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G.V.GRIGORYAN, R.P.GRIGORYAN, I.V.TYUTIN

RENORMALIZATION OF N=4 SUPERGAUGE THEORY

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Г.В.ГРИГОРЯН, Р.П.ГРИГОРЯН, И.В.ТЮТИН

ПЕРЕНОРМИРОВКА N =4 СУПЕРКАЛИБРОВОЧНОЙ ТЕОРИИ

Доказано сохранение суперсимметрии N =4 после перенормировки суперсимметричной янг-миллсовской теории.

Ереванский физический институт
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The conservation of N=4 supersymmetry after the renormalization of N=4 supersymmetric Yang-Mills theory is proved.

Yerevan Physis Institute

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RENORMALIZATION OF N=4 SUPERGAUGE THEORY

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1. The supergauge N=4 model [1] had been recently widely discussed in literature. The model is notable, in particular, because it has a vanishing three-loop Gell-Mann-Low β-function [2-6]. However the question whether the model can be supersymmetrically renormalized was not so far considered. This paper is devoted to the investigation of this problem. The proof is based on the analysis of super Ward identities for $\Gamma(\Phi)$, the generating functional of the one particle irreducible (proper) vertices. In sec.2 the derivation of supersymmetric Ward identities is presented. Sec.3 contains the results of the diagrammatic analysis of the divergences of the vacuum expectation values contributing to the Ward identities. Sec.4 presents the most general expression for the structure of divergences of the $\Gamma(\Phi)$ that respects the gauge invariance, while sec.5 contains the analysis of super Ward identities. The investigations of secs.2-5 were carried out in the Wess-Zumino gauge. In sec.6 the supersymmetric (N=1) gauge is discussed.

We prove the renormalizability on the assumption of the existence of the regularization that conserves the supersymmetric properties of the theory.

2. The free Lagrangian of the N=4 supergauge model has the form [1]

$$S_0 = \frac{1}{g^2}\Big\{ -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \frac{1}{4}\left(\nabla^{ab}_\mu \varphi^b_{ij}\right)\left(\nabla^{\mu,ac} \varphi^{c,ij}\right) +$$

$$+ i\bar{\chi}_i^a \gamma^\mu \nabla_\mu^{ab} \chi^{i,b} + \frac{i}{\sqrt{2}} f^{abc} \left[-\chi^{a,i} C L \chi^{j,b} \varphi_{ij}^c - \bar{\chi}_i^a C R \bar{\chi}_j^b \varphi^{c,ij} \right] -$$

$$- \frac{1}{16} f^{abc} \varphi_{ij}^b \varphi_{\kappa n}^e f^{adf} \varphi^{d,ij} \varphi^{f,\kappa n} \Big\}$$

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c, \quad \nabla_\mu^{ab} = \delta^{ab} \partial_\mu + f^{acb} A_\mu^c \quad (1)$$

$$\varphi_{ij}^a = -\varphi_{ji}^a, \quad \varphi_{ij}^{a*} = \varphi^{a,ij} = \frac{1}{2} \varepsilon^{ij\kappa e} \varphi_{\kappa e}^a, \quad i, j = 1, 2, 3, 4$$

$$L = \frac{1}{2}(1 + \gamma_5), \quad R = \frac{1}{2}(1 - \gamma_5)$$

Here A_μ^a , $\chi^{a,i}$, φ_{ij}^a are multiplets of gauge, spinor and scalar fields, respectively, which transform as the adjoint representation of the gauge group, f^{abc} are structure constants of the gauge group; moreover, the fields $\chi^{a,i}$, $\bar{\chi}_i^a$ and φ_{ij}^a transform (over the indices i , j) as representations 4, $\bar{4}$ and 6 of the SU(4) group; C is the charge conjugation matrix, g is interaction constant.

