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CLASSICAL AND QUANTUM MECHANICS OF NON-ABELIAN GAUGE FIELDS

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КЛАССИЧЕСКАЯ И КВАНТОВАЯ МЕХАНИКА НЕАБЕЛЕВЫХ КАЛИБРОВОЧНЫХ ПОЛЕЙ

В работе исследуется классическая и квантовая механика неабелевых калибровочных полей как без спонтанного нарушения, так и со спонтанным нарушением симметрии. Рассматривается фундаментальная подсистема классической механики Янга-Миллса (КМЯМ). Показано, что фундаментальная подсистема КМЯМ является К - системой Колмогорова и следовательно обладает сильными статистическими свойствами. Найдены также интегрируемые подсистемы, к которым близка (в смысле КАМ - теории) классическая механика Янга-Миллса-Хиггса (КМЯМХ). Обсуждаются квантовомеханические свойства К - систем.

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CLASSICAL AND QUANTUM MECHANICS OF NON-ABELIAN GAUGE FIELDS

Classical and quantum mechanics of non-Abelian gauge fields is investigated both with and without spontaneous symmetry breaking. The fundamental subsystem (FS) of the Yang-Mills classical mechanics (YMCM) is considered. It is shown to be a Kolmogorov K-system and, hence, to have strong statistical properties. Integrable systems are also found, to which in terms of KAM-theory the Yang-Mills-Higgs classical mechanics (YMHCM) is close. Quantum-mechanical properties of K-system and their relation to the problem of confinement are discussed.

Yerevan Physics Institute

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CLASSICAL AND QUANTUM MECHANICS OF NON-ABELIAN GAUGE FIELDS

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To my father's memory

1. Introduction

In refs. [1,2,3] authors have investigated classical equations of motion of non-Abelian gauge fields, properties of both separate solutions and the system as a whole. In particular, they have considered the solutions which in a certain coordinate system depended on the time ($A^a_\mu(t)$) only. It has been shown that the system described by such potentials ($A^a_\mu(t)$) is reduced to the discrete nonlinear mechanical system (YMCM) with the Hamiltonian [2]

$$H_{YM}=\sum_{a,i}\frac{1}{2}\left(\dot{A}^a_i\right)^2+\frac{g^2}{4}\left[\left(A^a_i A^a_i\right)^2-\left(A^a_i A^a_j\right)^2\right] \quad (1.1)$$

and coupling equations

$$n^a=\varepsilon^{abc}A^b_i\dot{A}^c_i \quad (1.2)$$

where $i,j,K=1,2,3$, $a,b,c=1,2,3$ are for the SU(2) group. Solutions of this system in an arbitrary coordinate system are nonlinear plane waves $A^a_\mu(K\cdot X)$ with a non-zero square of the wave vector $K^2=\mu^2$ [2,3].

The general picture of the variation of the color amplitudes A^a_i in time is characterized by alternate rapid oscillations and decrease in some color amplitudes and growth in others, color "beats" [2].

The strong instability of trajectories of this system with respect to small variations of initial conditions in phase space has led the authors of ref. [2,1] to a conclusion on the stochasticity and nonintegrability of YMCM. Characteristically it has at least a countable set of periodical trajectories [2].

In the last paper of these authors [1] analogous properties of classical gauge systems with a spontaneously broken symmetry are investigated. Such a gauge system with isodoublet breaking of SU(2) group in the gauge $A_0^a = 0$ is described by the Hamiltonian [1]

$$H = H_{YM} + \frac{1}{2}\left(\dot{\sigma}^2 + \dot{B}_a^2\right) + \tag{1.3}$$

$$\frac{g^2}{2}\left(A_i^a A_i^a\right)\left[\frac{B_a^2}{2} + \left(\frac{\sigma}{\sqrt{2}} + \eta\right)^2\right] + \lambda^2\left[\frac{B_a^2}{2} + \left(\frac{\sigma}{\sqrt{2}} + \eta\right)^2 - \eta^2\right]^2$$

and coupling equations

$$\varepsilon^{abc} A_i^b \dot{A}_i^c - \frac{\eta}{\sqrt{2}}\dot{B}_a + \frac{1}{2}\left[\sigma \dot{B}_a - B_a \dot{\sigma} - \varepsilon^{abc} B_b \dot{B}_c\right] = 0 \tag{1.4}$$

where η is the vacuum expectation value of the scalar field

$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} iB_1 + B_2 \\ \sqrt{2}\eta + \sigma - iB_3 \end{pmatrix} \tag{1.5}$$

Since the field φ should coincide with φ_{vac} at large $|x|$ then this implies for uniform fields that $\sigma = B_a = 0$

In [1] a two-dimensional case was investigated when $A_1^1 \equiv x$, $A_2^2 \equiv y$ and all the other components A_i^a are zero

$$H = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) + \frac{g^2\eta^2}{2}\left(x^2 + y^2\right) + \frac{g^2}{2}\left(xy\right)^2 . \qquad (1.6)$$

This system is characterized by one parameter

$$\mathcal{T} = \left(\frac{g}{2}\right)^2 \left(\frac{\eta}{\mu}\right)^4 , \qquad (1.7)$$

it being shown in [1] that at $\mathcal{T}_c \simeq 0.15$ there occurs a phase transition in the following sense: at large values of $\mathcal{T}$ the system is close to the integrable one, whereas at $\mathcal{T} < \mathcal{T}_c$ the motion is stochastic, just as for $\mathcal{T} = 0$ ($g \neq 0$, $\eta = 0$). This phenomenon is well illustrated by figs. 1–3 ($\mathcal{T}_1 = 4.84$, $\mathcal{T}_2 = 0.35$, $\mathcal{T}_c \simeq 0.15$) from ref. [1] , which present cross sections of phase trajectories in the four-dimensional space ($x, \dot{x}, y, \dot{y}$) with a plane $\alpha = (y, \dot{y})$ at $\dot{x}(\alpha) > 0$.

