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RESONANCE ACCELERATION OF CHARGED PARTICLES BY A SURFACE WAVE ARISING AT TOTAL INTERNAL REFLECTION

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РЕЗОНАНСНОЕ УСКОРЕНИЕ ЗАРЯЖЕННЫХ ЧАСТИЦ ПОВЕРХНОСТНОЙ ВОЛНОЙ, ВОЗНИКАЮЩЕЙ ПРИ ПОЛНОМ ВНУТРЕННЕМ ОТРАЖЕНИИ

В целях поисков возможных путей увеличения темпа ускорения заряженных частиц рассмотрен вариант резонансного ускорения поверхностной волной, возникающей в вакууме при полном внутреннем отражении света от границы с диэлектриком и затухающей в направлении нормали к границе раздела. Предложено использовать поляризованную лазерную волну, волновой вектор которой ориентирован так, чтобы можно было обеспечить фазовую устойчивость ускоряемых частиц: именно, направление распространения образующейся поверхностной волны должно составлять черенковский угол с направлением скорости заряженных частиц, движущихся вдоль плоской поверхности раздела. Для обеспечения фазовой устойчивости и высокого темпа ускорения включается дополнительное постоянное магнитное поле, перпендикулярное поверхности раздела и направлению скорости частиц. Вертикальное равновесие обеспечивает магнитное поле, параллельное поверхности раздела и также перпендикулярное направлению скорости частиц. Это поле, однако, для обеспечения вертикальной устойчивости должно менять знак от одного участка тракта ускорения к другому одновременно со скачкообразным изменением фазы лазерной волны. Несмотря на то, что поверхностная волна экспоненциально затухает на расстояниях от поверхности раздела порядка длины волны лазера, в режим ускорения может быть захвачено (независимо от длины волны лазера) до 10^{11} частиц/импульс. Параметры поляризации и угол падения лазерной волны подбираются из условий оптимизации темпа ускорения при разумных значениях напряженностей дополнительных постоянных магнитных полей и с учетом электрической прочности диэлектрика. Показано, что использование существующих лазеров, созданных для целей термоядерного синтеза, может обеспечить темп ускорения порядка 100МэВ/м. Дальнейшее усовершенствование технологии (в частности, создание мощных пикосекундных лазеров и сильных, быстро изменяющихся в пространстве постоянных магнитных полей), по-видимому, позволяет ожидать при использовании предлагаемого способа достижения темпа ускорения до 10 ГэВ/м, что на три порядка превышает практически достигнутый и используемый в настоящее время темп ускорения.

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RESONANCE ACCELERATION OF CHARGED PARTICLES BY A SURFACE WAVE ARISING AT TOTAL INTERNAL REFLECTION

In search of possible ways to enhance the charged particle acceleration rate we have considered a version of resonance acceleration by a surface wave that occurs in vacuum at total internal reflection from a dielectric surface and vanishes towards the normal to the surface. It is suggested to use a polarized laser wave with the wave vector oriented so that one could provide the phase stability of accelerated particles, i.e. the direction of propagation of the surface wave is to make up the Cerenkov angle with the direction of the velocity of charged particles moving along the surface. In order to provide phase stability and high gradients of acceleration, an additional constant magnetic field normal to the surface and the particle velocity is applied. Although the surface wave vanishes exponentially at distances of the order of the laser wavelength from the flat interface, sufficient number of particles per pulse can be captured in the acceleration mode (irrespective of the laser wavelength). The polarization parameters as well as the laser wave incidence angle are chosen from the acceleration rate optimization conditions for reasonable values of the additional constant magnetic fields strengths with respect to the dielectric breakdown limit. The application of existing lasers designed and constructed for thermonuclear fusion purposes (for example, TIR-I) is shown to provide an acceleration gradient of the order of 100 MeV/m. After further improvements in technology the suggested method will allow to obtain acceleration rates as high as 1.0 - 10 GeV/m.

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1. Introduction

The phenomenon of total internal reflection of electromagnetic waves passing through the interface of an optically denser medium into a rarer one is well known ever since one's school-days. Far less popular is the fact that at total internal reflection in the optically rarer medium a surface wave arises (see, e.g. [1-4]), whose field components have significant values only in the immediate proximity to the interface and fall-off exponentially according to the law

$$\exp\left\{-\frac{2\pi z}{\lambda}\sqrt{\varepsilon\sin^2\varphi-1}\right\} \tag{1.1}$$

where z is the distance along the normal to the plane surface, λ is the wavelength in the rarer medium (vacuum in what follows), φ is the incidence angle which is larger than the critical angle φ_g , $\sin\varphi_g = 1/\sqrt{\varepsilon}$, $\varepsilon \equiv \varepsilon(\omega)$ is the dielectric constant of the first, optically denser medium. Note that if the wave vector $\vec{K}$ of the incident wave lies in the XOZ plane, the surface wave in the second medium travels along the X axis with a phase velocity $c(\sqrt{\varepsilon}\,\sin\varphi)^{-1}$, i.e. the effective refraction index for it will be

$$1 < n = \sqrt{\varepsilon} \sin\varphi , \quad \varphi_g < \varphi < \pi/2 \tag{1.2}$$

Thus, the surface wave created in vacuum at total internal reflection is a slowed-down one, which makes it particularly attractive for the purposes of laser acceleration of particles. The averaged per cycle energy flux of the surface wave in the direction OX is nonzero. Therefore, if above the surface in the region where the surface wave field strength is still high, a charged particle is ejected with a velocity $V = c\beta$ at an angle Θ with respect to the OX axis determined from the Cerenkov condition

$$\cos\Theta = \frac{1}{n\beta} ; \quad n\beta > 1 \tag{1.3}$$

the particle will be in phase with the surface wave. It follows from (1.2) and (1.3) that the lower bound on the particle Lorentz factor is

$$\gamma > (1 - n^{-2})^{-1/2} = \gamma_n \tag{1.4}$$

To analyze the feasibility of particle acceleration by a surface wave and the stability of its motion, let us write down the solutions of Maxwell equations for a surface wave in vacuum with a wave vector

$$\vec{K} = \frac{\omega}{c}\left(n\vec{e}_1 + i\sqrt{n^2-1}\,\vec{e}_3\right) , \quad \vec{K}^2 = \omega^2/c^2 \tag{1.5}$$

and arbitrary polarization defined by complex parameters α_1 and α_2

$$\vec{E}(\vec{r},t) = E_0 \, \mathrm{Re}\left[\alpha_1(\vec{e}_1 + i\gamma_n\vec{e}_3) + \alpha_2\vec{e}_2\right]\exp\left\{i(\vec{K}\vec{r} - \omega t)\right\},$$

$$\vec{H}(\vec{r},t) = -E_0\frac{n}{\gamma_n}\,\mathrm{Re}\, i\left[\alpha_2(\vec{e}_1 + i\gamma_n\vec{e}_3) + \frac{\gamma_n^2}{n^2}\alpha_1\vec{e}_2\right]\exp\left\{i(\vec{K}\vec{r} - \omega t)\right\} \tag{1.6}$$

The field (1.6) is not transverse and the scalar product $\vec{E}\cdot\vec{H} \neq 0$, even though the wave travels in vacuum. At $\alpha_1 = 0$ the wave is transverse-electrical, at $\alpha_2 = 0$ transverse-magnetic.