The action (1) is invariant under gauge transformations

$$\delta g A_\mu^a = \nabla_\mu^{ab} \lambda^b, \quad \delta_g \chi^{a,i} = f^{acb} \chi^{c,i} \lambda^b, \quad \delta g \varphi_{ij}^a = f^{acb} \varphi_{ij}^c \lambda^b \quad (2)$$

(λ^b is the infinitesimal gauge parameter), and also under global supersymmetry transformation [1]

$$\delta_s A_\mu^a = i \left(\bar{\alpha}_i \gamma_\mu L \chi^{a,i} - \bar{\chi}_i^a R \gamma_\mu \alpha^i \right)$$

$$\delta_s \varphi_{ij}^a = i\sqrt{2} \left(\bar{\alpha}_j R C \bar{\chi}_i^a - \bar{\alpha}_i R C \bar{\chi}_j^a - \varepsilon_{ij\kappa e} \alpha^\kappa C L \chi^{a,e} \right), \quad (3)$$

$$\delta_s\left(L\chi^{a,\kappa}\right)=\frac{1}{2}\sigma_{\mu\nu}\,G^{\mu\nu,a}L\alpha^{\kappa}+\sqrt{2}\left(\nabla_{\mu}^{ab}\varphi^{b,\kappa e}\right)\gamma^{\mu}RC\bar{\alpha}e+$$
$$+f^{abc}\varphi^{b,\kappa n}\varphi^{c}_{ne}\,L\alpha^{e},$$
$$\delta_s\left(\bar{\chi}_i^{\,a}R\right)=-\frac{1}{2}\bar{\alpha}_i\,R\,G^{\mu\nu,a}\,\sigma_{\mu\nu}-\sqrt{2}\left(\nabla_{\mu}^{ab}\varphi^{b}_{ij}\right)\alpha^{j}CL\gamma^{\mu}+$$
$$+f^{abc}\varphi^{b}_{in}\,\varphi^{c,ne}\,\bar{\alpha}e\,R$$

(α^i , $\bar{\alpha}_i$ are spinors, infinitesimal Grassmann parameters of supertransformations, transforming as 4 and $\bar{4}$ of the SU(4) group).

Henceforth for the set of vector A_μ^a , scalar φ_{ij}^a and spinor $\chi^{a,i}$, $\bar{\chi}_i^{\,a}$ fields the notation $\phi^{a,b}=(A_\mu^a,\varphi_{ij}^a,\chi^{a,i},\bar{\chi}_i^{\,a})$ will be used.

Consider the generation functional Z of Green's functions of the fields $\phi^{b,B}$:

$$Z=\int d\phi\exp\left\{\frac{i}{\eta}\left[L_0-\frac{\alpha}{2}t^at^a-i\eta\,\mathrm{Sp}\ln M^{ab}(\phi)+J_B^b\phi^{b,B}\right]\right\}=$$
$$=\int d\phi\,dc\,d\bar{c}\,\exp\left\{\frac{i}{\eta}\left[L_0-\frac{\alpha}{2}t^at^a+\bar{c}^aM^{ab}(\phi)c^b+J_B^b\phi^{b,B}\right]\right\},\qquad(4)$$

$$t^a=\partial_\mu A^{\mu,a},\quad M^{ab}(\phi)=\frac{\delta t^a}{\delta\phi^{d,B}}R^{B,db}=\partial_\mu\nabla^{\mu,ab}$$

where L_0 is given by (1), C , $\bar{C}$ are ghost fields, $R^{B,ab}$ are generators of the gauge transformations of the fields $\phi^{a,B}$, $(-\frac{\alpha}{2}t^at^a)$ is the gauge fixing term.

Prior to the derivation of supersymmetric Ward identities, one important fact must be stated. Under the change of variable $\phi^{b,B}\to\phi^{b,B}+\delta_s\phi^{b,B}$

in the functional $Z(J)$, $\delta_s \phi^{b,B}$ being given by formulae (3), the expressions for t^a and M^{ab} naturally transform into $t'^a = t^a(A_\mu + \delta_s A_\mu)$ and $M'^{ab} = M^{ab}(A_\mu + \delta_s A_\mu)$, respectively. It turns out, however, that the expression for M'^{ab} coincides with the one constructed using the function t'^a. In other words, for the theory under consideration the result of the supersymmetry transformations (3) is equivalent to a transformation of the gauge. Using this fact and the invariance of L_0 under the gauge transformations (2), we change the variables in (4)

$$\phi^{b,B} \to \phi^{b,B} + \bar{\delta}\phi^{b,B},$$
$$\bar{\delta}\phi^{b,B} = \delta_s \phi^{b,B} + \delta_g \phi^{b,B} = \delta_s \phi^{b,B} - R^{B,bc} \mathcal{D}^{cd} \delta_s t^d \qquad (5)$$