The purpose of the present paper is the further investigation of the gauge systems (1.1) and (1.3). Let's first note that in refs. [1,2,3] such two- and three-dimensional subsystems of (1.1) and (1.3) are considered, when the coupling equations (1.2) and (1.4) are identically satisfied. It is the case when the matrix A_i^a is diagonal in indices a, i . It has seemed important to consider the general case when all the components of the matrix A_i^a are not zero and coupling equations are taken into account. In a recent paper by two authors [17] new compact variables * have been introduced,

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* It is implied that their variation range lies on the compact manifold, in this case on a sphere (see sec. 2).

where the general system looks much simpler and has a natural analogy with the solid body mechanics. In the second section the necessary information from ref. [17] is presented and at the same time a method of describing YMCM by means of compact variables is developed. The concept of FS (2.12) of YMCM inducing stochastic properties of a complete system is introduced.

In section 3 the YMCM is considered from the viewpoint of ergodic theory. The need in such an investigation followed from refs. [1,2] . As has been mentioned, it has been proven in these papers that trajectories of FS (2.12) have a strong instability which leads to the fact that the phase trajectory chaotically fills up the energy surface $E = \text{const}$. It is intuitively clear what is implied by chaotic or stochastic motion, but it is important to define these concepts more strictly.

In the ergodic theory, or otherwise, the theory of dynamic systems (DS) the classification of motions as the increase of chaotic-stochastic properties is obtained: they are ergodic systems, systems with weak mixing, with mixing, with n-fold mixing and, finally, K-systems. It is important to establish which class of DS the YMCM belongs to. It is also clear that if the arbitrary trajectory uniformly fills up the hypersurface $E = \text{const}$, then there is no sense in searching for concrete solutions of the system. Instead one should investigate the global properties of DS as a whole. It turns out that if it is established which class of DS the YMCM belongs to, then it will be possible to find out these global properties.

Section 3 is planned as follows. First, the necessary information from the classification of DS is presented. Then we consider the sufficient conditions satisfying which one may answer the question: which class of DS the system with the given Hamiltonian H belongs to ? In papers by Hadamard, Hedlund, Hopf, Krylov, Anosov, Sinai et al.[9-12] fairly general criteria for the above question were obtained.

Using these sufficient conditions and the concept of structural stability in the narrow sense (see sec. 3, p. 24) one may prove that the YMCM is equivalent (homeomorphic) to the billiards system of Krylov-Sinai with hyperbolic walls (fig. 4) and that it is a Kolmogorov K-system[8]. Physically the strong statistical properties of the YMCM are due to the instability arising at the scattering of phase trajectory on the convex inside hyperbolic boundary of fig. 4. In this case the convex inside boundary plays the scattering role of negative curvature (see Appendix B).

After this fact being established, all the global properties of K-systems are assigned to the YMCM. The YMCM possesses exponentially compressing and extending transverse fiberings equivalent to appropriate fiberings of the hyperbolic billiards, positive entropy of Kolmogorov [8], the strongest statistical properties, K-mixing. Finally, there is one property which we would like to especially dwell on. In 1931 Koopman[7] obtained a brilliant result indicating that to each classical DS corresponds a one-parametric group of unitary operators whose spectral properties characterize the DS. Since it has been proven by Kolmogorov that K-systems have a countably-multiple Lebesgue spectrum, it turns out that the group of unitary operators U_{YM} corresponding to the YMCM has a uniform countably-multiple Lebesgue spectrum.

Thus, the YMCM is not only a nonintegrable system but also possesses the strongest statistical properties which are described by concepts of the ergodic theory.

In section 4 gauge systems with spontaneous symmetry breaking are investigated. It is shown by introducing convenient variables of the type angle-action that at $\mathfrak{I} \gg \mathfrak{I}_c$ the system (1.6) is close to the integrable system (in terms of KAM-theory) with the Hamiltonian (4.20) whose phase picture looks like fig. 1. This result is of interest because the perturbation Hamiltonian H_1 (4.3) consists of five terms of the same order. The fact that there is a close integrable subsystem is obtained by averaging over rapid variables. The phase transition in the YMH system to the disorder phase is shown to be due to the infinite sequence of rapidly growing bifurcations.

Results of sections 3 and 4 are substantially of theoretical-nonperturbative nature owing to the following: if H_0 is the Hamiltonian of the integrable system, and $\frac{1}{\mathfrak{I}}$ H_1 the Hamiltonian of perturbation, and if by means of a canonical transformation $(pq) \rightarrow (p', q')$ one manages to reduce the Hamiltonian $H_0 + \frac{1}{\mathfrak{I}} H_1$ to $H_0'(p') + \frac{1}{\mathfrak{I}^2} H_1'(p', q')$ then during the time $t \sim \mathfrak{I}$ we shall have an approximated solution with an error of the order $1/\mathfrak{I}$. That is the classical perturbation theory describes the system during a finite time interval, therefore, any statement concerning the nature of the system behavior at $t \to \infty$ has a theoretical-nonperturbative character and cannot be obtained in any finite order of perturbation theory*.

In the conclusive fifth section quantum-mechanical properties of the YM mechanics and their relation to the problem of confinement are discussed.

2. Fundamental Subsystem of YMCM

As is shown in ref. [17] the Hamiltonian (1.1) has an obvious mechanical interpretation. Due to SO(3)⊗SO(3) symmetry are preserved the moments

$$n^a = \varepsilon^{abc} A_i^b \dot{A}_i^c \tag{2.1}$$

$$m_i = \varepsilon_{ijk} A_j^a \dot{A}_k^a \tag{2.2}$$

with (2.1) coinciding with coupling equation (1.2) at a constant external density of color charge. If the new variables O_1, E, O_2 are introduced by means of the relation

$$A = O_1 \; E \; O_2^T \tag{2.3}$$

where

$$E = \begin{pmatrix} x(t) & & 0 \\ & y(t) & \\ 0 & & z(t) \end{pmatrix} \tag{2.4}$$

and O_1 and O_2 are orthogonal matrices, then the Hamil-

* The creation of a convergent perturbation theory (KAM-theory etc.) is a vivid example of the achievements of the classical theory of DS. In the quantum mechanics a similar result is lacking despite the recent progress.