In the coordinate system (x', y', z', t') moving with the wave along the X axis direction ($\vec{e}$) with velocity c/n (static wave system - SWS) the fields (1.6) transform into

$$\vec{E}' = \vec{e}_1(1-\gamma_n)(\vec{e}_1\vec{E}) + \gamma_n\left(\vec{E} + \frac{1}{n}[\vec{e}_1\cdot\vec{H}]\right) = E_0 e^{-\zeta} \mathrm{Re}\,\alpha_1(\vec{e}_1 + i\vec{e}_3)e^{i\phi}, \tag{1.7}$$

$$\vec{H}' = \vec{e}_1(1-\gamma_n)(\vec{e}_1\vec{H}) + \gamma_n\left(\vec{H} - \frac{1}{n}[\vec{e}_1\vec{E}]\right) = -\frac{n}{\gamma_n} E_0 e^{-\zeta} \mathrm{Re}\, i\alpha_2(\vec{e}_1 + i\vec{e}_3)e^{i\phi}, \tag{1.8}$$

where

$$\zeta = \frac{\omega'}{c} z \ , \quad \phi = \frac{\omega'}{c} x', \qquad \omega' = \omega\sqrt{n^2-1} \tag{1.9}$$

Just as it should be, fields $\vec{E}'$ and $\vec{H}'$ are time-independent in SWS; polarization parameters α_1 and α_2 are coupled respectively with the magnitudes of the electric and magnetic fields. The condition of particle vertical equilibrium requires ζ = const, the Cerenkov resonance condition requires phase ϕ stability; hence, in virtue of (1.9), the equilibrium particle in SWS can move only in direction $\vec{e}_2$ with a velocity

$$\vec{\beta}' = \gamma_n\sqrt{\beta^2 - n^{-2}}\,\vec{e}_2 \ , \qquad \gamma' = \gamma/\gamma_n \ , \tag{1.10}$$

which follows also from Lorentz transformations of the particle velocity V . Particles moving with velocity (1.10) in SWS, as it follows from (1.7), do not gain longitudinal acceleration from a surface wave. However, the required electric field component directed along $\vec{e}_z$ in SWS can be obtained by introducing a constant vertical magnetic field b_3 in the lab system directed against the Z axis. Then, in virtue of (1.7), an electric field arises in SWS

$$\vec{E}_3' = \frac{\gamma_n}{n} \vec{e}_z b_3 \tag{1.11}$$

that provides longitudinal acceleration. On the other hand, the field b_3 in the lab system is necessary to maintain phase equilibrium in the wave accelerating phase. The magnitude of b_3 will be determined from this requirement.

The vertical equilibrium evidently requires a field normal to the particle trajectory and parallel to the dielectric surface in the lab system. However for the stability of this equilibrium, the field must be alternating, changing its sign from one section along the acceleration line to another. As it follows from equations of motion, this circumstance in turn demands a jump of the laser wave phase.

Note that the vertical equilibrium conditions are similar to those considered also in another version of laser acceleration based on the Smith-Purcell inverse effect [5], when a particle passing over a diffraction grating at a distance of approximately λ from it is accelerated by the field of the laser waves incident upon the grating under definite angles [6-9]. Here also, in order to maintain vertical equilibrium, an alternating-sign magnetic field parallel to the grating plane and normal to the particle trajectory is needed with a simultaneous change of the laser wave phase [10].

The above-described qualitative picture of the possibility of stable acceleration of charged particles in a surface wave field should be supplemented by a few remarks.

The first one implies that the surface wave fields (1.6) are obtained under the assumption of both stationarity of the total internal reflection process and infinite extension of the dielectric surface and the incident wave front.

In this case the average energy flux normal to the interface of two media is zero. The wave energy flows out alternately through the interface into vacuum and back into the dielectric at some other place of the surface in the same amount, so that the average flux through the surface is zero, but the average flux along the surface in the plane of incidence is nonzero, as it was already mentioned.

For particle acceleration we shall however deal with space-restricted beams of incident electromagnetic waves, and the dielectric breakdown limit at high field strengths requires short pulses.

The total reflection of a finite cross section light beam was investigated for the first time by J.Picht in 1929 and, with a different method, by E.Noether in 1931 [1,4]. These authors showed that formulae for the fields change in such a way that the average energy flux normal to the interface in this case is nonzero. Changes of surface wave fields due to the finiteness of the time interval have not been treated.

The account of both abovementioned circumstances will impose certain restrictions on the space-time dimensions of the laser pulses, or demand at least a quantitative revision of the values of the amplitudes of the surface waves and the external magnetic fields.

Another factor violating the balance of energy fluxes will be the Cerenkov radiation of particles moving in vacuum above a dielectric surface.

The account of this radiation, whose properties are extensively studied (see, e.g. a review by B.M. Bolotovsky [11] and references therein), will result in the limitation of the number of accelerated particles. The estimations of relevant beam loading effects in the suggested acceleration scheme were carried out by one of the authors (H.N.) and reported at the meeting on

Laser Acceleration held in the end of September, 1982 at Nor Amberd near Yerevan. It is important to mention that vertical equilibrium conditions are not so restrictive when beam loading effects are taken into account.

Both in the case of applying the Smith-Purcell inverse effect [5] and in the case under consideration, acceleration takes place in the region spaced from the surface at distances of the order of a laser wavelength, i.e. 10 microns in case of a CO_2-laser. At first sight, this limitation seems severe, decreasing sharply the intensity of the accelerated beams. On the other hand, works of the Novosibirsk group (A.N.Skrinsky et al. [12]) and the Stanford group (B.Richter et al. [13,14]) devoted to designing linear colliding beams - linear colliders - gradually force us to think that it is possible and even necessary to have beams of micron and even submicron diameter in the interaction region in order to obtain luminosities required by physics needs.

In case of single collisions, the beam-beam interaction is of no significance and the diminution of beam diameters becomes principally possible.

Besides, as it follows from (1.1), the beam height above the surface in the considered version of laser acceleration can be effectively adjusted by varying the incidence angle φ .

The idea to use a surface wave arising at total internal reflection of elliptically polarized laser radiation for the particle acceleration purposes was proposed at Yerevan Physics Institute in 1971 [15]. However the configuration of fields with space-variable polarization parameters considered in [15] does not provide a significant stability region in the phase space and gives a low rate of acceleration.

The possibility to use the evanescent waves arising on a dielectric surface at total internal reflection for laser-driven acceleration is mentioned also in the works of J.Lawson [7,16].

Table 1.4. lists the result of calculations of the main physical parameters of the set-up: the average acceleration rate $\langle d\mathcal{E}/d\mathcal{Z}\rangle$, vertical $\mathcal{B}_3 = \mathcal{B}\cos\psi$ and horizontal $\mathcal{B}_\perp = \mathcal{B}\sin\psi$ constant magnetic fields, the length ℓ of the section with a definite sign of field $\mathcal{B}_\perp$. One can see from the table that using the CO_2-laser TIR-I of the Atomic Energy Institute and the Scientific Research Institute for Electrophysical Apparatus [17] with 1 nsec pulse length, 900 cm^2 aperture, 10^3 J energy and total number of shots $\sim 10^3$ (prior to glass change) may provide the acceleration gradient of $\sim$ 100 MeV/m at a field $\mathcal{B} \sim 1$ T and length $\ell \sim 0.1$ cm with KCl taken as a dielectric (to be replaced after 10^3-fold exploitation [18]). As is seen from the table, the acceleration rate can be enhanced 100 times, i.e. up to 10 GeV/m if the pulse length is decreased down to 1 nsec, magnetic field $\mathcal{B}$ increased up to tens T and ℓ is decreased to about 0.01 cm.

The following sections of this work contain basic arguments and calculations of physical parameters listed in Table 1.4.