Here $\delta_s \phi^{b,B}$ are given by (3), $\delta_g \phi^{bB}$ denote transformations (2) with a gauge parameter

$$\lambda^b = -\mathcal{D}^{bc} \delta_s t^c,$$

$\mathcal{D}^{bc}$ being defined by the equation

$$M^{ab} \mathcal{D}^{bc} = \delta^{ac}$$

Under this change of variables t^a does not change. Besides, it is easy to show that the Jacobian of this change of variables ($\phi^{b,B}$ by $\phi'^{b,B}$) in the functional integral is compensated by the change of the summand $-i Sp \ln M^{ab}(\phi)$ in the exponent.

Thus, after the change of variables (5) the generating functional Z is given by

$$Z=\int d\phi \exp\left\{\frac{i}{\eta}\left[L_0-\frac{\alpha}{2}t^a t^a - i\eta S_p \ln M^{ab} + J_B^b \phi^{b,B} + \right.\right. \tag{6}$$

$$\left.\left. + J_B^b \bar{\delta}\, \phi^{b,B}\right]\right\}$$

From (6) follows the Ward identity

$$J_B^b\left[\delta_s \phi^{b,B} - R^{B,bc}\mathcal{D}^{cd}\,\delta_s t^d\right] z(J) = 0 \tag{7}$$

In the identity (7) the exchange of $\phi^{b,B}$ by $\frac{\eta}{i}\frac{\delta}{\delta J_B^b}$ in the operator in the brackets is presumed.

Introducing now the generating functional $\Gamma(\phi)$ of proper vertices

$$\Gamma(\phi) = -i\eta \ln z(J) - J_B^b\, \phi^{b,B}$$

$$\phi^{b,B} = -i\eta \frac{\delta}{\delta J_B^b} \ln z(J)$$

we write down (7) in terms of $\Gamma(\phi)$

$$\frac{\delta \Gamma}{\delta \phi^{b,B}} < \delta_s \phi^{b,B} - R^{B,bc}\mathcal{D}^{cd}\,\delta_s t^d > = 0$$

Here $<\ldots\ldots>$ is the vacuum expectation, which is a function of ϕ and which is obtained from the operator in the brackets of identity (7) by replacement

$$\phi^{b,B} \rightarrow \phi^{b,B} + i\eta(-1)^{P_B(P_B+1)}\left[(\Gamma'')^{-1}\right]^{bd,BD}\frac{\delta_\Lambda}{\delta \phi^{d,D}}$$

where $\delta_L/\delta\phi^{d,D}$ is a left derivative, P_B is the Grassmann parity of the field $\phi^{b,B}$. and $\left[(\Gamma'')^{-1}\right]^{bd,BD}$ denotes the matrix reverse to

$(\Gamma'')^{bd}_{BD} = \delta^2\Gamma/\delta\phi^{b,B}\delta\phi^{d,D}$. In the following it will be convenient to turn from the functional $\Gamma(\phi)$ to $\bar{\Gamma}(\phi)$ defined by the relatio

$$\bar{\Gamma}(\phi) = \Gamma(\phi) + \frac{\alpha}{2} t^c t^a$$

Using the relation

$$< \mathcal{D}^{ab} P(\phi) > = -\frac{i}{\eta} < c^a \bar{c}^b P(\phi) >,$$

where $P(\phi)$ is an arbitrary functional of the fields $\phi^{b,B}$, we obtain for the functional $\bar{\Gamma}(\phi)$ the identity

$$\frac{\delta\bar{\Gamma}}{\delta\phi^{b,B}} < \delta_s \phi^{b,B} > + \frac{i}{\eta} \frac{\delta\bar{\Gamma}}{\delta\phi^{b,B}} < R^{B,bc} c^c \bar{c}^d \delta_s t^d > = 0. \tag{8}$$