tonian (1.1) will be rewritten as [17]

$$H_{YM}=\frac{1}{2}\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)+\frac{g^2}{2}\left(x^2y^2+y^2z^2+z^2x^2\right)+T_{YM} \qquad (2.5a)$$

$$T_{YM}=\frac{1}{2}\sum_{a,i}\left(I_i\Omega_i^2+I_a\omega_a^2-2I_{ai}\Omega_i\omega_a\right) \qquad (2.5b)$$

where

$$\begin{aligned} &I_1=y^2+z^2, \quad I_2=x^2+z^2, \quad I_3=x^2+y^2, \\ &I_{11}=2yz, \quad I_{22}=2xz, \quad I_{33}=2xy, \\ &I_{ai}=0 \quad \text{for} \quad i\neq a \end{aligned} \qquad (2.6)$$

and Ω_i and ω_a are the vectors of angular velocity dual to appropriate antisymmetric tensors [17]

$$\omega=-\dot{O}_1^T\ O_1, \qquad \Omega=-\dot{O}_2^T\ O_2. \qquad (2.7)$$

Projections of the momenta (2.1-2) on the "movable" set of coordinates have the form $n^a=O_1^{ab}N^b$, $m_i=O_{2ij}M_j$ [17]

$$N_a=I_a\omega_a-I_{ai}\Omega_i, \quad M_i=I_i\Omega_i-I_{ia}\omega_a \qquad (2.8)$$

and obey the generalized Euler equations [17]

$$\frac{d\vec{N}}{dt}+\left[\bar{\omega}\times\vec{N}\right]=0, \quad \frac{d\bar{M}}{dt}+\left[\bar{\Omega}\times\bar{M}\right]=0 \qquad (2.9)$$

The variables x, y, z satisfy the following equations:

$$\begin{aligned} &\ddot{x}+x\left(y^2+z^2\right)+x\left(\Omega_2^2+\Omega_3^2+\omega_2^2+\omega_3^2\right)-2z\Omega_2\omega_2-2y\Omega_3\omega_3=0 \\ &\ddot{y}+y(x^2+z^2)+y(\Omega_1^2+\Omega_3^2+\omega_1^2+\omega_3^2)-2z\Omega_1\omega_1-2x\Omega_3\omega_3=0 \\ &\ddot{z}+z(x^2+y^2)+z(\Omega_1^2+\Omega_2^2+\omega_1^2+\omega_2^2)-2y\Omega_1\omega_1-2x\Omega_2\omega_2=0 \end{aligned} \qquad (2.10)$$

Thus eqs. (2.9-10) together with the relations (2.8), (2.6) completely determine the YMCM which has three independent single-valued integrals [17]

$$H_{YM}\,, \quad M^2 = M_i M_i\,, \quad N^2 = N_a N_a \tag{2.11}$$

The following analogy with the mechanics of solid body is rather useful. As is known the kinetic energy of the solid body is

$$T^{solid} = \frac{1}{2}\sum_i I_i \Omega_i^2$$

and due to SO(3) symmetry the latter preserves the square of the total momentum M^2 . If the inertia tensor I_i is independent of time, $\dot{I}_i = 0$, the energy is preserved as well, which permits to completely integrate the Euler equation [15] otherwise ($\dot{I}_i \neq 0$) it would be impossible, since [17]

$$\frac{dT^{solid}}{dt} = -\frac{1}{2}\sum_i \dot{I}_i \Omega_i^2 \tag{2.11a}$$

Similarly in (2.5) we have a kind of "gauge" body with time-dependent inertia moments (2.6) which rotates in usual and internal spaces (precession of spin and isospin), and it is essential that due to nondiagonal elements of the inertia tensor I_{ai} there is a correlation between these rotations.

We shall call the system with the Hamiltonian

$$H_{FS} = K_{FS} + U_{FS} = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{g^2}{2}(x^2y^2 + y^2z^2 + z^2x^2) \tag{2.12}$$

the fundamental subsystem of the YMCM. Then

$$H_{YM} = H_{FS} + T_{YM} \tag{2.13}$$

As a whole the picture is this: one may imagine the variables x, y, z to be the coordinates of a material point performing compound motion in a three-dimensional space limited by an equipotential surface

$$U_{eff} = U_{FS} + T_{YM} = const \tag{2.14}$$

At $T_{YM} = 0$ ($\varphi_i = \omega_a = 0$) it has the form shown in fig. 4. If $\varphi_i, \omega_a \neq 0$, then $T_{YM} > 0$ and the equipotential surface lies within that shown in fig. 4 and has a somewhat more complicated form and, besides, is time-dependent. It is convenient to present the rotation in the following way. Since M^2 and N^2 (2.11) are conserved, one may present the vectors M_i and N_i as points on spheres with radii M^2 and N^2.

So we have a very obvious description of the YMCM when it is necessary to follow three points, two of which "run" over the sphere surface and describe the rotation, and the third "runs" in the space limited by equipotential surface (2.14).

As it will be shown in the next section, the basic role in the formation of stochastic properties of the YMCM is played by the equipotential surface of the FS (2.12) which has a negative curvature, in the sense that it is convexed into the region of the motion of the point with the coordinates (x, y, z). As for the FS of the YMCM (2.12) itself, it is a Kolmogorov K-system and, hence, possesses the strongest statistical properties.

3. Yang-Mills Classical Mechanics as a Kolmogorov K-System

Consider the YMCM from the viewpoint of ergodic theory.

It has been shown in refs. [1,2] that trajectories of the FS (2.12) are strongly unstable with respect to small changes in initial conditions and that this system is nonintegrable. Phase trajectories chaotically fill the energy surface $H_{fs} =$ const. It is intuitively clear what is implied by chaotic or stochastic motion, but it is important to formulate them more definitely. In ergodic theory [4-7] these concepts are definitely formulated and the classification of the motion as the increase of chaotic-statistical properties is obtained.

The aim of this section is to show that the FS of the YMCM (2.12) possesses the strongest statistical properties from all known ergodic theories, that it is a Kolmogorov K-system, and to describe its global properties. Let's first present the necessary information from ergodic theory [4-7].