2. Equilibrium Conditions and Mechanisms of Dynamic Stability

Let's consider the equations of motion of a particle with charge e and mass m moving in the field (1.6) in the presence of an external magnetic field

$$\vec{\mathcal{B}} = -\vec{e}_3 \mathcal{B}_3 + [\vec{e}_3 \cdot \vec{e}\,'] \mathcal{B}_\perp + \vec{e}\, \mathcal{B}_\parallel \tag{2.1}$$

where

$$\vec{e} = \frac{1}{n\beta} \vec{e}_1 + \sqrt{1 - \frac{1}{n^2\beta^2}}\, \vec{e}_2 \tag{2.2}$$

is the direction of particle velocity $\vec{\beta}$ in the lab. system connected with the dielectric, $\mathcal{B}_3$, $\mathcal{B}_\perp$ - vertical and horizontal (sign-changing) magnetic fields, $b_\parallel$ is the magnetic field component parallel to the particle velocity. Fields b_3 and $b_\perp$, perpendicular to the equilibrium trajectory, are strong, whereas field $b_\parallel$ is a weak residual longitudinal one occurring for technical reasons. Neglecting the radiation friction force (the equilibrium trajectories in the ultrarelativistic case are straight lines), we shall obtain the following equations of motion:

$$\frac{d\vec{\beta}\gamma}{dt} = \frac{eE_r}{mc} e^{-\zeta} Re\, e^{i\phi} \Big\{ \alpha_1 \big(\vec{e}_1 + i\gamma_n \vec{e}_3 - \frac{i\gamma_n}{n} [\vec{\beta}\cdot\vec{e}_2]\big) + \tag{2.3}$$

$$+ \alpha_2 \big(\vec{e}_2 - \frac{in}{\gamma_n} [\vec{\beta}\cdot(\vec{e}_1 + i\gamma_n \vec{e}_3)]\big)\Big\} + \frac{e}{mc} [\vec{\beta}\cdot\vec{b}] ,$$

where $\zeta = \frac{\omega}{c} z\sqrt{n^2-1}$ is the particle vertical displacement from the dielectric surface, $\phi = \frac{\omega}{c}(nx - ct)$ is the surface wave phase at particle location. For particles in equilibrium, taking $\zeta = \zeta_s$ = const, $\phi = \phi_s$ = = const and taking into account (2.1), (2.2), (2.3) can be rewritten in the form

$$\vec{e}_1 \frac{d\gamma}{dt} + \vec{e}_2 \frac{d\gamma\sqrt{n^2\beta^2-1}}{dt} = \frac{enE_o}{mc} e^{-\zeta_s} Re\, e^{i\phi_s} a \big(\vec{e}_1 + \frac{i}{\gamma_n}\vec{e}_3\big) + \tag{2.4}$$

$$+ \frac{e}{mc} \big[n\beta b_\perp \vec{e}_3 + b_3 \big(\vec{e}_2 - \vec{e}_1 \sqrt{n^2\beta^2-1}\big)\big] , \quad a = \alpha_1 + \sqrt{n^2\beta^2-1}\,\alpha_2$$

Projection of (2.4) on $\vec{e}_3$ axis yields the vertical equilibrium condition

$$\beta b_\perp = -\frac{E_o}{\gamma_n} e^{-\zeta_s} Re\, i e^{i\phi_s} a \tag{2.5}$$

Two remaining projections on $\vec{e}_1$ and $\vec{e}_2$ axes yield the phase equilibrium condition

$$b_3 = \frac{n^2-1}{n} E_0 e^{-\zeta_s} Re\, e^{i\phi_s} a \Big/ \sqrt{n^2\beta^2-1} \qquad (2.6)$$

and the acceleration rate for particles at equilibrium

$$\frac{d\gamma}{dt} = \frac{\sqrt{n^2\beta^2-1}}{n^2-1} \; \frac{eE_3}{mc} = \frac{eE_0}{mcn} e^{-\zeta_s} R_e e^{i\phi_s} a \qquad (2.7)$$

These conditions are easily generalized for the case when all the quantities are slowly (compared with the laser wavelength and frequency) varying functions of coordinates and time, in particular, functions of the equilibrium particle energy. (Despite the high acceleration rate $\lesssim$ 10 GeV/m, the Lorentz factor change along a wavelength is $\Delta\gamma \sim q = eE_0/(mc\omega) \ll 1$). Our purpose is to clarify, in principle, the consistency of the suggested scheme, therefore we do not consider such technically important aspects as the permissible surface curvature radii, laser pulse spectral composition, scattered fields, etc., where slow variations of the scheme parameters are essential.

Note however that conditions (2.5), (2.6) allow alternations of the sign of $b_\perp$ simultaneously with sharp changes of phase Φ_s (phase jumps), the value and sign of field $b_\smile$ being unaltered.

It is convenient to carry out the stability analysis in the wave rest frame. For particles moving near an equilibrium trajectory we consider the linearized equation of motion in SWS assuming $\vec{r}' \to \vec{r}'_s + \vec{g}$, $\vec{g} = -\vec{e}_1 g_1 + \vec{e}_2 g_2 + \vec{e}_3 g_3$. Taking into account equilibrium conditions (2.5), (2.6) as well as relations (1.7), (1.8), (2.1), (2.2), we derive a set of equations for the components of g

$$\left(\gamma'^3 \dot{g}_2\right)^{\cdot} = \frac{e}{mc}\left(\dot{g}_1 h_3 - \dot{g}_3 \hat{h}\right) \qquad (2.8)$$

$$(\gamma'\dot{g}_1)^{\cdot} + \frac{e}{mc}\left(\omega' b_\perp \frac{\gamma_n}{n}\sqrt{1+n^2\beta'^2/\gamma_n^2}\, g_1 - \omega'\beta'\gamma_n b_3 g_3 + h_3\dot{g}_2 - h_\parallel \dot{g}_3\right) = 0,$$

$$(\gamma'\dot{g}_3)^{\cdot} + \frac{e}{mc}\left(-\omega'\beta'\gamma_n b_3 g_1 - \omega' b_\perp \frac{\gamma_n}{n}\sqrt{1+n^2\beta'^2/\gamma_n^2}\, g_3 - \hat{h}\dot{g}_2 + h_\parallel \dot{g}_1\right) = 0$$

where the dot means differentiation with respect to time t', and the following notations are introduced:

$$h_3 = \frac{n}{\gamma_n} E_0 e^{-\zeta_s} \mathrm{Re}\, e^{i\phi_s}\alpha_2 - \gamma_n b_3$$

$$\hat{h} = \frac{n}{\gamma_n} E_0 e^{-\zeta_s} \mathrm{Re}\, i e^{i\phi_s}\alpha_2 - \frac{b_\parallel - n\beta' b_\perp/\gamma_\parallel}{\sqrt{1+n^2\beta'^2/\gamma_n^2}}, \qquad (2.9)$$

$$h'_\parallel = \frac{b_\perp + n\beta' b_\parallel/\gamma_n}{\sqrt{1+n^2\beta'^2/\gamma_n^2}}$$

In most practically important cases the invariant dimensionless parameter $q = eE_0(mc\omega)^{-1}$ of field strength is much less than unity. Thus, at $eE_0 \sim 3\cdot 10^8$ eV, $\lambda \sim 10^{-3} \div 10^{-4}$ cm we have $q \sim 10^{-2} \div 10^{-3}$. (Note that the magnetic fields must be of the order of 100 T, so that practically all materials are ruptured after single use). It is senseless to use fields in which matter ionization takes place during time $\sim 1/\omega$, i.e. at any wavelength we require that $q < 1/137$. This restriction allows one to apply for the stability analysis the perturbation theory using q as the expansion parameter.

Equilibrium conditions (2.5), (2.6) and the acceleration rate (2.7) which are rewritten in SWS in the form of

$$\beta'\gamma_n b_3 = E_0 e^{-\zeta_s} \mathrm{Re}\, e^{i\phi_s}(\alpha_1 + \alpha_s n\beta'/\gamma_n) \qquad (2.10)$$

$$\gamma_n b_1/n \cdot \sqrt{1+n^2\beta'^2/\gamma_n^2} + E_0 e^{-\xi_s} \mathrm{Re}\, e^{i\phi_s} i(\alpha_1 + \alpha_2 n\beta'/\gamma_n) = 0,$$

$$\dot{\gamma}' = \frac{e b_3 \beta' \gamma_n}{mcn},$$

show that coefficients of the system (2.8) remain practically constant during a time interval typical of variations of g_2 and g_3. Indeed, introducing a dimensionless variable $\Omega t'$, where Ω is the corresponding characteristic frequency, we are convinced that the only choice of Ω, for which these coefficients are arranged in the order of decreasing of small parameter q/γ' beginning with unity at second derivatives, consists in taking $\Omega \sim \omega\sqrt{q/\gamma'}$. Here, according to (2.10), terms with the first derivative, responsible for adiabatic oscillation damping, and coupling coefficients of phase (g_2) and transverse (g_1, g_3) oscillations $\sim\sqrt{q/\gamma'} \ll 1$, whereas coefficients at second and zero derivatives are of the order of unity. The absence of zero derivatives in the equation for g_2 allows one to neglect the variation of g_2 during times $\sim\gamma^2/\Omega$

