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DEPENDENCE OF THE SYNCHROTRON RADIATION BRIGHTNESS ON THE PARAMETERS OF ELECTRON STORAGE RING LATTICE

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И.П.КАРАБЕКОВ, К.М.КАРАПЕТЯН, В.М.ЦАКАНОВ,
Г.Р.ПЕТРОСЯН

ИССЛЕДОВАНИЕ ЗАВИСИМОСТИ ЯРКОСТИ СИНХРОТРОННОГО ИЗЛУЧЕНИЯ ОТ ПАРАМЕТРОВ ОПТИЧЕСКОЙ СИСТЕМЫ НАКОПИТЕЛЯ ЭЛЕКТРОНОВ

В работе проведено исследование яркости синхротронного излучения в зависимости от частот бетатронных колебаний и структуры ячейки периодичности. Введен критерий яркости, позволяющий оптимизировать параметры накопителя. Машинный эксперимент на БЭСМ-6 полностью подтвердил полученные теоретические результаты.

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The synchrotron radiation brightness versus the betatron oscillation frequency and the lattice structure is investigated. The brightness criterion permitting to optimize the storage ring parameters is introduced. The machine test at BESM-6 has confirmed the obtained theoretical results.

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Yerevan 1983

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A dedicated storage ring for the synchrotron radiation is specified with the brightness of the source [1,2,3]. For the given values the particles energy E , stored current I , bending radius ρ and mean radius of the machine R the maximum brightness can be achieved by the choice of betatron oscillation frequences ν_x, ν_z and lattice structure defining the forms and values of the amplitude functions $\alpha(s), \beta(s), \gamma(s)$.

The development of the correlations between the space and angular distributions of the electrons σ_x, σ_x', σ_z, σ_z' and similar parameters of the photon distribution $\sigma_{x\gamma}$, $\sigma_{x'\gamma}$ is complicated according to the lack of the corresponding analytical relations. That is why the criterion of the betatron oscillation values selection [2,3] corresponds to the minimum of the electron beam cross-section dimensions, which requires a supplementary investigation. Besides, for the present there is no qualitative determination of the source brightness, which permits to define the source optimization as a whole.

This report presents the results allowing to find the analytical criteria for the optimization of the betatron oscillation values and the lattice parameters for the dedicated storage ring.

The source brightness, which is a storage ring parameter, is independent of the beam collimation, and, hence, we can chose as such a parameter the photon density at the origin of the coordinates X_o, X'_o. In fact, let the photons emitted from the equivalent source be arbitrarily distributed in the phase plane X_o, X'_o at the point S_o of the design orbit, the tangent to which at a particular point coincides with the core of a symmetrical 2d wide collimator spaced at ℓ from S_o. The projection of the slit at the phase plane (fig. 1b) is obtained by a linear transformation

$$\begin{aligned} X_o &= x - x'\ell , \\ X'_o &= x' . \end{aligned} \tag{1}$$

Photons emitted from the limited plane unit of the projection, of the slit shape the SR beam. The collimated system with arbitrary parameters ℓ and $2d$ gives the photon coordinates $X_o = 0,\ X'_o = 0$ in SR beam. To achieve the maximum photon density at the origin of the coordinates X_o, X'_o we have optimized the storage ring parameters of the periodical magnetic system. The calculation of the source brightness, i.e. the photon density at $(0, 0)$ point is carried out by summing up the photon number at the shaping area of the beam of emitted photons. The radial oscillation x, x' of the electron distribution function at the phase surface at any point of the asimuth coordinate S is known (see [1]) to be determined by the normal rule

$$P_\varepsilon(x,x') = \frac{1}{2\pi\sigma_x\sigma_{x'}\sqrt{1-r^2}}\, e^{-\frac{1}{2(1-r^2)}\left(\frac{x^2}{\sigma_x^2} - \frac{2r x x'}{\sigma_x\sigma_{x'}} + \frac{x'^2}{\sigma_{x'}^2}\right)} \quad (2)$$