The ergodic theory investigates the statistical properties of the measure-preserving mapping T^t of the phase space M on itself. The set of ordinary differential equations of the Hamiltonian dynamics

$$\dot{q}_i = \frac{\partial H}{\partial p_i}\ , \qquad \dot{p}_i = -\frac{\partial H}{\partial q_i} \qquad (3.1)$$

gives such a mapping T^t of the phase space M on itself* since to each point $(p, q) \equiv X \in M$ is determined an appropriate

* The right hand side of (3.1) determines in the phase space M the vector field called the vector field of phase velocity.

point $(p,q)_t \equiv X_t \in M$ and due to the Louiville theorem the initial phase volume is preserved.

The mapping T^t forms a one-parametric group called phase flow. An important result of the ergodic theory [4-7] is the existence of the time average (average along the trajectory)

$$\lim_{t \to \infty} \frac{1}{t} \int_0^t f(T^t x)\, dt \stackrel{def}{=} \bar{f} \qquad (3.2)$$

where f is the arbitrary function in M. If $f(x) = \chi_A(x)$ is the characteristic function of the set $A \subset M$ ($\chi_A = 1$ if $x \in A$ and is zero if $x \bar{\in} A$), then from the Birkhoff theorem (3.2) follows the existence of the mean number of hits in any measured set A for almost every point (x).

Let us introduce the classification of various types of flows in the phase space M which characterizes the flows by the increase of their statistical properties. These are ergodic systems, systems with weak mixing, with mixing, with n-fold mixing and, finally, K-systems.

The DS is called ergodic if all invariant sets of the mapping T^t are sets of zero measure or sets of total measure M.

The concept of ergodicity, undecomposability of mapping or metrical transitivity is a natural requirement that the given mapping T^t mixes the points of M well enough. It is clear that if there is an invariant set B ($T^t B = B$) whose measure differs from 0 and $\mu[M]$ then the DS may be considered acting independently on B and its supplement $M \setminus B$, i.e. the mapping does not mix the points B and $M \setminus B$

and, hence, the flow is not ergodic.

Let, for example, the Hamiltonian system (3.1) have a conserving integral $\Phi(p,q)$, then the condition $\Phi(p,q) = \text{const}$ picks out the invariant subset and, hence, the flow T^t is not ergodic. Ergodicity thereby means that the DS has no conserving integrals.

In particular, it follows from ref. [1,2] that the FS (2.12) is an ergodic system. It is its weakest statistical property. We shall later show that it has much stronger statistical properties.

For the arbitrary mapping T^t the space M (with an accuracy up to the set of zero measure) may be represented as a sum of the counting number of invariant subsets, on each of which T^t is already ergodic [6]. Thus, every DS T^t may be decomposed to ergodic components. Thus, in the previous example, already on the subset $\Phi(p,q) = \text{const}$ the flow T^t is ergodic.

For ergodic systems the Birkhoff theorem allows to prove the equality of time averages to space ones [4-7]

$$\lim_{t\to\infty} \frac{1}{t} \int_0^t f(T^\tau x)\, d\tau = \int_M f(x)\, d\mu \tag{3.3}$$

Let $f = \chi_A$ then it follows from (3.3) that

$$\int_M f(x)\, d\mu = \mu[A]$$

hence, for ergodic systems the trajectory of almost every point $x \in M$ falls in any measurable subset of the positive measure and stays in this set for a time in mean proportional to its measure. The equality (3.3) may be considered as another definition of ergodicity. From (3.3) follows that for arbitrary functions $f(x)$ and $g(x)$ in M we have

$$\lim \frac{1}{t}\int_0^t d\tau \int_M f(Tx^\tau)\, g(x)\, d\mu = \int_M f(x)\, d\mu \cdot \int_M g(x)\, dx \tag{3.4}$$

or if $f = \chi_A,\ g_t = \chi_B$ then

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t d\tau\, \mu\left[T^\tau A \cap B\right] = \mu\left[A\right]\cdot\mu\left[B\right] \tag{3.5}$$

that is, any two sets A and B are statistically independent only on the average.

Let us determine the DS with stronger statistical properties. The flow T^t has a property of <u>weak mixing</u>, if for arbitrary functions f and g in M we have [4-7]

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t d\tau\left[f(T^\tau x)\, g(x)\, d\mu - \int_M f(x)\, d\mu \int_M g(x)\, d\mu\right]^2 = 0 \tag{3.6}$$

and the property of <u>mixing</u>, if simply

$$\lim_{t\to\infty} \int_M f(T^t x)\, g(x)\, d\mu = \int_M f(x)\, d\mu \int_M g(x)\, d\mu \tag{3.7}$$

Let now again $f = \chi_A,\quad g = \chi_B$ then from (3.7)

$$\lim_{t\to\infty} \mu\left[T^t A \cap B\right] = \mu\left[A\right]\cdot\mu\left[B\right]. \tag{3.8}$$

hence, during its motion the arbitrary set A will all the time traverse the set B , the measure of the part of A that at the moment t has fallen in B being asymptotically proportional to that of B , i.e. A is uniformly mixed all over the phase space.

Let us cite an example used by Gibbs explaining these definitions. Add some vermouth into a vessel with gin (proportions 10% to 90%, respectively). Ergodicity means that after a fairly long time there will be on the average 10% of vermouth in an arbitrary part of the vessel, though it is not excluded that in definitely far moments of time the liquid in an arbitrary part of the vessel may be either too strong or too sweet, whereas the mixing means that after a fairly long time the liquid in an arbitrary part of the vessel <u>should</u> contain 10% of vermouth.

It is easy to show that if the flow T^t has a weak mixing, then it is ergodic, if it is mixing, then it is both ergodic and weakly mixing. The opposite is incorrect: there are ergodic systems without weak mixing, systems with weak mixing, without mixing and systems with mixing.