In the same notations the gauge Slavnov-Taylor identity, which is a result of the invariance of L_o under transformations (2), reads

$$\frac{\delta\bar{\Gamma}}{\delta\phi^{b,B}} < R^{B,\hat{b}c} c^c \bar{c}^d > = 0. \tag{9}$$

Using the identity (9) the second summand in (8) can be rewritten in terms of quantities dscribed by one particle irreducible Feynman diagrams. To this end, introduce the notations $<\dots>^c$ and $< \quad >^{OPI}$ for the connected and one particle irreducible parts of $<\dots>$. Let us define, besides,

$$f^{abd} < \phi^{b,B} c^d \bar{c}^c \delta_s t^c >^{OPI} \equiv \eta\, T_\phi^{a,B}$$

$$< c^b \bar{c}^c \delta_s t^c >^c \equiv < c^b \bar{c}^c > T^c_{\delta_s t}$$

$$< R^{B,ba} c^a \bar{c}^c >^c \equiv < c^a \bar{c}^c > T_1^{B,ba}$$

where $T_{\phi}^{a,b}$, $T_{\delta_s t}^{c}$, $T_1^{B,ba}$ are one particle irreducible diagrams. Taking into account that

$$<R^{B,ba}C^a\bar{C}^c\delta_s t^c> = <R^{B,ba}C^a\bar{C}^c>\left(\delta_s t^c + T_{\delta_s t}^c\right) + \eta T_{\phi}^{b,B}$$

and using (9), the identity (8) can be transformed into

$$\frac{\delta\Gamma}{\delta\phi^{b,B}}\left[\delta_s\phi^{b,B} + \eta <<\delta\phi^{b,B}>>\right] = 0 \tag{10}$$

where

$$<\delta_s\phi^{b,B}> = \delta_s\phi^{b,B} + \eta <\delta_s\phi^{b,B}>^{0\,PI},$$

$$\ll\delta\phi^{a,B}\gg = <\delta_s\phi^{a,B}>^{0\,PI} + i\,T_{\phi}^{a,B}.$$

Let us now exploit (10) for the analysis of the structure of the divergent parts of the functional $\bar{\Gamma}(\phi)$. To do that, we must make use of the loop decomposition of the quantities entering (10)

$$\bar{\Gamma} = S_0 + \eta\bar{\Gamma}_1 + \eta^2\bar{\Gamma}_2 + \ldots ,$$

$$<<\delta\phi^{b,B}>> = <<\delta\phi^{b,B}>>_1 + O(\eta),$$

and also detach their divergent and finite parts

$$\Gamma_1 = \bar{\Gamma}_{1fin} + \bar{\Gamma}_{1div},$$

$$\ll\delta\phi^{b,B}\gg_1 = \ll\delta\phi^{b,B}\gg_{1fin} + \ll\delta\phi^{b,b}\gg_{1div}$$

The identity for the divergent part of $\bar{\Gamma}_1(\phi)$ following from (10), then reads

$$-\frac{\delta \Gamma_{1div}}{\delta \phi^{b,B}} \delta_s \phi^{b,B} + \frac{\delta S_0}{\delta \phi^{b,B}} \langle\langle \delta \phi^{b,B} \rangle\rangle_{1div} = 0 \tag{11}$$

The expression (11) is the equation for the $\overline{\Gamma}_1 div$ and we'll proceeed now with solving it.

3. To find out the structure of vacuum expectation values one has to analyze the diagrams, taking into account the SU(4) symmetry, the momentum dependence, the γ -matrices structure, the CP invariance *. The divergence index of the diagrams $\langle\langle \delta\phi^{b,B} \rangle\rangle$ is equal to $\omega = d_{\phi^{b,B}} + \frac{1}{2} - N - \frac{3}{2} n$, where $d_{\phi^{b,B}}$ is the dimension of the field $\phi^{b,B}$, N, n are the numbers of external boson and fermion lines, respectively.