One σ contour of this distribution is described by the equation of the ellipse

$$\frac{x^2}{\sigma_x^2} - \frac{2r x x'}{\sigma_x\sigma_{x'}} + \frac{x'^2}{\sigma_{x'}^2} = 1 - r^2 , \quad (3)$$

where r is the correlation coefficient, $\sigma_x, \sigma_{x'}$ is the electron distribution dispersion on the phase surface. Using Kurant-Snider's invariant the formula (3) may be presented as

$$\gamma_x(S)x^2 + 2\alpha_x(S)xx' + \beta_x(S)x'^2 = E_x , \quad (4)$$

where $\alpha_x, \beta_x, \gamma_x$ are Twiss' elements matrix of storage ring periodical magnetic system, E_x is the radial oscillation emittance related to σ_x and $\sigma_{x'}$ by the expressions

$$\begin{aligned} \sigma_x(S) &= (E_x \cdot \beta_x(S))^{1/2}, \\ \sigma_{x'}(S) &= (E_x \cdot \gamma_x(S))^{1/2} . \end{aligned} \quad (5)$$

Taking into account (4), the formula (2) takes the form

$$P_\varepsilon(x,x') = \frac{1}{2\pi E_x}\, e^{-\frac{1}{2E_x}\left(\gamma_x x^2 + 2\alpha_x xx' + \beta_x x'^2\right)} . \quad (6)$$

The distribution of photons emitted by the stored electrons at the unit arc dS from $dx\,dx'$ surface in the dX' angle, according to (1) looks like

$$P_\gamma(x,x') = I_0 P_\varepsilon(x,x') \frac{N_K''(X'-x',\lambda)}{I}\, dx\, dx'\, dX' , \quad (7)$$

where I_0 is the electron current density at the center of the cross-section distribution of $P_\varepsilon(x)$, I is the strength of stored current, N''_κ is the polarized photon number at the surface orbit as defined by the formula (1):

$$N''_\kappa(\psi,\lambda)=\frac{3\kappa e I dS}{20\pi\rho\hbar}\gamma_0^2\left(\frac{\lambda_c}{\lambda}\right)^2\left(1+\gamma_0^2\psi^2\right)^2 K_{2/3}\left[\frac{\lambda_c}{2\lambda}\left(1+\gamma_0^2\psi^2\right)^{3/2}\right], \qquad (8)$$

where $\psi = X' - x'$ is the angle between the electrons movement and the emitted photon directions, λ_c is the characteristic length of the SR spectrum wave, $\gamma_0 = E/mc^2$, $K_{2/3}$ is Bessel's function, $\hbar$ is Plank's constant.

To obtain the total distribution of the radiation $P_\gamma(X_0, X_0')$ at the phase surface X_0, X_0' in the orbit center, where the SR beam is shaped, the expression (7), allowing for (6) and (8), should be integrated in the limits of the range S_1, S_2 and over x' in the limits of $\pm\infty$.

The coordinates $X(S), X'(S)$ are transformed into X_0, X_0' using the expressions

$$X_0 = X(S) - (S-S_0)\,X'(S) + \frac{(S-S_0)^2}{2\rho}, \qquad (9)$$

$$X_0' = X'(S) + \frac{S-S_0}{\rho} .$$

Taking into account (9), the expression for the density distribution of the radiation of an equivalent source takes the form

$$P_\gamma(X_0,X_0')=\frac{I_0\,dX_0\,dX_0'}{2\pi E_x}\int_{S_1}^{S_2}\int_{-\infty}^{\infty}\frac{N''_\kappa\left(X_0'-\frac{\Delta S}{\rho}-x'\right)}{I}\,e^{-\frac{\left(X_0+\Delta S X_0'-\frac{3(\Delta S)^2}{2\rho}\right)^2\gamma_x+2\alpha_x\left(X_0+\Delta S X_0'-\frac{3(\Delta S)^2}{2\rho}\right)x'+\beta_x x'^2}{2E_x}}\,dx'\,ds \qquad (10)$$

In particular, for the source brightness we have

$$P_\gamma(0,0)=\frac{A_0\,dX_0\,dX_0'}{2\pi E_x} \qquad (11)$$

where

$$A_0 = I_0 \int_{S_1}^{S_2} e^{\frac{9(S-S_0)^4}{8E_x}\gamma_x} \left[\int_{-\infty}^{\infty} \frac{N_x^{*}\left(\frac{S-S_0}{\rho} - x', \lambda\right)}{I} e^{\frac{3(S-S_0)^2 \alpha_x x' - \beta_x x'^2}{2E_x}} dx' \right] dS \quad (12)$$

To obtain the qualitative value of the brightness and determine its dependence on ν_x and ν_z we must numerically integrate (10) and (11) using preliminary tabulated values $\alpha_x(S), \beta_x(S), \gamma_x(S)$ and E_x for ν_x, ν_z selected values.