The DS has an n-fold mixing if for any functions $f_0, f_1 \cdots \cdots f_n$ in M we have [7]

$$\lim_{t_1 \ldots t_n \to \infty} \int f(x) f_1(T^{t_1} x) \cdots f_n(T^{t_1 + \cdots + t_n} x) d\mu = \prod_{i=0}^{n} \int_M f_i(x) d\mu \qquad (3.9)$$

and if $f = \chi_{A_0}, \ldots f_n = \chi_{A_n}$ then

$$\lim_{t_1 \ldots t_n \to \infty} \mu [A_0 \cap T^{t_1} A_1 \cap \ldots \cap T^{t_1 + \cdots + t_n} A_n] = \prod_{i=1}^{n} \mu [A_i]$$

The ordinary mixing (3.7-8) is a 1 multiplicity mixing.

Let us finally introduce the concept of K-flow which we shall need later. K-systems were introduced by Kolmogorov in ref. [8] where they were called quasiregular. K-systems have still stronger statistical properties than those mentioned above. Unfortunately, it is impossible to define the K-flow in terms of concepts used above.

The flow T^t is called K-flow, if there is such a measurable split-up $\xi_0 = \{C\}$ of the space M ($\bigcup_{C \in \xi_0} C = M$, $C'_{\xi_0} \cap C''_{\xi_0} = \phi$) that

$$1. \quad T^t \xi_0 \geq \xi_0 \qquad \text{at } t > 0 .$$

$$2. \quad \bigvee_{-\infty}^{+\infty} T^t \xi_0 = \mathcal{E} \tag{3.10}$$

$$3. \quad \bigwedge_{-\infty}^{+\infty} T^t \xi_0 = \nu$$

where $\mathcal{E}$ is the split-up to separate points M, ν is the split-up consisting of M itself or empty set ϕ, $\bigvee_\alpha \xi_\alpha$ is the upper bound and $\bigwedge_\alpha \xi_\alpha$ the lower bound of the split-ups ξ_α. The inequality between split-ups is defined as follows: $\xi \leq \eta$ (ξ is not smaller than η), if each element of $C_\xi \in \xi$ is a combination of a number of elements of $C_\eta \in \eta$. The upper and lower bounds are understood by means of this ordering relation.

The meaning of this definition is the following. Let M be the region on the plane covered by the split-up ξ_0 (see fig. 5). The first condition of (3.10) implies that at the mapping T^t there arises a new split-up $T^t \xi_0$ which is always smaller. The second and third conditions mean that during the evolution of the DS from $-\infty$ to $+\infty$ split-ups will arise, whose intersections consist simply of the points M (they are the same as the upper bound), and their lower bound is equal to M.

It is intuitively clear from the very definition of (3.10) that there occurs a very strong mixing. Indeed the K-flow has a mixing of any multiplicity n [6]. At the same time there are DS having mixings of any multiplicity, which are not K-systems. K-systems have very good statistical properties (in a certain sense the best). There is a fine example of a K-system in Appendix A.

Let us now assume that a DS is considered whose Hamiltonian is given. Will it be an ergodic, mixing or K-system ? Are there sufficient conditions by means of which one can answer this question ? The importance of this question lies in the fact that we want to define what class of DS the YMCM belongs to, i.e. what maximum statistical properties it has !

In the papers by Hadamard, Hedlund, Hopf, Krylov, Anosov, Sinai et al. [9-12] such sufficient conditions were obtained. These authors have investigated the behavior of geodesic flows on Riemannian manifolds of negative curvature. They proved that the geodesic flow on the manifold of a constant negative curvature and on the surfaces of the variable of negative curvature has a strong instability and that it is a K-flow. Therefore, the negativity of the curvature of the manifold M is the sufficient condition for DS to be a K-system.

On the other hand, it follows from the Maupertuis principle that trajectories of the system (3.1) are geodesic flows of some Riemannian metrics [13]. If the Hamiltonian is written as

$$H = \frac{1}{2}\left(\frac{dS}{dt}\right)^2 + U(q)$$

$$ds^2 = dq_1^2 + \dots + dq_n^2 \tag{3.11}$$

and the Riemannian metrics is given in the form

$$d\rho^2 = [E - U(q)]\, ds^2 \tag{3.12}$$

where E is the total energy, then the negativity of the curvature in the region of configuration space $U(q) < E$ will be the sufficient condition for the system (3.11) to be a K-system.

Consider for simplicity the two-dimensional FS (2.12) when $M_i = N^a = 0$ (a more general case is considered at the end of this section)

$$H = \frac{\dot{q}_1^2 + \dot{q}_2^2}{2} + \frac{g^2}{2}(q_1 q_2)^2. \tag{3.13}$$

The metrics (3.12) has the form

$$d\rho^2 = \lambda(q_1 q_2)\, ds^2 = \left[E - \frac{g^2}{2} q_1^2 q_2^2\right]\left(dq_1^2 + dq_2^2\right) \tag{3.14}$$

The curvature in the region $U(q) < E$ is

$$R = -\frac{1}{2\lambda}\Delta \ln\lambda = \frac{(q_1^2 + q_2^2)\left(E + \frac{g^2}{2} q_1^2 q_2^2\right)}{\left[E - \frac{g^2}{2} q_1^2 q_2^2\right]^3} > 0, \tag{3.15}$$

$$\Delta = \frac{\partial^2}{\partial q_1^2} + \frac{\partial^2}{\partial q_2^2},$$

and it is strictly positive. Thus, the sufficient condition is not satisfied in this case. But this only comes to indicate that the phase trajectories are stable, if considered only in the intervals between two scatterings on the boundary $U(q) =$

const (see fig 4a). As it will be shown later, the instability detected in [1,2] develops due to the scattering on the equipotential boundary $U_{FS}(q)=$ const. For this purpose special criteria should be used obtained in papers mentioned above[9-12,32].

The investigation of geodesic flows on the manifold of negative curvature[9-12] has shown that in the formation of statistical properties of K-systems an important role is played by the so-called exponentially compressing and expanding transverse fibers [15]. In particular, if the curvature is negative, then the existence of appropriate fibers is proven. The negativity of curvature is a sufficient but not a necessary condition. Therefore, the way to find out when the DS is a K-system lies through the construction and investigation of transverse fibers.