The result of the analysis is that

$$\langle\langle \delta A^a_\mu \rangle\rangle = \eta a \delta_s A^a_\mu + \eta a_1 (\bar{\alpha}_K \gamma_\mu \chi^{K,a}_L - \bar{\chi}^a_{R,K} \gamma_\mu \alpha^K) + O(\eta^2),$$

$$\langle\langle \delta \varphi^a_{ij} \rangle\rangle = \eta a \delta_s \varphi^a_{ij} + \eta b_1 (\bar{\alpha}_i CR \bar{\chi}^a_j - \bar{\alpha}_i CR \bar{\chi}^a_i - \varepsilon_{ijKe} \alpha^K C_L \chi^{a,e}) + O(\eta^2)$$

$$\begin{aligned} \langle\langle \delta \chi^{a,K} \rangle\rangle = \eta a \delta_s \chi^{a,K} + \eta \big[(& u_1 \sigma_{\mu\nu} F^{\mu\nu,a} + u_2 \partial_\mu A^{\mu,a} + \\ & + t_1^{abc} \sigma_{\mu\nu} A^b_\mu A^c_\nu + t_2^{abc} A^b_\mu A^{\mu,c} + t_3^{abc} \varphi^{b,ij} \varphi^c_{ij}) \alpha^K_L + \\ & + (u_3 \partial_\mu \varphi^{a,Ke} + t_4^{abc} \varphi^{b,Ke} A^c_\mu) \gamma^\mu RC \bar{\alpha}_e + \\ & + t_5^{abc} \varphi^{b,Kj} \varphi^c_{je} \alpha^e_L \big] + O(\eta^2) \end{aligned} \tag{12}$$

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* The definition of C transformations for gauge theories is given in 7

where a , a_1, b_1 , u_1 , u_2 , u_3 are divergent constants, t_α^{abc} are divergent quantities with invariant isotopic structure. The structure of divergences of $\langle\langle \delta \bar{\dot{\chi}}_K^a \rangle\rangle$ is not written down since it, due to the CP invariance (which like any global symmetry holds for the vacuum expectations), can be derived from $\langle\langle \delta \chi^{a,K} \rangle\rangle$ by the Dirac conjugation of fields (and not of coefficients). Also due to the CP invariance we have only by one divergent constant in each of the one-loop approximations of $\delta \dot{A}_\mu^a$ and $\delta \dot{\varphi}_{ij}^a$.

4. The general structure of Γ_{1div} which respects the gauge symmetry was deduced in [9]

$$\Gamma_{1div} = \frac{1}{g^2}\Big[-\frac{1}{4}\delta Z_1 (G_{\mu\nu}^a)^2 + i\delta Z_2 \bar{\chi}_i^a \gamma_\mu \nabla^{\mu,ab} \chi_L^{b,i} +$$
$$+ \frac{1}{4}\delta Z_3 (\nabla_\mu^{ab} \varphi_{ij}^b)(\nabla^{\mu,ac}\varphi^{ij,c}) + \delta \tilde{Z}_{\varphi^2} \varphi_{ij}^a \varphi^{ij,a} - \qquad (13)$$
$$- \frac{i}{\sqrt{2}} \delta Z_{\chi^2\varphi}^{abc} (\chi^{ai} C L \chi^{jb} \varphi_{ij}^b + \varphi^{ij,c} \bar{\chi}_i^a C R \bar{\chi}_j^b) -$$
$$- \frac{1}{16}\left(\delta Z_{\varphi^4}^{bcdf} \varphi_{ij}^b \varphi^{ij,d} \varphi_{k\ell}^c \varphi^{k\ell,f} + \delta \tilde{Z}_{\varphi^4}^{bcdf} \varphi_{ij}^b \varphi^{jkd} \varphi_{kc}^c \varphi^{\ell i,f} \right) +$$
$$+ \frac{1}{2}\delta Z_A \Big\{ i \bar{\chi}_i^a \gamma_\mu f^{acb} A^{\mu,c} \chi^{b,i} + \frac{1}{2}(\nabla_\mu^{ab} \varphi_{ij}^b) f^{adc} A^{\mu,d} \varphi^{ij,c} -$$
$$- \frac{1}{2} G^{\mu\nu,a} \left[F_{\mu\nu}^a + 2 f^{abc} A_\mu^b A_\nu^c \right] \Big\} \Big]$$

