\mu(\varepsilon)$ in the bare mass. We could include this constant in the Wilson loop $W(C_{xx})$. This would renormalize $W(C)$ in this particular path integral, but for some other amplitudes other factors will be regularized. Say the Green's functions of two vector quark currents, $\bar{q}\,\gamma_\mu\, q$ (the quark-gluon vacuum polarization by the photon) will also be expressed as a path integral of $W(C)$ with a different weight factor. Here no renormalization constants are required since the vector current has no anomalous dimension. It was implied above that $N = \infty$. In higher orders in $1/N$ the $1/N$ corrections to the bare coupling $g_o(\varepsilon)$ inside the Wilson loop has to be taken into account. These corrections will cancel with corrections from path integrals involving extra quark loops. So there are delicate cancellations of divergences in loop dynamics. Divergences appear all the way from the bare lagrangian to observable quantities, and the Wilson loop is only an intermediate stage on this route. Unfortunately the cutoff has to be kept finite to the end. At the same time the relevant loops are not regular. As it follows from the above arguments the velocities $\dot{x}$ fluctuate as in Brownian motion. So we need the Wilson loop factors in the regularized theory (i.e finite ε) but for irregular loops (i.e. loops with cusps and self-intesections). As we shall shortly see, there are no singularities at cusps or self-intersections in the regularized theory so Wilson loops are well defined. There is only one real problem in such an approach. This is the problem of regularization ambiguities, related to the phase transition problem. The ambiguity in the regularization procedure has to be utilized, rather than avoided. We have to find the regularization which has the smooth local limit. The phase transition should be absent. In lattice regularization this is not quite so. It is difficult to see asymptotic freedom and quark confinement at the same time. Also, violation of Lorents symmetry and the absence of dimensional transmutation is annoying in the lattice regularization. Something better can be done with the loop equation than introducing a hypercubic lattice.

Analytic Regularization of the Loop Equation

The most convenient regularization for perturbative gauge theory is the famous dimensional regularization, which was in fact implied above. However it turns out to be ill defined beyond perturbation theory. For example, it misses such an important phenomenon as the axial anomaly, which requires explicit point splitting. The version of the point splitting procedure closest to dimensional regularization is analytic regularization. Now we are going to adapt analytic regularization for the loop equation. The idea is to replace the propagator $K^{-2} \to K^{-2-2\varepsilon}$ in a gauge invariant manner. The regularization will consist in analytic continuation in ε from the domain of convergence. The number of dimensions $d=4$ will be unchanged, so the loop functionals will be well defined. Only the coefficient functionals i.e. traces of gluon Green functions will become nonsingular at coinciding points. With the gluon propagator $K^{-2\varepsilon-2}$ in planar graphs the above line with a cross in (3.51) will correspond to $K^{-2\varepsilon}$ since the vertex (3.47) will remain K^2 as before. At finite ε the points x and y would be split with the weight$\sim\varepsilon(x-y)^{2\varepsilon-4}$ instead of the δ-function. Naturally there should arise extra graphs to compensate the violation of gauge invariance at the point splitting. In the old formulation it corresponds to path factors $U(\Gamma_{xy})$ in the adjoint representation where Γ_{xy} is some path between x and y. In the loop equation the paths Γ_{xy}, Γ_{xy} should be added to close the loops

$$W(C_{xy}) \to W(C_{xy}\,\Gamma_{xy}^{-1}) \tag{3.54}$$

$$W(C_{yx}) \to W(C_{yx}\,\Gamma_{xy}) \tag{3.55}$$

From the point of view of gauge invariance these paths are arbitrary but space symmetry is violated for arbitrary paths. There is a natural choice of these paths preserving the symmetry. Namely they may be chosen to coinside with the path of the gluon. The extra vertices arising from the factor $(\mathcal{U}(\Gamma_{xy})^{dj}$ now can be interpreted as gluon interactions. Roughly speaking this is the same as replacing the δ-function by a covariant derivative in the adjoint representation $\nabla_\mu = \partial_\mu + [A_\mu,$ raised to the non integral power $(-\nabla_\mu^2)^{-\varepsilon}$.