Anosov and Sinai [11,12,14] succeeded in constructing and investigating a wide class of DS with transverse fibers. They are $\mathcal{Y}$ systems of Anosov [11] and Sinai's scattering "billiards" [15], which due to the existence of fibers with required properties are K-systems. We must show that the FS (2.12) has exponentially compressing and expanding transverse fibers which are homeomorphic to fibers of hyperbolic K-billiards of Sinai and, hence, the FS is a K-system.

In our case there is no need in a strict definition of fibers, it is enough to have an obvious idea of them. Consider the integral curves of equations

$$\dot{x}=-x, \quad \dot{y}=y$$

near the solution $x=y=0$ (see fig. 6).

To describe the arbitrary trajectory near the solution $x = y = 0$ one should follow how its coordinates x, y vary together with time in the plane perpendicular to t. It is easy to see that solutions with initial values lying on the X-axis form a family which at $t \to \infty$ exponentially approximates to the solution $x = y = 0$, whereas solutions with initial values on the Y-axis behave in the same way when the time direction changes. The first family of solutions forms an exponentially compressing transverse fiber, the second an expanding one. The DS is a y system of Anosov, if the phase curves near any datum behave like integral curves near the solution $x = y = 0$ in the example considered above. For such systems Anosov has constructed transverse fibers and proved that they are K-systems.

In Sinai's papers [15] transverse fibers have been systematically used to prove that the given DS is a K-system. In particular, he has considered the motion of elastic balls in a box with elastic and convex inside walls, a scattering billiards [32], and by means of transverse fibers it has been proven to be a K-system. Unlike y systems of Anosov, Sinai's scattering billiards has no uniform instability. However, the instability arising at the scattering on the convex inside boundary is already sufficient to form strong statistical properties like those of K-systems. In this case the scattering role of the negative curvature is played by the convex inside boundary*.

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* In Appendix B an example is given used by Arnold [6] illustrating this statement.

Let us come back to the Yang-Mills equations. As we have seen, the curvature in the range $U_{FS}(q) < E$ is positive, i.e. geodesic lines are stable there and the system is not uniformly unstable, as was the case with Y systems. But in spite of this, the equipotential boundary $U_{FS} = E$ (see fig. 4)

$$(q_1 q_2)^2 + (q_2 q_3)^2 + (q_3 q_1)^2 = const$$

is convex inside and has a negative curvature ! As is the case with Sinai's billiards, just this circumstance provides strong statistical properties of this system, K-mixing. Below it will be proven that transverse fibers of the FS (2.12) have the same structure as in the hyperbolic K-billiards of Sinai and, hence, the FS (2.12) is a K-system.

To prove the equivalency (homeomorphism) of these systems, one should introduce the concept of the structural stability in the narrow sense. The concept of coarseness or structural stability of DS was introduced by Andronov and Pontryagin [18]. The dynamical system T^t is called coarse, if for any system S^t fairly close to T^t * (in the sense of C^1 metrics $\rho(f,g) = \int |f-g| \, d\mu$ [19]) there exists a mutually single-valued and continuous transformation (homeomorphism) close to the identity transformation (in the sense of C^0 metrics $\rho(f,g) = \sup|f-g|$ [19]), transforming T^t system trajectories into those of S^t system, the direction of the motion along the trajectory being conserved (the velocity of motion may be various, i.e. for example, periodical trajectories

* By the distance between DSs is implied the distance in the metrics C^1 between the vector fields generating these systems (right hand parts in (3.1)).

corresponding to each other may have different motion periods).

Anosov has proven that any Y system is coarse, hence, so is the geodesic flow on the manifolds of negative curvature.

Thus, despite the complexity of the picture of phase curves, each of which is exponentially unstable, at a transition to a close vector field defined by the right hand part of (3.1) all this picture with everywhere dense phase curves and an infinite number of periodical trajectories stays almost unchanged. The FS (2.12), as we have noted, is not a Y system and, possibly, is not coarse. However, as we shall show below, it is stable with respect to special perturbations and is coarse in the narrow sense.

We shall call the system T^t structurally stable in the narrow sense, if it is structurally stable with respect to perturbations only along some curve in the C^1 topology and not in the whole vicinity. In fig. 7 the $\mathcal{E}$-vicinity of the system T^t in the C^1 topology is represented by a circle, and the $\mathcal{E}$-vicinity along the curve by a fragment of the curve AB within the circle.

Consider the following perturbation of the system (2.12)

$$U_{FS} \to U_{FS}^{\alpha} = \frac{1}{2}\left[(q_1 q_2)^{\alpha} + (q_2 q_3)^{\alpha} + (q_3 q_1)^{\alpha}\right] \qquad (3.16)$$

The parameter α defines the curve in the space of vector fields determining the DS. At $\alpha = 2$ the (3.17) coincides with the system (2.12) (in fig. 7 we denote it by A), and at $\alpha = \infty$ (the point B in the space of vector fields in fig. 7) it is Sinai's billiards with absolutely elastic walls depicted in fig. 4.

The main statement of this section is that at the change of α from two to infinity, phase trajectories are homeomorphic to each other despite the large change of α ! We have made sure in this calculating periodical and nonperiodical trajectories of the system (3.17) with a computer and then comparing them to trajectories of the billiards system with $\alpha = \infty$ constructed by means of laws of elastic reflection from absolutely elastic walls (fig. 4)

$$U_{FS}^{\alpha=\infty}(q) = const$$

The above fact yields three significant conclusions: firstly, the strong instability of the YMCM develops at the scattering of the trajectory on the equipotential boundary of negative curvature (fig. 4); secondly, the structure of exponentially compressing and expanding transverse fibers of the FS of the YMCM (2.12) is similar to that of Sinai's billiards system with hyperbolic walls of fig. 4, and hence, finally, the FS of the YMCM (2.12) is a K-system of Kolmogorov and possesses strong statistical properties.

The properties enumerated above are those of the global characteristics of DS. There are two more global characteristics of classical systems which are essential for the understanding of quantum mechanics of K- and YM-systems.

To each DS T^t one may juxtapose the number $h(T)$ called the Kolmogorov entropy [8]. The quantity h is positive and at various T^t may have values from 0 to $+\infty$.