In obtaining (13) the SU(4) and CP invariances of Γ were taken into account, the consequence of the latter being the equality of the coefficients of the structures $\chi^2\varphi$ and $\bar{\chi}^2\varphi$.

5. Substituting the expressions (12), (13) into identity (11) and analysing the structures in it we derive the relations between δZ and divergent parts of $\langle\langle \delta\phi \rangle\rangle_{1div}$

$$\delta \tilde{Z}_{\varphi^2} = 0 \ , \quad u_2 = 0 \ , \quad t_2^{abc} = 0 \ , \quad t_3^{abc} = 0 \tag{14}$$

$$t_1^{abc} = \eta_1 f^{abc} , \quad t^{abc} = \eta_2 f^{abc} , \quad t_5^{abc} = \eta_3 f^{abc}$$

where η_1 , η_2 , η_3 are divergent constants.

Moreover, we have

$$\delta Z^{abc}_{\chi^2\varphi} = f^{abc} \, \delta Z_{\chi^2\varphi}$$

$$\delta Z^{bcdf}_{\varphi^4} = f^{abc} f^{adf} \, \delta Z_{\varphi^4}$$

$$\delta \tilde{Z}^{bcdf}_{\varphi^4} = f^{abc} f^{adf} \, \delta \tilde{Z}_{\varphi^4} .$$

The latter equality leads to the relation

$$\delta \tilde{Z}^{bcdf}_{\varphi^4} \varphi^b_{ij} \varphi^{jk,d} \varphi^c_{kc} \varphi^{f,\ell i} = -\frac{1}{2} \delta \tilde{Z}_{\varphi^4} f^{abc} f^{adf} \varphi^a_{ij} \varphi^{ij,d} \varphi^c_{kc} \varphi^{k\ell,f}$$

If we introduce new notations

$$\delta Z_1 = -2\delta Z_g$$

$$\delta Z_2 = \delta Z_\chi - 2\delta Z_g$$

$$\delta Z_3 = \delta Z_\varphi - 2\delta Z_g$$

$$\delta Z'_{\varphi^4} = \delta Z_{\varphi^4} - \frac{1}{2} \delta \tilde{Z}_{\varphi^4}$$

then the remaining relations are as follows

$$\delta Z_{\chi^2\varphi} = \delta Z_\chi + \frac{1}{2} \delta Z_\varphi - 2\delta Z_g \tag{15}$$

$$\delta Z'_{\varphi^4} = 2\left(\delta Z_\varphi - \delta Z_g\right)$$

$$a_1 = \frac{1}{2}\left(\delta Z_\chi - \delta Z_A\right)$$

$$b_1 = \frac{i}{\sqrt{2}}\left(\delta Z_\varphi - \delta Z_x\right)$$

$$u_1 = \frac{1}{4}\left(\delta Z_A - \delta Z_x\right)$$

$$\eta_1 = \frac{1}{4}\left(2\delta Z_A - \delta Z_x\right)$$

$$u_3 = \frac{1}{\sqrt{2}}\left(\delta Z_\varphi - \delta Z_x\right)$$

$$\eta_2 = \frac{1}{\sqrt{2}}\left(\delta Z_\varphi + \delta Z_A - \delta \dot{Z}_x\right)$$

$$\eta_3 = \delta Z_\varphi - \frac{1}{2}\delta Z_\chi$$

It is easy to see that $S_R = S - \eta \Gamma_{1div}$, owing to the relations (14), (15), coincides with the one-loop approximations of $S_{1R} = S(Z_\phi^{1/2}\phi, Z_g g)$