This implies that in the presence of instability of the non-perturbed system, which according to (2.8) has the form of

$$\ddot{g}_1 + \hat{m} g_1 - m_3 g_3 = 0, \qquad \hat{m} = \frac{e m' b_1 \gamma_n}{mcn\gamma'} \sqrt{1 + n^2\beta'^2/\gamma_n^2},$$

$$\ddot{g}_3 - m_3 g_1 - \hat{m} g_3 = 0, \qquad m_3 = \frac{e\omega'\beta' b_3 \gamma_n}{mc\gamma'}, \tag{2.11}$$

the deviations g, g_3 increase exponentially with a time constant $\sim\sqrt{\gamma'(q\omega'^2)^{-1}}$. During this time corrections from terms $\sim\sqrt{q/\gamma'}$ will amount to less than 10% in the nonrelativistic case (i.e. when in the lab. system the equilibrium particle moves with a velocity $V \sim c/n$), and less than 0.1% in the ultrarelativistic case. Thus stability of motion nearby an equilibrium trajectory is determined basically by the presence in Eqs.(2.11) of oscillating solutions. One can easily verify that in the case $\hat{m}$ = const (m_3 is practically always constant, otherwise the acceleration will not take place), among characteristic roots of Eq.(2.11) there is a positive one, i.e. $g \sim \exp\left\{+\left(\hat{m}^2 + m_3^2\right)^{1/4} t'\right\}$, and stability is absent. A similar trouble occurred at a time in strong-focusing accelerator theory and now we know well how to handle it [19].

We deem in this case that $\hat{m}$ must not be sign-constant over the whole acceleration line. In the accepted flat surface model, slow variations of the wave phase, polarization and amplitude for m_3 = const and $\hat{m}$ $\neq$ const cannot take place. Consider therefore the case when the magnetic field $b_\perp$ is sign-variable, which according to (2.5) leads to the requirement of phase jumps, so the equilibrium conditions aren't violated. Besides, we have to choose the values of the accelerating section lengths and the values of fields $b_\perp$, b_3 on them. We shall take the coefficients constant within the limits of one section. In this approximation real non-uniformity of laser pulse amplitudes and magnetic fields can be taken into account by selection of the model values of coefficients $\hat{m}$, m_3 which result in the same deviations $\vec{q}$ in one section as real fields do.

The methods of analysing Eqs.(2.11) as well as the choice of the focusing structure, i.e. the path lengths and field amplitudes on them are well studied in the theory of conventional strong-focusing accelerators [19]. However the presence of strong coupling (m_3) of vertical and horizontal oscillations results in a quantitative change in the focusing structure parameters, so these structures have to be recalculated. Another distinctive feature of the considered laser-driven accelerator is that the free gaps, on which $b_3 = 0$, are extremely undesirable because this reduces the acceleration rate. Therefore, structures of the type FODO in the laser-driven accelerator are replaced by structures with $b_\perp = 0$ for the "0" type gaps, whereas $b_3 \neq 0$ and particles are accelerated on all the elements of the structure, the acceleration rate in the "0" sections being even higher than that in focusing "F" and defocusing "D" sections. It is seen from (2.8), (2.9) that the choice of polarization parameters affects both acceleration rate and adiabatic variation of the oscillation amplitude. This circumstance permits also to optimize the acceleration rate in order to increase beam intensity and reduce the accelerator cost for a given focusing structure.

Note that the two complex quantities α_1 and α_2 are connected by one condition if the value of the time-averaged electromagnetic wave flux in the medium is fixed (assuming that in vacuum the flux before entering the medium is equal to $cE_0^2/8\pi$, and the reflection is neglected).

This condition is as follows

$$|\alpha_1|^2 \gamma_n^2 (1+\varepsilon^{-1}) + |\alpha_2|^2 = \frac{4}{\sqrt{\varepsilon}} \frac{\varepsilon - n^2}{\varepsilon - 1} \tag{2.12}$$

3. FD Type Focusing Structure

As an example, we consider in what follows the simplest FD type structure with sections of equal length ℓ_1 along the $\vec{e}_1$ axis. The time interval t'_e in SWS during which an equilibrium particle passes through this section in lab system is defined by the formula

$$ct'_e = \ell_1 \sqrt{n^2 - 1} \tag{3.1}$$

Introducing a new independent variable $\tilde{\epsilon}$

$$t' = \tilde{\epsilon}\left(\frac{mc\gamma'}{e b_3 \beta' \omega' \gamma_n}\right)^{1/2} \tag{3.2}$$

we rewrite (2.11) as

$$\frac{d^2 g}{d\hat{\epsilon}^2} + Mg = 0, \quad g - \begin{pmatrix} g' \\ g_3 \end{pmatrix}, \quad M = \begin{pmatrix} tg\psi & -1 \\ -1 & -tgt \end{pmatrix}, \quad tg\psi = \frac{b_\perp}{b_3}\sqrt{\frac{\gamma^2 - 1}{\gamma^2 - \gamma_n^2}}, \tag{3.3}$$

where $b_\perp$ = const within one section and changes its sign within the adjacent one. Noticing that the transformation

$$g = U\tilde{g}, \quad U = \begin{pmatrix} 1 & 1 \\ -e^{-\alpha} & e^{\alpha} \end{pmatrix}, \quad sh\alpha = tg\psi, \quad \cos\psi = 1/ch\alpha \tag{3.4}$$

diagonalizes the matrix

$$M = U^{-1}\begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix} U / \cos\psi \tag{3.5}$$

and taking $\tilde{\epsilon} = \delta\sqrt{\cos\psi}$ in (3.3), we shall derive a solution of Eq.(2.3)

$$g = UCU^{-1}g_0 + USU^{-1}\left.\frac{dg}{d\delta}\right|_{\delta=0}, \quad C = \begin{pmatrix} \cos\delta & 0 \\ 0 & ch\delta \end{pmatrix}, \quad S = \begin{pmatrix} \sin\delta & 0 \\ 0 & sh\delta \end{pmatrix}, \tag{3.6}$$

where δ is counted from the beginning of the section; g_0, $dg/d\delta|_{\delta=0}$ are the initial displacements g_1, g_3 and their derivatives with respect to δ. Here the length ℓ_1 depends on δ according to the formula

$$\ell_1 = \frac{c\delta}{\sqrt{n^2-1}}\left(\frac{mc\gamma'\cos\psi}{e b_3 \beta' \omega' \gamma_n}\right)^{1/2} = \delta\left(\frac{mc^3\gamma\cos\psi}{e b_3 \omega n^2\sqrt{n^2\beta^2-1}}\right)^{1/2} \tag{3.7}$$

A characteristic equation of the transition matrix through two adjacent sections of the same length δ has the form

$$\Lambda^4 - \Lambda^3 a_1 + \Lambda^2 a_2 - \Lambda a_1 + 1 = 0,$$

$$a_1 = 2\cos^2\psi\,(\mathrm{ch}\,2\delta\cos 2\delta + 2\,\mathrm{tg}^2\psi\,\mathrm{ch}\,\delta\cos\delta) \qquad (3.8)$$

$$a_2 = \cos^4\psi\,[2 + (1+\cos^{-2}\psi)^2\,\mathrm{ch}\,2\delta\cos 2\delta +$$

$$+ 8\,\mathrm{tg}^2\psi\,\mathrm{ch}\,\delta\cos\delta + \mathrm{tg}^4\psi\,(\cos 2\delta + \mathrm{ch}\,2\delta + 3)].$$

Values of ψ and δ for which all the roots $|\Lambda_i| = 1$ (i = 1, 2, 3, 4) correspond to the case of stable motion. For arbitrary ψ, δ the solutions of Eq.(3.8) are represented in the form $\Lambda_{1,2} = e^{\pm i\mu^+}$ and $\Lambda_{3,4} = e^{\pm i\mu^-}$, where $\mu^\pm$ are either real or complex conjugate numbers, the stability being absent in the latter case. Real numbers $\mu^\pm$ are dimensionless frequencies of "betatron oscillations", i.e. show the fraction of 2π covered by the length 2δ. From (3.8) we find for these frequencies

$$\cos^2\left(\frac{\mu^\pm}{2}\right) = \frac{1}{4}\cos^2\psi\left[\mathrm{ch}\,\delta - \cos\delta \pm \sqrt{(\mathrm{ch}\,\delta + \cos\delta)^2 + 2\,\mathrm{tg}^2\psi\,(1+\mathrm{ch}\,\delta\cos\delta)}\,\right]^2 \qquad (3.9)$$