Let us now make these ideas more precise. First of all it is convenient to consider the integrated version of the loop equation

$$L\,W(C) = \lambda \int_C dx_\mu \int_{C_{xx}} dy_\mu\ \delta(x-y)\,W(C_{xy})\,W(C_{yx}) \tag{3.56}$$

The original local version represented a vector equation for the scalar functional $W(C)$, so it was overcomplete. There were certain consistency relations following from the identities (2.). The R.H.S. satisfied the self consistency relations due to conservation of the vector current $j_\nu(C_{xx})$ in the sence of loop calculus $(\partial^\nu(x)\, j_\nu\,(C_{xx}) = 0)$. Point splitting in the original equation should respect self-consistency conditions. There is no real problem, since there are no anomalies in the vector current but it leads to unnecessary complications. The scalar equation (3.56) is, in principle, equivalent to the overcomplete system, but it is more convenient for point splitting.

As a first step in the gauge invariant point splitting procedure let us introduce the heat propagation kernel $K_1(x,y)$

$$K_{T}(x-y) = \langle x | \exp(-T\hat{P}^2) | y \rangle \tag{3.57}$$

$$P_\mu = i\frac{\partial}{\partial x_\mu} \tag{3.58}$$

At vanishing proper time T, it reduces to a δ-function, so it may serve as a regularized definition of the δ-function. However, it would be more convenient to introduce a dimensionless cutoff $\mathcal{E}$ by integrating the heat kernel over proper time with the corresponding weight.

$$\mathcal{D}_\varepsilon(x-y) = \varepsilon\int_0^\infty dT\,T^{\varepsilon-1} K_T(x-y) \tag{3,59}$$

This function interpolates between the Feynman propagator ($\mathcal{E}$ =1) and the δ-function ($\mathcal{E}=0$). At finite $\mathcal{E}$ it corresponds to the propagator $\Gamma(\mathcal{E}+1)(P^2)^{-\mathcal{E}}$ with a branch point rather than a pole. The Feynman integrals with such propagators are defined as analytic continuation in $\mathcal{E}$ from the domain of convergence.

Next recall the well known path integral representation of the heat kernel

$$K_T(x-y) = \int_{x(0)=x}^{x(T)=y} \mathcal{D}x(t)\,\exp\left(-\int_0^T dt\,\tfrac{1}{4}\dot{x}^2\right) \tag{3.60}$$

We shall use a special notation for such a path integral (including the proper time integration)

$$\int_0^\infty dT \int_{x(0)=x}^{x(T)=y} \mathcal{D}x(t) \exp\left(-\int_0^T dt \tfrac{1}{4}\dot{x}^2\right) \equiv \int \mathcal{D}\Gamma_{xy} \tag{3.61}$$

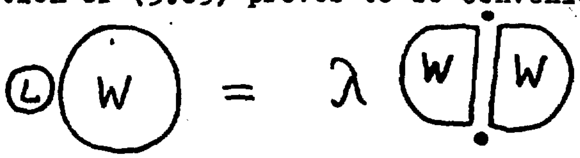
With this notation

$$\mathcal{D}_\varepsilon(x-y) = \int \mathcal{D}\Gamma_{xy}\, \varepsilon T^{\varepsilon-1} \tag{3.62}$$

The reader is warned that sometimes other definitions are used in the literature of the path integral, in particular in the author's papers. We use this one here, since it is most traditional and (therefore) the easiest to memorize. Now we are ready to write down the regularized loop equation

$$L W(C) = \varepsilon\lambda \int_C dx_\mu \int_C dy_\mu \int \mathcal{D}\Gamma_{xy} T^{\varepsilon-1} W(C_{xy}\Gamma_{yx}) W(C_{yx}\Gamma_{xy}) \tag{3.63}$$

The iteration of this equation would produce regularized perturbation theory. By construction the first iteration yields the one gluon graph with propagator $\Gamma(\varepsilon+1)(K^2)^{-\varepsilon-1}$. In higher orders extra vertices will arize. The following graphical representation of (3.63) proves to be convenient.