Analytical criteria which may limit the optical frequency searches, can be obtained in the following way. According to (9), for small ΔS the emission for all points corresponding to the ordinates x_0 of the electron distribution is summed up at each point X'_0 on the ordinate of X_0 when inequality $\Delta S\left(x' + \frac{\Delta S}{\rho}\right) << \sigma_x$ is fulfilled and the values of $\alpha_x(S), \beta_x(S), \gamma_x(S)$ could be taken unchangeable. Thus, the emission at any point X_0, X'_0 at the SR equivalent source phase plane is obtained by integrating the distribution (2) over x_0. Confining to one σ contour for the direct ellipse case $(\alpha_x(S_0) = 0)$ the photon distribution function at the phase surface X_0, X'_0 will be described by

$$P_\gamma(X_0, X'_0) = \frac{1}{2\pi\sigma_x} e^{-\frac{X_0^2}{2\sigma_x^2}} \phi\left(\sqrt{1 - \frac{X_0^2}{\sigma_x^2}}\right), \quad (13)$$

where $\phi\left(\sqrt{1 - \frac{X_0^2}{\sigma_x^2}}\right)$ is the probability integral, being a fast falling function, which will vanish at $X_0^2 = \sigma_x^2$. For this reason the expected photon summed distribution will be narrower than the electron distribution along the X_0 core. This conclusion has a principle meaning, for it permits to approximate the SR source by a point source.

According to the formulae (11) and (13) it follows that the source brightness is inversely proportional to the radial oscillations emittance E_x which is determined by the amplitude and dispersion functions and their derivatives for the given structure of the gradient period [4] :

$$\frac{\sigma_x^2(S)}{\beta_x(S)} = E_x = C_\gamma < \mathcal{H} >_{Mag} \qquad (14)$$

where

$$C_\gamma = \frac{55}{32\sqrt{3}} \cdot \frac{\hbar \gamma_0^2}{m c \rho_0} \qquad (15)$$

$$<\mathcal{H}>_{Mag} = \frac{1}{2\pi\rho_0} \int\limits_{Mag} \frac{1}{\beta_x}\left(\eta^2 + \left(\beta_x \eta' - \frac{1}{2}\beta_x' \eta\right)^2\right) dS \qquad (16)$$

In (16) the integration is taken over only those parts of the design orbit which are in the bending magnets.

Using a good approximating for η -functions [4]

$$\eta(S) \approx \left(\frac{\alpha R}{\nu_x}\right)^{1/2} \beta_x^{1/2}(S) \qquad (17)$$

we shall obtain

$$E_x \approx C_\gamma \frac{\alpha R}{\nu_x} \approx C_\gamma \frac{\bar{\beta}_x}{\nu_x^2} \qquad (18)$$

From (18) it follows that the maximum brightness corrsponds to the maximum value of $\frac{\nu_x^2}{\beta_x} = \xi$. Hence we can take ξ as a brightness criterion. The analytical expression for ξ parameters of FODO symmetrical structure can be obtained by approximate formulae for $\beta_{x\,max}, \beta_{x\,min}$ [5]

$$\beta_{x\,\substack{max\\min}} = 2L\,\frac{1\pm\sin(\mu/2)}{\sin(\mu)} \qquad (19)$$

where $\mu = \frac{2\pi\nu}{N}$, $2L$ is the cell length, N is the total number of cells in the ring. From (19) by a good approximation the quantity $\bar{\beta}_x$ equals:

$$\bar{\beta}_x = \frac{2L}{\sin(\mu)}\ , \qquad (20)$$

and the expression for ξ is read

$$\xi = \frac{N^2}{8\pi^2 L}\cdot\mu^2\sin\mu\ . \qquad (21)$$

Replacing $L = \frac{2\pi R}{N}$ in (20) we shall obtain

$$\xi = \frac{N^3}{8\pi^3 R}\,\mu^2\sin\mu\ , \qquad (22)$$

where R is the mean radius of the storage ring. Analyzing (21) for the given values of N and R we have the maximum source brightness at $\mu \approx 131^{\circ}$. According to (18) the absolute value of ξ also allows to determine the quantitative measure of the source brightness. As is seen from (22) the source brightness at the given R is proportional to the total number of cells. So for the generation of SR beams in dedicated storage rings, it is necessary to provide the maximum value of N . To compare the brightness of a number of operational, constructed and designed SR sources we present the values of ξ in Table 1.