Regions of stable values of parameters ψ, δ are defined by the condition $\cos^2(\mu^\pm/2) \leq 1$, namely

$$\frac{\mathrm{ch}\,\delta - \cos\delta}{1-\mathrm{ch}\,\delta\cos\delta} + \sqrt{\left(\frac{\mathrm{ch}\,\delta - \cos\delta}{1-\mathrm{ch}\,\delta\cos\delta}\right)^2 - 1} < \frac{1}{\cos\psi} < \begin{cases} \infty, & \text{при } -1 < \mathrm{ch}\,\delta\cos\delta < 1 \\ \sqrt{\dfrac{\mathrm{ch}^2\delta + \cos^2\delta - 2}{-2(1+\mathrm{ch}\,\delta\cos\delta)}}, & \text{при } \mathrm{ch}\,\delta\cos\delta < -1 \end{cases} \qquad (3.10)$$

The least stable value of $\mathrm{tg}\,\psi \approx 2.26$ occurs at $\delta \approx 2.16$. For these values the length ℓ of the sections along the direction (2.2) of motion of an equilibrium particle, taking into account (3.7) is given by

$$\ell = n\beta\ell_1 = \delta\sqrt{\frac{mc^3\gamma\beta^2\cos\psi}{eb_3\omega\sqrt{n^2\beta^2-1}}} \approx 1.57\sqrt{\frac{mc^3\gamma\beta^2}{eb_3\omega\sqrt{n^2\beta^2-1}}} \tag{3.11}$$

For the remaining stability regions concentrated around $\delta \sim K\pi + \pi/2$ for $K > 1$ the least value of $\operatorname{tg}\psi \sim \frac{1}{2}\operatorname{ch}(K\pi + \pi/2)$, which results on the one hand, in increasing the field $b_\perp$ if γ is not too close to the threshold value γ_n, and on the other hand, decreases significantly the acceleration rate. For this reason the regions with $K > 1$ are not considered below. The region with the least stable value $\operatorname{tg}\psi$ is shown in the Figure.

Formulae (2.11), (3.3 - 3.11) hold, if the relative variation in equilibrium particle energy over the length ℓ is negligibly small. Using (2.7) and (3.11), the following restriction on ℓ_3 is obtained

$$\frac{eb_3}{mc\omega} \ll \frac{(n^2-1)^2\,\gamma}{\delta^2\cos\psi\sqrt{n^2\beta^2-1}}\,. \tag{3.12}$$

Since the right-hand side of this inequality has a minimum with respect to n^2 at $n^2 = (4-\beta^2)(3\beta^2)^{-1}$, which is essential near the threshold $\gamma \approx \gamma_n \sim 1$, then the inequality (3.12) can be rewritten in the form

$$\frac{ab_3}{mc\omega} \ll 16\left(3^{3/2}\beta^4\gamma^2\delta^2\cos\psi\right)^{-1} \sim 1{,}63\beta^{-4}, \quad b_3 \ll 1{,}75\cdot 10^4\,\frac{m\beta^{-4}}{m_e\lambda_{\text{mkm}}}\,\text{Tc} \tag{3.13}$$

4. Optimization of Acceleration Conditions

We will not dwell on a more detailed study of such properties of the set (2.8) as the coupling of phase motions and transverse oscillations, adiabatic variation of the oscillation amplitude and other slow effects, for such an expansion of the problem is beyond the limits of our approximations for our simple model and requires a simultaneous study of a number of problems dealing both with matter behaviour in strong fields and ways of their generation and utilization. Nevertheless, it is possible and necessary to estimate the optimal parameters of the model accelerator. This concerns, first of all, the acceleration rate.

The connection between the magnetic fields b_3, $b_\perp$, the section length ℓ and parameters δ, ψ, i.e. relations (3.3), (3.11), remain the same for a focusing structure of any type (FODO, unequal interval lengths, etc.). Taking into account also that the beam acquires a spatial-modulated structure during acceleration and hence the wave velocity (i.e. parameter n) must not change along a considerable number of line sections, it is possible to obtain a general formula for the mean acceleration rate, by summing up the energy gains in all the sections of the focusing structure element (of periodicity) and dividing then the overall gain by the total length of the sections. Then, with account of (2.7), (3.11) the following formula is derived

$$\left\langle \frac{d\gamma}{d\mathfrak{z}} \right\rangle = \frac{e\sqrt{n^2\beta^2-1}}{mc^2\beta(n^2-1)} \quad \frac{\sum\limits_i \delta_i \sqrt{b_3^{(i)}\cos\psi_i}}{\sum\limits_j \delta_j \sqrt{\cos\psi_j / b_3^{(j)}}} , \tag{4.1}$$

where superscripts i, j number the sections. Equilibrium conditions (2.5), (2.6) allow to rewrite this formula through the wave polarization parameters. For each section they are expressed with account of (2.12) through two independent real parameters χ, $\phi_1 - \phi_2$

$$\alpha_1 = 2e^{i(\phi_1-\phi_s)} \sin\chi\, \varepsilon^{1/4} (\varepsilon^2-1)^{-1/2} (\varepsilon - n^2)^{1/2} / \gamma_n , \tag{4.2}$$

$$\alpha_2 = 2e^{i(\phi_2-\phi_3)}\cos\chi\,\varepsilon^{1/4}(\varepsilon-1)^{-1/2}(\varepsilon-n^2)^{1/2},$$

where $0<\chi<\pi/2$; an arbitrary common phase is added to simplify the calculations. Substitution of (4.2) into the equilibrium conditions (2.5), (2.6) allows to express the magnetic fields through the independent polarization parameters:

$$b_3 = \frac{2E_0 e^{-z_s}\sqrt{\varepsilon-n^2}}{\varepsilon^{1/4}(\varepsilon-1)^{1/2}}\,\frac{n}{\gamma_n}\left(\frac{1}{\varkappa}\sin\chi\cos\phi_1+\cos\chi\cos\phi_2\right) \quad (4.3)$$

$$b_\perp = \frac{2E_0 e^{-z_s}\sqrt{\varepsilon-n^2}}{\varepsilon^{1/4}(\varepsilon-1)^{1/2}}\,\frac{\sqrt{n^2\beta^2-1}}{\beta\gamma_n}\left(\frac{1}{\varkappa}\sin\chi\sin\phi_1+\cos\chi\sin\phi_2\right),$$

$$\varkappa = \gamma_n(1+\varepsilon^{-1})^{1/2}\sqrt{n^2\beta^2-1}\,.$$

One may conclude from (4.1) that the acceleration rate grows with b_3 and reaches a maximum at ψ = const (see (3.3)) for $\phi_1=\phi_2=\psi$, $\mathrm{tg}\,\chi = 1/\varkappa$. Thus the acceleration rate optimized on polarization is given by

$$\left\langle\frac{d\varepsilon}{dz}\right\rangle = eE\,\mathcal{F}(\xi)<\cos\psi>,\quad <\cos\psi> = \sum_i \delta_i\cos\psi_i\Big/\sum_j\delta_j, \quad (4.4)$$

$$\mathcal{F}(\xi) = \exp\left\{-2\pi\xi\sqrt{n^2-1}\right\}\frac{2\sqrt{n^2\beta^2-1}}{n\beta\varepsilon^{1/4}}\sqrt{\frac{\varepsilon-n^2}{\varepsilon-1}(1+\varkappa^{-2})},$$

where the factor $\mathcal{F}(\xi)$ does not depend on the focusing structure type, weakly depends on β , has no singularity in the typical for the Cerenkov resonance point $\beta=1/n$ and produces some surface form-factor still dependent on the equilibrium particle relative height $\xi = z/\lambda$. Varying the incidence angle ψ for a given value of ξ , one can find out such a value of n at which $\mathcal{F}(\xi)$ is at maximum. This maximum can be enhanced by choosing the dielectric constant; however the dielectric breakdown limit varies with ε. A numerical analysis (see Table 1) shows that in any case $\mathcal{F}(\xi) \leq 0.975$. At large ξ, $\beta = 1$ we must have $n - 1 \ll 1$, hence

$$\mathcal{F}(\xi) \approx \frac{2}{\varepsilon^{1/4}} \sqrt{\frac{2\varepsilon+1}{\varepsilon+1}} \sqrt{n^2-1} \, \exp\left\{-2\pi \xi \sqrt{n^2-1}\right\} \tag{4.5}$$