(3.64)

Here the functional integration over paths Γ_{xy} corresponding to the double line and the integration over the end points x, y is implied. Each window corresponds to the W-factor of the corresponding loop. We refer to such a graph as a "glassed" graph. Note that only planar graphs can be glassed. We have already dealt with glassed graphs in the random matrix model, in which they were only a technical device. Here the graphs have to be understood in the usual Feynman sense of summing over all histories the corresponding amplitudes. The iterations of this equation can be performed in an integral form

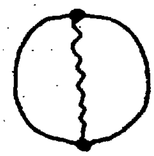
$$W = 1 + \lambda L^{-1}\, W \, W \qquad (3.65)$$

In the first order on the right

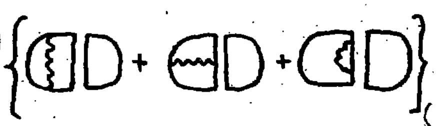
$$1\,1 = \int dx_\mu \int dy_\mu \int \frac{d^4K}{(2\pi)^4}\, \frac{e^{iK(x-y)}}{(K^2)^{1+\varepsilon}}\, \Gamma(\varepsilon+1) \qquad (3.66)$$

The application of the inverse operator (with the Euclidean boundary conditions) reduces to multiplication by K^{-2} in momentum space i.e.

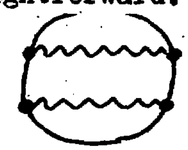
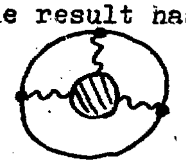
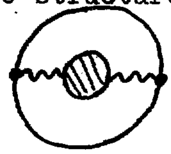
$$W^1 = L^{-1}\, 1\,1 = \int dx_\mu \int dy_\mu \frac{d^4K}{(2\pi)^4}\, e^{iK(x-y)}\, \frac{\Gamma(\varepsilon+1)}{(K^2)^{1+\varepsilon}}$$

$$\equiv$$  (3.67)

In the next order we find

$$W = 2L^{-1}\, W'\, 1 = 2L^{-1}\Big\{$$  $$\Big\} \qquad (3.68)$$

where the wavy line denotes the regularized gluon propagator $\delta_{\mu\nu}(K^2)^{-1-\varepsilon}\Gamma(\varepsilon+1)$. Application of L^{-1} here is a bit more tedious, but streightforward. The result has the structure

$$W^{(2)} = 2$$  $+$  $+$  (3.69)

The regularization 3-point vertex and the self-energy part can be found by applying L to (3.69) and using the graphic rules (3.46). We omit these calculations, since for practical purposes of perturbative Q.C.D. analytic regularization is less convenient than dimensional regularization. However it is acceptable from the theoretical point of view and it is particularly convenient for the investigation of the loop equation.

Self Consistent Area Law

Let us start the investigation of the loop equation in the most interesting infrared domain. Note that the equation can be reduced to a universal form by renormalizing the Wilson loop factor

$$W(C) = \frac{1}{\lambda}\, \omega(C) \tag{3.70}$$

The renormalized factor ω satisfies the equation without λ. The bare coupling enters the initial condition.

$$\lambda = \omega(1) \tag{3.71}$$

We may treat this condition as a definition of the bare coupling $\lambda_0(\varepsilon)$.

Clearly this constant is irrelevant. The physical scale σ can be defined as a coefficient in front of the area in the area law which we expect to arise as a self consistent asymptotic solution at large C.

$$\ln \omega(C) \to -\sigma |S_c| \tag{3.72}$$

Here S_c is the minimal surface bounded by C, and $|S_c|$ is the area of this surface. The analytic regularization of the area can be given.