$$S_{1R} = S - \eta \Gamma_{1div} + O(\eta^2) ,$$

while one-loop approximation of new supersymmetry transformations $\delta_{S1R}\phi^{b,B}$ coincides with $\delta_S \phi^{b,B} - \eta \ll \delta\phi^{b,B} \gg_{1div}$

$$\delta_{S1R}\,\phi^{b,B} = \frac{Z_S}{Z_\phi^{1/2}}\,\delta_S \phi^{b,B}\left(Z_\phi^{1/2}\,\phi,\; Z_g\, g\right), \qquad (16)$$

$$\delta_{S1R}\,\phi^{b,B} = \delta_S\,\phi^{b,B} - \eta \ll \delta\phi^{b,B} \gg_{1div} + O(\eta^2)$$

Here $Z_s = 1 - \eta a$, $Z_\phi = 1 - \eta \delta Z_\phi$, $Z_g = 1 - \eta \delta Z g$.

Let us now start with a new Lagrangian (with the Lagrangian of the ghost fields, which is defined by the gauge symmetry)

$$S_{1R,eff} = S_{1R} - \frac{\alpha}{2} (\partial_\mu A^{\mu,a})^2 + \bar{C}^a \partial_\mu R^{R\mu ab} C^b, \tag{17}$$

where $R^{R;Bba} = Z_c Z_{\phi^{b,B}}^{-1/2} R^{B,ba}(Z_\phi^{1/2}\phi)$. Construct a generating functional of Green's functions

$$Z(J) = \int d\phi dc d\bar{c} \exp\left\{\frac{i}{\eta}\left(S_{1R} + J_B^b \phi^{b,B}\right)\right. \tag{18}$$

and integrate over the ghost fields. Changing in (18) the integration variable

$$\phi^{b,B} \to \phi'^{b,B} = \phi^{b,B} + \delta_{S1R} \phi^{b,B} + \delta_{g1R} \phi^{b,B}$$

where $\delta_{g1R} \phi^{b,B} = - R^{RBba} \mathcal{D}_R^{ac} \delta_{1R} t^c$

$$\mathcal{D}_R^{ab} = \left(\partial_\mu R^{R\mu ab}\right)^{-1}$$

we come to the Ward identity

$$J_B^b \left[\delta_{S1R} \phi^{b,B} + \delta_{g1R} \phi^{bB}\right] Z(J) = 0$$

Going througn the step analogous to those leading to the formula (10), we obtain for the $\bar{\Gamma}$ functional

$$\left(\delta_{1R}\phi^{b,B} + \eta \ll \delta \phi^{bB} \gg\right) \frac{\delta \bar{\Gamma}}{\delta \phi^{bB}} = 0$$

where

$$\ll \delta\phi^{\ell B} \gg \; = \; < \delta^{SIR} \phi^{\ell, B} >^{OPI} + i T_{\phi}^{\ell, B} ,$$

$$T^{\ell}_{\phi^{a,B}} = f^{\ell a d} < \phi^{a, B-d} C C^{c} \delta_{SR} t^{c} >$$

Inserting the loop decomposition of $\bar{\Gamma} = S_0 + \eta \bar{\Gamma}_{1fin} + \eta^2 \bar{\Gamma}_2 + O(\eta^3)$ ($\bar{\Gamma}_{1fin}$ is finite because we deal with the action (17)), and also taking into account that

$$\delta_{1R} \phi^{\ell B} = \delta_S \phi^{\ell B} - \eta \ll \delta \phi^{\ell B} \gg_{1div} + \eta^2 \delta_2 \phi^{\ell B} + O(\eta^3),$$

$$\ll \delta \phi^{\ell B} \gg \; = \; \ll \delta \phi^{\ell B} \gg_1 + \eta \ll \delta \phi_2^{\ell B} \gg + O(\eta^2)$$

we obtain for the divergent parts in the two-loop approximation

$$\delta_S \phi^{\ell B} \frac{\delta \bar{\Gamma}_{2div}}{\delta \phi^{\ell B}} + \ll \delta \phi^{\ell B} \gg_{2div} \frac{\delta S_0}{\delta \phi^{\ell B}} = 0 \tag{19}$$

where $\ll \delta \phi^{\ell, B} \gg_{2div} = \ll \delta \phi_2^{\ell, B} \gg_{div} + \delta_2 \phi^{\ell, B}$