It is easy to show that for an asymmetrical FODO system the criterion of brightness ξ is:

$$\xi = \frac{N^3}{\pi^3 R} \quad \frac{\mu_x^2 \sin \mu_x}{8 + \cos \mu_x - \cos \mu_z} \qquad (23)$$

where μ_z is the vertical betatron shift per cell. From (22) is seen that the source brightness slightly depends on μ_z. So th optimization of the number of vertical oscillations per turn is achieved with the minimization of $\beta_{z\,max}$ which defines the amplitude of vertical betatron oscillations. In accord with the calculations for $\mu_x = 131^\circ$, $\beta_{z\,max}$ reaches its minimum value nea $\mu_z = 90^\circ$. For this case the region of the optimum choice of μ_z is in the vicinity of $\mu_z \approx 110^\circ$, according to the resonance diagram [4].

According to (12) the source brightness also depends on the beam formation length ΔS. This can be easily obtained from the condition

$$\frac{\Delta S}{\rho} = \sigma_{x'} \quad , \qquad (24)$$

or taking into account (5)

$$\Delta S = \rho \left(E_x \cdot \gamma_x (S_0) \right)^{1/2} \qquad (25)$$

As is seen from (25), the source brightness is achieved at γ_{ma} points. To test the obtained analytical criteria the summary photon current was computed, using the formula (10), for the parameters of the designed storage ring ERSINE of the Yerevan Physics Institute [6]. In Table 2 we give the values of brightness P_{00} and the criteria of ξ for diagonal pairs of ν_x and ν_z values as well as the values of electrons and photons distribution parameters $\sigma_{\Im\lambda}^2$, σ_γ^2. The data confirm the correctness of brightness criterion choice. From the data in columns $\sigma^2_{эл}$ and σ^2_{γ} it is seen that the distribution of the equivalent SR photon source narrows down as compared with the electron distribution obtained by the formula (13).

The dependences of P_{oo}, ξ, $\beta_{x\,max}$, $\beta_{x\,min}$, $\beta_{z\,max}$ on the vertical betatron oscillation ν_z are presented in Table 3. The dependence of P_{oo} on ν_z coincides with the dependence $\xi(\nu_z)$ determined by the formula (23) for the criterion of brightness of the asymmetrical FODO structure.

The dependence of $P_{oo}(S)$ and $\xi(S)$, respectively on the values of $\beta_x(S)$ and $\gamma_x(S)$ for ν_x = 5.82 and ν_z = 5.8 is presented in Table 4. As is seen from the table, the brightness criterion, when $\bar{\beta}_x$ is replaced by $\beta_x(S)$, reproduces on the change of $P_{oo}(S)$ along the azimuthal coordinates. This allows one to use the criterion of $\xi(S)$ for the cell optimization with a view to achieve the required source brightness at the given points of the orbit.

Table 1

Facility	ν_x	ν_z	$\beta_{x\,max}$ (м)	$\beta_{x\,min}$ (м)	$\xi = \frac{\nu_x^2}{\beta_x}$
ESF [7]	26.2	13.7	35	1.2	37.92
VEPP-4 [8]	9.2	9.2	12	1.5	12.54
DORIS [9]	7.2	5.3	13.5	1.4	6.96
NSLS [10]	9.7	5.7	26	1.4	6.87
KEK [11]	6.25	5.25	12.25	2.56	5.275
ERSINE [6]	5.82	4.85	17.55	1.2	3.613
SPEAR [12]	5.2	5.1	14	2.5	3.277
VUV Ring [13]	3.32	1.32	12.5	1.1	1.62
Daresbury [14]	3.25	2.25	32.5	0.8	0.634

Table 2

ν_x	ν_z	$\sigma^2_{эл} \cdot 10^6$ (м^2)	$\sigma_\gamma^2 \cdot 10^6$ (м^2)	$\beta_{x\,max}$ (м)	$\beta_{x\,min}$ (м)	$\rho_{00} \cdot 10^{20}$ ($\gamma \cdot$м$^{-1} \cdot$рад$^{-1}$)	$\xi = \frac{\nu_x^2}{\beta_x}$
6.85	6.8	6.113	0.375	32.958	0.572	1.88	2.799
6.18	6.15	4.005	0.567	21.525	0.932	2.273	3.401
5.82	5.8	3.816	0.720	18.612	1.123	2.314	3.468
5.2	5.15	4.217	1.208	15.114	1.527	2.198	3.348
4.8	4.85	4.996	1.718	13.901	1.818	2.027	2.931
4.2	4.1	7.107	3.119	12.571	2.355	1.718	2.251
3.85	3.8	9.489	4.598	12.233	2.370	1.498	2.03