$$|S|_\varepsilon = \frac{\varepsilon}{2\pi} \int_S d\sigma_{\mu\nu} \int_S d\sigma_{\mu\nu} \left((x-y)\right)^{\varepsilon - 1} \tag{3.73}$$

here

$$d\,\sigma_{\mu\nu}(x) = t_{\mu\nu}(x)\, d^2 y_{\parallel} \tag{3.74}$$

is the tensor area element in the local tangent plane. The tangent tensor $t_{\mu\nu}(x)$ is normalized to unity

$$\frac{1}{2} t^2_{\mu\nu} = 1 \tag{3.75}$$

At $\varepsilon \to 0$ the domain of $y \to x$ contributes

$$\frac{\varepsilon}{\pi} \int t_{\mu\nu}(y)\, d^2 y_{\parallel} \left((x-y)^2\right)^{\varepsilon - 1} \quad \varepsilon \to 0 \tag{3.76}$$
$$\to t_{\mu\nu}(x)$$

Hence at $\varepsilon = 0$ the standard definition

$$|S| = \int_S d^2 y_{\parallel} \tag{3.77}$$

is recovered. Let us now demonstrate that the area law does indeed satisfy the loop equation at large C . Substitute the following ansatz into the loop equation

$$\omega(c) = exp(-\sigma |S_c|_\varepsilon)\, g(c) \tag{3.78}$$

Only the exponential factor has to be differentiated for large C

$$L\,\omega(c) \to \sigma y(c)\, e^{-|S_c|^\sigma} L\, |S_c|_\varepsilon \tag{3.79}$$

A simple calculation yields

$$L\,|S_c|_\varepsilon = \frac{\varepsilon}{2\pi} \int_C dx_\nu \int_{|S_c|} d\sigma_{\mu\nu}(y) \frac{\partial}{\partial x_\mu} \left((x-y)^2\right)^{\varepsilon - 1} \tag{3.80}$$

On the right of (3.63) we find that the exponential factors cancel (at $\mathcal{E}=0$)

$$\exp(-\sigma|S_1|-\sigma|S_2|+\sigma|S_c|) \rightarrow 1 \tag{3.81}$$

It was implied here that the path Γ_{xy} varies at the minimal surface S_c. Then the sum of areas of the two windows.

$S_1 + S_2 = S_c$. For the regularized area this is not quite so, but we expect that at $\mathcal{E}\rightarrow 0$ the dominant paths Γ_{xy} vary along the minimal surface. The transverse components $x^{\perp}_{\mu}(\mathcal{E})$ can be taken into account perturbatively. In the Gaussian approximation we shall obtain some determinant

$$\int \mathcal{D}x^{\perp}(\varepsilon)\ \exp(-\sigma|S_1|-\sigma|S_2|) \tag{3.82}$$

$$\rightarrow e^{-\sigma|S_c|}\int \mathcal{D}x^{\perp}\exp\left(-\int_0^T dt\left(-\tfrac{1}{4}\dot{x}^2+\delta^2(S_1+S_2)\right)\right)$$

$$=\exp\left(-\sigma|S_c|\right)\Delta\left(\Gamma_{xy}, C\right)$$

We shall not work out the details here, but in principle this determinant is calculable. The resulting equation for the pre exponential factor $g(C)$ reads

$$\sigma\ g(C)\int_C dx_\nu \int d\sigma_{\mu\nu}(y)\frac{\partial}{\partial y_\mu}(x-y)^{2(\varepsilon-1)} = \mathrm{tr}\int_C dx_\mu \int_{C_{xx}} dy_\mu$$

$$\int \mathcal{D}\Gamma_{xy}\, T^{\varepsilon-1}\Delta(\Gamma_{xy}, C)\, g(C_{xy}\Gamma_{xy}^{-1})\, g(C_{yx}\Gamma_{xy}) \tag{3.83}$$

$\Gamma_{xy}^{\prime\prime}$ C-Si

At present we do not know how to solve this equation; even the determinant Δ is unknown. But maybe some approximation to this equation will be found in the future. At least we see that the area law is self-consistent. Note that this mechanism of the area law is explicitly Lorentz invariant and exhibits dimensional transmutation. The same glassed graph which describes gluon exchange at small distances, leads to confining forces at large distances. So we have a smooth interpolation between asymptotic freedom and confinement.

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