Note that all one-loop subdiagrams of $\ll \delta \phi^{\ell B} \gg$ are finite (due to (16)), and hence $\ll \delta \phi^{\ell, B} \gg_{2div}$ is a local functional of the fields. Thus the structure of the divergences of vacuum expectations in the two-loop approximation coincides with the structure of one-loop divergences of vacuum expectations. Moreover, the structure of Γ_{2div} coincides with that of Γ_{1div} [9].

Hence, the equation (19) which coincides with the one-loop equation (11) leads to relations between δZ and divergent quantities in $\ll \delta_{SIR} \phi^{\ell B} \gg_{2div}$

that followed from (11). Acting by induction we obtain that the renormalized action is

$$S_R = S\left(Z_\phi^{1/2}\,\phi,\, Z_g\, g\right)$$

which is invariant under the new supersymmetry transformations

$$\delta_{SR}\,\phi^{b,B} = \frac{Z_S}{Z_\phi^{1/2}}\,\delta_S\,\phi^{b,B}\left(Z_\phi^{1/2}\,\phi,\, Z_g\, g\right)$$

Thus the supersymmetric (multiplicative) renormalization of N=4 supergauge theory on the Wess-Zumino gauge is proved.

6. In papers [3,4] the action of N=4 supergauge theory is expressed in terms of N=1 superfields V and φ^i. The explicit expression of the action which we'll write down as $S^{SS}(V,\varphi^i, g)$, is of no importance to us. The thing that matters is that the action of sec.2 may be obtained from the action [3,4] simply by imposing the Wess-Zumino's gauge condition. Thus the action (1) and the action $S^{SS}(V,\varphi^i, g)$ may be considered as the same action under different gauge conditions.

To construct the renormalized action, corresponding to S^{SS} in the supersymmetric linear gauge, we make use of the result of Ref. [10], according to which the renormalized actions of the gauge theories in different gauges are connected via a (canonical) change of variables (for details, see 10). For the case of linear gauge we obtain making use of the dimensional analysis (i.e. with the account of the divergence index of the diagrams defining the change of variables) and supersymmetry, that the total renormalized action is where Z_g is the charge renormalization constant, which coincides with Z_g in the Wess-Zumino gauge (Z_g is gauge-independent [10]), Z is a renormalization constant, $F_R(V)$ is a local function of V (V is nonlinearly renormalized, since its dimension is equal to zero); S_g is the gauge fixing term (which is supersymmetric, quadratic over V and coinciding with the initial S_g); S_{gh} is the Lagrangian of the ghost field, which is obtained via usual Faddeev-Popov rules and generators

$$R_R^{B,ba} = Z_c \frac{\delta \phi^{bB}}{\delta \phi'^{dD}} R^{D,da}(\phi')$$

where Z_c is the ghost superfields renormalization constant, $\phi^{b,B} = (V, \varphi^i)$ (in this case), $\phi'^{bB} = (F_R(V), Z^{1/2}\varphi)$ R^{Bba} are the generators of the supergauge transformations of the initial action S^{SS}.

Moreover, there is a relation between the renormalization constant specific for N=1 supersymmetry (that the vertices of the chiral field $\sim(\varphi^i)^3$ do not renormalize [11])

$$Z^{3/2} = Z_g^2$$

Note that the explicit two-loop calculations in the Feynman gauge disclose [3-6] that the theory is finite

$$F_R = V, \quad Z = Z_g = 1$$

and, of course, $Z = Z_g = 1$ in three loops [3,4].

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Г.В.ГРИГОРЯН, Р.П.ГРИГОРЯН, И.В.ТЮТИН

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