Choosing the angle φ in (1.2) so that $\sqrt{n^2-1} = 1/(2\pi\xi)$, we shall get $\mathcal{F}(\xi) = (\pi e \varepsilon^{1/4} \xi)^{-1} \sqrt{2\varepsilon+1}/\sqrt{\varepsilon+1} \lesssim 0.176/\xi$, i.e. a power instead of an exponential decrease of the acceleration rate. This circumstance may allow one to adjust the device for large heights of the equilibrium tra-jectory ($z \sim 10 \div 100 \lambda$).

5. Limitations of the Acceleration Rate Due to the Dielectric Breakdown Limit

Experimental investigations of typical laser material breakdown [20] by high-power light pulses of $\tau > 10$ nsec duration show that 30% of rupture occurs after the passage of one pulse with an integral flux satisfying the relation

$$J \sim J_c = J_m \ln(e\tau/\tau_m), \tag{5.1}$$

where J_m, τ_m are characteristic constants for each material. For example, for silica glass ED-2 $J_m \approx 3.64$ J/cm^2, $\tau_m \approx 0.63$ ps. If $M > 1$ pulses are required, then the flux J of each of them is to be reduced according to

$$M \approx (J_c/J)^2, \qquad 1 < M < 10^5, \tag{5.2}$$

where for ED-2 glass $\tau \approx 2.45$. Note also that for MgF_2 films $\mathfrak{J}_m \approx 58.7\ J/cm^2$, for KCl samples $\mathfrak{J}_m \approx 170\ J/cm^2$ and $300\ J/cm^2$ for surface and volume rupture, respectively. These values are obtained by recalculating (5.1) with $\tau_m = 0{,}63$ ps the experimental data in nanosecond (MgF_2 [20]) and microsecond (KCl [18]) ranges.

For pulses of arbitrary duration with an amplitude satisfying the constraints (5.1), (5.2) we present E_o in the form

$$E_o = E_m\, g(\tau, M), \quad E_m = \left(\frac{8\pi \mathfrak{J}_m}{c\tau_m}\right)^{1/2}, \quad g(\tau, M) = \left(\frac{\tau_m}{\tau M^{1/2}} \ln \frac{e\tau}{\tau_m}\right)^{1/2} \tag{5.3}$$

The value of E_m, in general, depends on the same atomic constants as $\mathcal{E}$ does, whereas the time factor $g(\tau, M)$ is defined mainly by the avalanche-type breakdown mechanism and sound generation. Numerical values of $g(\tau, M)$ are given in Table 1.4, together with the optimal values of n, $\mathcal{E}$ and $\mathcal{F}(\xi)$. The numerical values of eE_m are, for MgF_2: 265 MeV/cm, for KCl: 451 MeV/cm.

6. Estimation of the Luminosity in the Colliding-Beam Case

The event rate ν for a collision of two identical beams of cross section S, with total number of particles $\mathcal{N}$ in each, is given by

$$\nu = \sigma \mathcal{N}^2 / S . \tag{6.1}$$

where σ is a cross section of the given interaction, ν/σ is the luminosity for the pulse-repetition rate of 1 Hz.

The permissible particle density N in one stability region is estimated by the magnitude of induced fields up to a numerical factor $\eta \sim 0.01 \div 0.1$

$$N = \eta \frac{E_m}{e\lambda} g(\tau, M) \mathcal{F}(\xi) \sqrt{n^2-1} \tag{6.2}$$

For example, in $\sim$ 1 MeV/cm fields for $\lambda \sim 1$ cm we obtain $N \sim \eta \cdot 10^{13} cm^{-3}$, which corresponds to the beam density in conventional accelerators. For laser-driven accelerators, increasing the field 30 times and reducing λ 10^4 times gives $N \sim \eta \cdot 3 \cdot 10^{18} cm^{-3}$. However the total number of particles $\mathcal{N}$ is explicitly λ -independent. Indeed, the transverse dimensions of the stability region are $\sim 0.1 \xi \lambda^2$, the longitudinal - of the order of $D/\cos\theta = nD$, where D is the acceleration path width, i.e. the width of the laser-illuminated magnetic track, finally, the total number of stability regions is $\sim c\tau/\lambda$. Thus, the total beam volume is $\sim 0.1\, n\, \xi\, c\tau D \lambda$ and λ cancels out when being multiplied by (6.2)

The acceleration path width D depends on the total energy of the laser pulse W by a relation

$$W = \frac{c\tau}{8\pi} E_m^2 g^2(\tau, M) \cdot D \cdot \mathcal{L} \cos\varphi \tag{6.3}$$

where the acceleration path length $\mathcal{L}$, according to (4.4), (5.3), is expressed through the particle energy $\mathcal{E}$ in the following way:

$$\mathcal{L} = \mathcal{E} \cdot \left[e E_m \mathcal{F}(\xi) g(\tau, M) \langle \cos\psi \rangle \right]^{-1} \tag{6.4}$$

Taking into account the fact that the beam cross section S decreases adiabatically with increasing energy as $\sim (\gamma_0/\gamma)^{1/2}$, and using Eqs.(6.1-6.4) we shall obtain the construction parameters for one accelerator D, $\mathcal{N}$, ν as functions of the pulse energy

$$D = \frac{8\pi W}{\mathcal{E}} \frac{e\mathcal{F}(\xi) \langle \cos\psi \rangle}{c\tau E_m g(\tau, M) \cos\varphi}, \tag{6.5}$$

$$\mathcal{N}=\frac{8\pi W}{\varepsilon}\cdot 0{,}1\cdot\eta\cdot\xi\cdot\mathcal{F}^2(\xi)\,n\sqrt{n^2-1}\,<\cos\psi>/\cos\varphi, \tag{6.6}$$

$$\nu=\sigma\left(\frac{8\pi W}{\lambda\varepsilon}\right)^2\cdot 0{,}1\cdot\eta^2\cdot\left(\frac{\gamma}{\gamma_0}\right)^{1/2}\cdot\xi\mathcal{F}^4(\xi)\,n^2(n^2-1)<\cos\psi>^2/\cos^2\varphi, \tag{6.7}$$

where parameter ξ is limited by the energy balance condition which is rewritten in the form

$$0{,}1\cdot\xi\cdot\eta<\cos\varphi\cdot\left(8\pi n\sqrt{n^2-1}\,<\cos\psi>\right)^{-1} \tag{6.8}$$

We shall write out also the physical parameters of the magnetic track - fields b_3 , $b_\perp$ and section length ℓ which according to (4.3) are calculated by formulae

$$b_3^{(i)}=E_m\, g(\tau,M)\,\mathcal{F}(\xi)\,\frac{\beta(n^2-1)}{\sqrt{n^2\beta^2-1}}\cos\psi_i\,,$$

$$b_\perp^{(i)}=E_m\, g(\tau,M)\,\mathcal{F}(\xi)\sqrt{n^2-1}\,\sin\psi_i\,, \tag{6.9}$$

$$\ell^{(i)}=\frac{c\delta_i}{\omega}\left(\frac{mc\omega\beta\gamma}{eE_m g(\tau,M)\mathcal{F}(\xi)(n^2-1)}\right)^{1/2}$$

The typical scale of advancement of particle physics researches into the high-energy region is determined by the total energy of the laser pulse W , if $\sigma=\sigma_0(\hbar c/\varepsilon)^2$ (cross section of particle pair production with mass $\sim\varepsilon/c^2$ in the S - state) provided that at least one event with this cross section is observed during M cycles of the accelerator operation. From (6.7) the following limitation is obtained

$$\mathcal{E} \lesssim \mathcal{E}_{W_0} G_M^{2/7}, \quad \mathcal{E}_{W_0} = \left(\frac{8\pi \hbar c W}{\lambda \mathcal{E}_0^{1/4}} \right)^{4/7}$$

$$G_M = 0{,}1 \cdot M \cdot \sigma_0 \cdot \eta^2 \cdot \xi \mathcal{F}^4(\xi) n^2 (n^2 - 1) < \cos\psi >^2 / \cos^2\varphi \tag{6.10}$$

For $W \sim 10^3$ J (TIR-I), $\mathcal{E}_0 \sim 1$ GeV, $\lambda \sim 10\,\mu$ we have $\mathcal{E}_{W_0} \sim 100$ GeV. Increasing W up to 10^5 J (Shiva-Nova-II) [17] for $\lambda \sim 1\,\mu$ the limit $\mathcal{E}_{W_0}$ increases up to 5 TeV. We did not take into account the possibility of additional focusing of the beams before the collision, which will raise the statistics.