K-systems possess positive entropy [8]. Since the FS of the YMCM is a K-system, it appears that its entropy is strictly positive.

$$h(T_{FS}) > 0 .$$

Secondly, to each DS T^t is juxtaposed a group of linear operators U^t

$$U^t f(x) = f(T^t x)$$

where $\{f(x)\}$ are the complex functions in M, summed up with the square $(f_2 \in L_2(M))$.

In 1931 Koopman has proven that if T^t is a measure preserving transformation, then the operators U^t form a one-parametric group of unitary operators [4,10,7]. The properties of the DS manifested as spectral invariants of the group U^t are called spectral properties. Ergodicity, weak mixing, mixing and K-mixing are spectral properties. In the case of K-flows one may calculate completely the spectrum of conjugated group of unitary operators U^t. It is a countably-multiple Lebesgue spectrum [8]. This implies, first, that the group U^t acting in the Gilbert space $L_2(M)$ has no invariant subspaces; second, there is such a countable set of functions $\{f_i(x)\}$ $i = 1,2,\ldots\ldots$ that $U^s f_i$, $-\infty < s < +\infty$ forms an orthogonal basis in $L_2(M)$. Thus, we manage to completely calculate the spectrum of the group of unitary operators U^t_{FS} conjugated with T^t_{FS} (2.12).

Up to now we have considered two- and three-dimensional FSs. Consider now a FS which arises in gauge theories with higher order symmetry groups. Let's define the N-dimensional FS as

$$H_{FS}^{(N)} = \sum_{i=1}^{N} \frac{\dot{q}_i^2}{2} + \frac{1}{4}\left[(q_1^2 + \dots + q_N^2)^2 - q_1^4 - \dots - q_N^4\right],$$

and calculate the curvature in the range of configurational space $U(q)<E$. According to (4) the metrical tensor is

$$g_{ij} = (E - U_{FS}^{(N)})\delta_{ij} = g(q_1 \dots q_N)\delta_{ij}$$

and the scalar curvature is

$$N^{-1}(N-1)^{-1} R_{FS}^{(N)} = -\frac{1}{Ng^2}\Delta g - \left(\frac{1}{4} - \frac{3}{2N}\right)\frac{(\nabla g)^2}{g^3} =$$

$$= \frac{(N-1)}{N}\ \frac{(q_1^2 + \dots + q_N^2)}{g^2} - \left(\frac{1}{4} - \frac{3}{2N}\right)\frac{\sum_{i=1}^{N} q_i^2 (q_1^2 + \dots + q_{i-1}^2 + q_{i+1}^2 + \dots + q_N^2)}{g^3}$$

It is seen from this expression for curvature that at $N \leq 6$, $R_{FS}^{(N)} \geq 0$. This implies that within the range $U_{FS}^{(N)}(q) < E$ trajectories are again stable, and the instability arises due to the scattering on the equipotential boundary $U_{FS}^{(N)}(q) = E$ of negative curvature, since the FS is homeomorphic to the scattering hyperbolic billiards. This fact cannot be established from (3.21), since it is valid beyond the equipotential boundary $U_{FS}^{(N)}(q) = E$ only, and doesn't take into account the boundary effects which are essential in this case.

A surprise arising at $N > 6$ is the emergence of the range with exponential instability defined by

$$\frac{N-1}{N} g(q_1^2 + \dots + q_N^2) < \left(\frac{1}{4} - \frac{3}{2N}\right)\sum_{i=1}^{N} q_i^2 (q_1^2 + \dots + q_{i-1}^2 + q_{i+1}^2 + \dots + q_N^2) \qquad (3.17)$$

since there $R_{FS}^{(N)} < 0$.

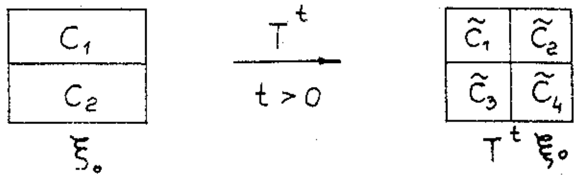

Fig. 5

From (3.17) follows that the range of exponential instability is along the equipotential boundary and broadens inside the range $U^{(N)}_{\bar{f}s} < E$ with $N \to \infty$! This comes to once again confirm the fact that the instability arises due to equipotential boundary.

In conclusion let us present the considerations indicating that statistical properties of the YMCM at the absence of external sources ($n^a = 0$) are caused just by the FS (2.12). To do so let us consider eq. (2.9) determining the motion of rotational degrees of freedom, assuming at the same time that the inertia moments (2.6), i.e. the variables x, y, z are independent of time. Whereas it is easy to understand from (2.11a) that T_{YM} will be a conserving quantity and that one can integrate eqs. (2.9) like Euler equations for a freely rotating solid body [13]. This is an obvious indication that rotational degrees of freedom in (2.9) do not induce statistical properties of the YMCM, and that these properties are caused by the FS (2.12).

In the next section classical gauge systems with spontaneously broken symmetry are considered and it is shown that when the value of the vacuum average of the scalar field φ is sufficiently large, the system doesn't manifest its statistical properties and is in a kind of order phase (it is close to the integrated system), and that with the decrease in $<\varphi>$ by way of an infinite sequence of rapidly growing bifurcations the system passes into the disorder phase.

4. Gauge Systems with Spontaneous Symmetry Breaking

In ref. [1] statistical properties of classical gauge systems with spontaneous symmetry breaking are investigated. In the present section these systems are analysed theoretically.