Table 3

ν_z	$\beta_{x\,max}$ (м)	$\beta_{x\,min}$ (м)	$\beta_{z\,max}$ (м)	$P_{00}\cdot10^{20}$ $(\gamma\cdot м^{-1} рад^{-1})$	$\sigma^2_{эл}\cdot10^6$ $(м^2)$	$\xi=\frac{\nu_x^2}{\beta_x}$
6.8	19.076	1.107	29.05	2.247	4.053	3.356
6.2	18.721	1.124	20.23	2.282	3.928	3.413
5.8	18.612	1.123	17.25	2.314	3.816	3.432
5.23	17.92	1.165	14.71	2.363	3.65	3.549
4.85	17.55	1.184	13.73	2.402	3.527	3.616
4.2	16.876	1.221	12.92	2.476	3.307	3.743
3.8	16.452	1.246	12.85	2.526	3.174	3.827
3.2	15.831	1.283	13.37	2.602	2.982	3.958

Table 4

Lattice Points		$\beta_x(s)$ (м)	$\gamma_x(s)$	$P_{00}\cdot10^{20}$ $(\gamma\cdot м^{-1} рад^{-1})$	$\xi=\frac{\nu_x^2}{\beta_x(s)}$
Bending magnet after F lens	entrance	14.8	0.89	1.93	2.289
	middle	10.87	0.88	2.31	3.1
	exit	7.45	0.89	2.87	4.54
Bending magnet after D lens	entrance	3.5	1.78	4.09	9.64
	middle	6.92	1.7	2.98	4.9
	exit	11.63	1.71	2.33	2.91

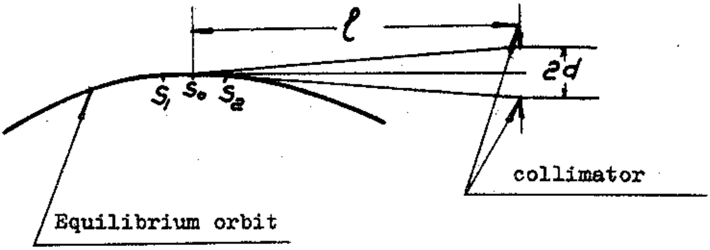
ℓ
$2d$
s_1
s_0
s_2
collimator
Equilibrium orbit

Fig. 1a

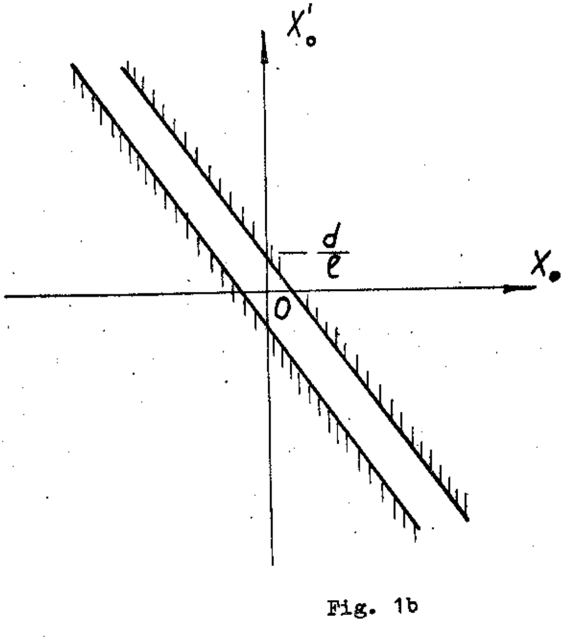
X_0'
$\frac{d}{\ell}$
X_0
0

Fig. 1b

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ИССЛЕДОВАНИЕ ЗАВИСИМОСТИ ЯРКОСТИ СИНХРОТРОННОГО ИЗЛУЧЕНИЯ ОТ ПАРАМЕТРОВ ОПТИЧЕСКОЙ СИСТЕМЫ НАКОПИТЕЛЯ ЭЛЕКТРОНОВ

(на английском языке, перевод Похсрарян А.М.)

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