7. Cost Optimization

Let us consider a procedure to optimize the number of "shots" M. The technically consistent formulation of the problem requires that the resource of all the systems and elements of the accelerator stand M -fold operation. For instance, the difference in breakdown thresholds of laser glass and the dielectric requires that the light should be focused after passing the last amplification phase. The amplification factor is to be chosen so that the cost of the glass should be equal to that of several magnetic tracks which are to be replaced during operation (or vice versa, if the tracks are more expensive).

Thus, in the first approximation the accelerator cost C is determined by the cost $C_1 (\mathcal{L}/\mathcal{L}_{min})^p$ of the magnetic tracks (here $\mathcal{L}_{min}$ is the length of a single track) and the laser cost $C_W \left(\frac{W}{W_{max}} \right)^q$ where W_{max} is the maximum energy of the pulse necessary to observe an event of energy $\mathcal{E}$ at $M = 1$. Noticing that according to (6.10) $W^2 M = W_{max}^2$ and minimizing the function

$$C = C_1 M^{\frac{p}{2\tau}} + C_w / M^{\frac{q}{2}} \tag{7.1}$$

over M , we find the optimal number of shots

$$M = \left(\frac{q\tau}{p} \frac{C_w}{C_1}\right)^{\frac{2}{q+p/\tau}} \tag{7.2}$$

and economy percent Q , compared with the single variant, in the form of

$$Q = \left(1 - \frac{C}{C_1 + C_w}\right) \cdot 100\% = \left[1 - \frac{(C_w/C_1)^{\frac{1}{R+1}}}{1 + C_w/C_1} \cdot \left(R^{\frac{1}{R+1}} + R^{-\frac{R}{R+1}}\right)\right] \cdot 100\% , \tag{7.3}$$

where $R = \tau q/p$. It is evident that (7.2), (7.3) make sense only for $C_w/C_1 > p \cdot (\tau q)^{-1}$, i.e. if the laser cost is sufficiently high. The pair of quantities $\{M^q, Q\}$ is tabulated (see Table 2) in the range $1/8 \le C_w/C_1 \le 8$, $R = 1 \div 8$. Note that M for modern lasers designed for thermonuclear research needs ranges from a few tens up to a few thousands [17] .

Table 1.1

Optimal values of the polarization coefficients $|\alpha_1|$, $|\alpha_2|$, formfactor $\mathcal{F}$ and parameters ε, n as functions of the equilibrium particle velocity β for $\xi = z/\lambda \to 0$.

β	0.28	0.396	0.512	0.628	0.744	0.86	$\gamma = 4$	$\gamma = 8$	$\gamma = 100$
$\mathcal{F}$	0.6305	0.7180	0.7751	0.8258	0.8733	0.9193	0.9691	0.9720	0.9750
ε	38.196	18.991	13.152	10.072	7.9190	6.2583	4.6536	4.8328	4.6251
n	3.5714	2.5253	2.1295	1.8961	1.7146	1.5632	1.4107	1.4129	1.4024
$\lvert\alpha_1\rvert$	0.6305	0.7180	0.6704	0.6009	0.5380	0.4781	0.4106	0.4082	0.4032
$\lvert\alpha_2\rvert$	0.0005	0.0010	0.4021	0.5915	0.7273	0.8433	0.9695	0.9700	0.9807

Table 1.2

Optimal values of ε , n , $\mathcal{F}$, $|\alpha_1|$, $|\alpha_2|$ as functions of ξ at $\beta = 1$.

ξ	ε	n	$\lvert\alpha_1\rvert$	$\lvert\alpha_2\rvert$	$\mathcal{F}(\xi)$
0.	4.78807	1.41190	0.40380	0.97627	0.97518
0.05	3.37161	1.24284	0.38985	1.05796	0.74698
0.1	2.76690	1.16815	0.36202	1.11386	0.60604
0.2	2.16982	1.09603	0.30595	1.19673	0.43760
0.4	1.68081	1.04278	0.22262	1.30605	0.27768

Table 1.3

Dielectric breakdown factor $g(\tau, M)$. The values of τ are in ps (10^{-12} s).

τ \ M	1	10	100	1000	10000
0.63	1	0.62506	0.39069	0.24421	0.15264
30	0.31957	0.19975	0.12486	0.07804	0.04878
100	0.19551	0.12220	0.07638	0.04774	0.02984
1000	0.07262	0.04539	0.02837	0.01773	0.01108

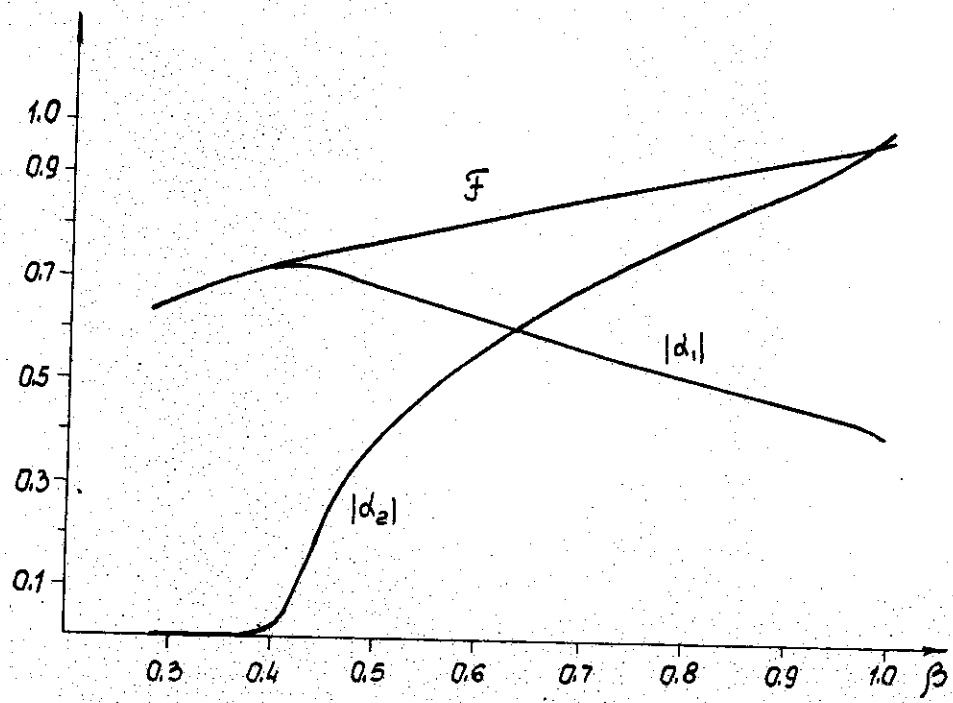


Diagram to Table 1.1

Table 1.4. Main physical parameters T [MeV/cm] , $\mathcal{B}$ [T] , ℓ_0 [cm] which define:

i) the average acceleration rate

$$\left\langle \frac{d\mathcal{E}}{d\mathcal{Z}} \right\rangle = T \langle \cos\psi \rangle, \quad T = eE_m\, g(\tau, M)\, \mathcal{F}(\xi);$$

ii) vertical $\mathcal{B}_3$ and horizontal $\mathcal{B}_\perp$ magnetic fields

$$\mathcal{B}_3 = \mathcal{B}\cos\psi, \quad \mathcal{B}_\perp = \mathcal{B}\sin\psi, \quad \mathcal{B} = E_m\, g(\tau, M)\, \mathcal{F}(\xi)\sqrt{n^2-1};$$

iii) length ℓ of the section with a fixed sign of $\mathcal{B}_\perp$

$$\ell = \ell_0\,\delta\left(\mathcal{E}_{GeV}\cdot\lambda_{mcm}\right)^{1/2}, \quad \ell_0 = 10^5\cdot\left(e\mathcal{B}\sqrt{n^2-1}\cdot 2\pi\right)^{-1/2}$$

for two values of the constant $eE_m = \{265:, 451\}$ MeV/cm. The values of the parameters are defined for an optimal choice of $\mathcal{E}$, n for $\xi = Z/\lambda = \{0:, 0.1; 0.4\}$ versus both pulse duration τ (in pico-seconds) and number of runs M .

All the quantities are weakly sensitive to $\mathcal{E}$ near the optimal values given in Table 1.2. Always $\gamma \gg \gamma_n$, i.e. the ultrarelativistic case is considered.

In order to calculate the necessary values of the physical parameters $\langle d\mathcal{E}/d\mathcal{Z}\rangle$, $\mathcal{B}_3$, $\mathcal{B}_\perp$, ℓ one should choose definite values of ψ and δ . Minimum of ψ and maximum of δ (the edge of the stability region) are mentioned in the text.

Table 1.4

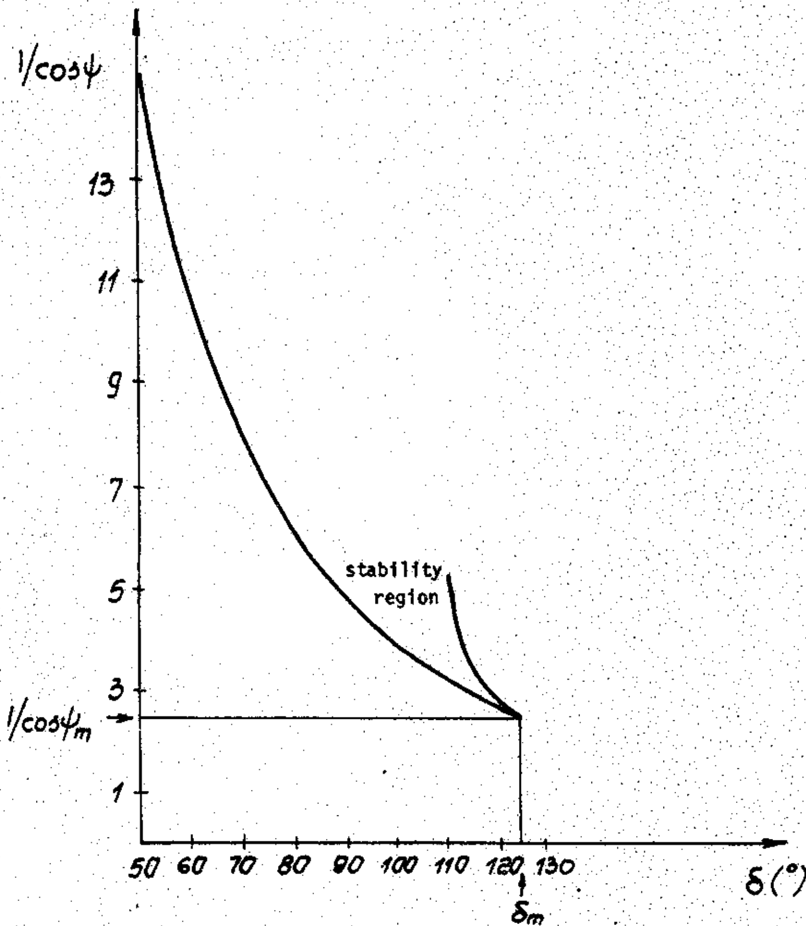
M =		1						10						1000					
eE_M =		451 MeV/cm			265 MeV/cm			451 MeV/cm			265 MeV/cm			451 MeV/cm			265 MeV/cm		
τ[cs]	ξ	T $\frac{MeV}{cm}$	bT	ℓ_o cm	T $\frac{MeV}{cm}$	bT	ℓ_o cm	T $\frac{MeV}{cm}$	bT	ℓ_o cm	T $\frac{MeV}{cm}$	bT	ℓ_o cm	T $\frac{MeV}{cm}$	bT	ℓ_o cm	T $\frac{MeV}{cm}$	bT	ℓ_o cm
	0	440	146	0.006	258	85.9	0.0079	275	91.3	0.0076	162	53.7	0.01	107	35.7	0.012	63	21	0.016
0.63	0.1	273	55	0.0126	161	32.3	0.0165	171	34.4	0.016	100	20.2	0.021	66.7	13.4	0.0256	39.2	7.89	0.033
	0.4	125	12.3	0.038	73.6	7.25	0.05	78.3	7.71	0.048	46	4.53	0.063	30.6	3.01	0.077	18	1.77	0.101
	0	141	46.7	0.0107	82.6	27.4	0.0139	87.9	29.2	0.0135	51.6	17.2	0.0176	34.3	11.4	0.0216	20.2	6.7	0.028
30	0.1	87.3	17.6	0.0224	51.3	10.3	0.0292	54.6	11	0.0283	32.1	6.46	0.0369	21.3	4.29	0.0452	12.5	2.52	0.059
	0.4	40	3.94	0.0675	23.5	2.32	0.088	25	2.47	0.0853	14.7	1.45	0.111	9.77	0.96	0.137	5.74	0.57	0.178
	0	86	28.6	0.0136	50.5	16.8	0.0178	53.7	17.9	0.0173	31.6	10.5	0.0225	21	6.98	0.0276	12.3	4.1	0.036
100	0.1	53.4	10.8	0.0286	31.4	6.3	0.0373	33.4	6.72	0.0362	19.6	3.95	0.0472	13	2.63	0.0578	7.67	1.54	0.075
	0.4	24.5	2.41	0.0862	14.4	1.42	0.113	15.3	1.51	0.109	8.99	0.886	0.142	5.98	0.59	0.175	3.51	0.35	0.228
	0	31.9	10.6	0.0224	18.8	6.23	0.0292	20	6.63	0.0283	11.7	3.9	0.037	7.8	2.59	0.0453	4.58	1.52	0.059
1000	0.1	19.8	3.99	0.0469	11.7	2.35	0.0612	12.4	1.51	0.0763	7.29	1.47	0.0774	4.85	0.975	0.0949	2.85	0.573	0.124
	0.4	9.09	0.896	0.142	5.34	0.527	0.185	5.68	0.56	0.179	3.34	0.329	0.234	2.22	0.219	0.286	1.3	0.129	0.374

All data is computed for optimal acceleration rate values of the dielectric constant ε and phase velocity parameter n.

Table 2

Cost optimization.

C_w/C_1 \ R	1	2	4	8
1/8				1; 0
1/4			1; 0	3.43: 2.8
1/2		1; 0	3.03; 4.3	11.8 ; 12.5
1	1; 0	2.52; 5.5	9.19; 17.5	40.3 ; 29
2	2; 5.7	6.35; 20.6	27.9; 36.8	138 ; 49
4	4; 20	16 ; 40	84.4; 56.5	474 ; 67
8	8; 37	40.3; 58	256 ; 72	1625 ; 80



Dependence of $1/\cos\psi$ on δ (in degrees).

The edge point of the stability region

δ_m = 123.72631^0 → 2.159431480 rad, →

$1/\cos\psi_m$ = 2.472995323, which corresponds to

ψ_m = 66.14847554^0, $\cos\psi_m$ = 0.4043679301,

$\sin\psi_m$ = 0.91459640.

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