Let us first consider the two-dimensional Hamiltonian (1.6)

$$H=\frac{1}{2}\left(p_1^2+p_2^2\right)+\frac{\omega^2}{2}\left(q_1^2+q_2^2\right)+\frac{g^2}{2}\left(q_1 q_2\right)^2 \qquad (4.1)$$

where $\omega^2=g^2\eta^2/2$. Let's introduce the variables J_i, φ_i

$$q_i=\left(2J_i/\omega\right)^{1/2}\sin\varphi, \quad p_i=\left(2J_i\,\omega\right)^{1/2}\cos\varphi \qquad (4.2)$$

$(i=1,2)$ where (4.1) will read

$$H=H_0+H_1, \quad H_0=\left(J_1+J_2\right)\omega$$

$$H_1=\frac{g^2}{2}\frac{J_1 J_2}{\omega^2}\left[1-\cos 2\varphi_1-\cos 2\varphi_2+\frac{1}{2}\cos\left(2\varphi_1+2\varphi_2\right)+\frac{1}{2}\cos\left(2\varphi_1-2\varphi_2\right)\right] \qquad (4.3)$$

As has been noted [1], the parameter $\Pi=\omega^4/g^2\mu^4$ completely characterizes the system. At $\Pi\to\infty$ the Hamiltonian H is equal to H_0 (harmonic oscillator). Trajectories of the system with the Hamiltonian H_0 are well known. The main problem is how they will behave at the presence of the perturbation H_1 , and $t\to\infty$.

It is convenient to consider, following Poincare and

Birkhoff [20,21], not the trajectories themselves in the phase space, but their successive intersections with two-dimensional cutting surface α. Let us determine the mapping T transforming α into itself as follows: fixing on α the point X, we let the trajectory out of it and follow it, till it again traverses α. The new point will be TX This Poincare mapping replaces the trajectory by an infinite set of points on α obtained at successive applications of the mapping $T^n X$ *. All the essential properties of the trajectory are reflected in the peoperties of this set, which we shall call a phase picture.

If the motion is periodic, the crossing occurs in a finite number of points; if it is limited by the tore surface, then the points are located on closed curves, and, finally, at ergodic motion the point chaotically wanders over the plane α.

The phase picture of the system (4.3) with the Hamiltonian H_0 in the plane $\alpha = (q_2, p_2)$ at $P_1(\alpha) > 0$ represents a circle of immobile points, and it is of interest for us how it will change at $\mathfrak{N} \neq \infty$.

Let us pass from the variables (4.2) to J_1, J_2, Φ_1, Φ_2

$$J_1 = \mathfrak{I}_1 + \mathfrak{I}_2, \qquad \Phi_1 = \frac{\varphi_1 + \varphi_2}{2},$$
$$J_2 = \mathfrak{I}_1 - \mathfrak{I}_2, \qquad \Phi_2 = \frac{\varphi_1 - \varphi_2}{2}, \tag{.4.4}$$

---

* This is a general construction allowing to compare each continuous flow T^t (see sec.2) to the cascade T^n, where n is the integer number.

then

$$H_o = J_1 \omega$$

$$H_1 = \frac{g^2(J_1^2 - J_2^2)}{8\omega^2}\left[1 - \cos(2\phi_1 + 2\phi_2) - \cos(2\phi_1 - 2\phi_2) + \frac{1}{2}\cos 4\phi_1 + \frac{1}{2}\cos 4\phi_2\right] \quad (4.5)$$

Since H_o is independent of J_2 the motion proceeds at one frequency ω

$$\dot{J}_1 = 0 , \qquad \dot{\phi}_1 = \frac{\partial H_o}{\partial J_1} = \omega ,$$
$$\dot{J}_2 = 0 , \qquad \dot{\phi}_2 = \frac{\partial H_o}{\partial J_2} = 0 . \qquad (4.6)$$

The variables J_1, ϕ_1 are called fast, and J_2, ϕ_2 slow. When the motion in the unperturbed system H_o is described by a smaller number of frequencies than the number of degrees of freedom (the system is "shifted" by conserving integrals) then [13]

$$\det \left\| \frac{\partial^2 H_o}{\partial J_x \partial J_m} \right\| = 0 \qquad (4.7)$$

This is a case of self-degenracy.

The answer to the question raised at the beginning of the section on the effect of the perturbation H_1 is given by the Kolmogoroc theorem [22]. If the perturbation H_1 is small, and the integrable system with the Hamiltonian H_o is not degenerate

$$\det \left\| \frac{\partial^2 H_o}{\partial J_x \partial J_m} \right\| \neq 0 \qquad (4.8)$$

then the majority of nonresonant tores* are slightly deformed.

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* The frequencies on appropriate tores are incommensurable (see [22]).

Trajectories of perturbed motion beginning on these deformed tores fill them everywhere densly and quasi-periodically. These tores form a closed set nowhere dense (between which slits are left filled up with the remnants of disintegrating resonance tores), which has a positive measure tending to that of the whole phase space when the perturbation $H_1 \to 0$ *.

If the Hamiltonian H_0 is degenerate, as in our case (4.7), then the perturbation will have a different result. The problems due to degeneracy were solved in Arnold's papers [25,26]. Below the results of perturbation theory for degenerate systems are used.

4a. Self-Degeneracy . Extraction of Age Part

The basic idea of classical perturbation theory for degenerate systems [26] consists in the averaging of the Hamiltonian H_1 (4.5) over the fast variable ϕ_1 and extracting the age part from H_1 [27]

$$\bar{H}_1 = \frac{1}{2\pi}\int_0^{2\pi} H_1(J_1, J_2, \phi_1, \phi_2)\, d\phi_1 = \frac{g^2(J_1^2 - J_2^2)}{8\omega^2}\left[1 + \frac{1}{2}\cos 4\phi_2\right] \quad (4.9)$$

or, in the variables $\mathcal{J}_1, \mathcal{J}_2, \varphi_1, \varphi_2$

$$\bar{H}_1 = \frac{g^2 \mathcal{J}_1 \mathcal{J}_2}{2\omega^2}\left[1 + \frac{1}{2}\cos(2\varphi_1 - 2\varphi_2)\right] \quad (4.10)$$

The periodical part $\tilde{H}_1$ is

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* This phenomenon was observed in various numerical experiments [23,24].

$$\tilde{H}_1 = H - \tilde{H}_1 = \frac{g^2 J_1 J_2}{2\omega^2}\left[\frac{1}{2}\cos(2\varphi_1 + 2\varphi_2) - \cos 2\varphi_1 - \cos 2\varphi_2\right] \quad (4.11)$$

Canonical equations with the Hamiltonian $H_o + \bar{H}_1$

$$\dot{j}_1 = 0 \quad , \qquad \dot{\phi}_1 = \omega + \frac{\partial \bar{H}_1}{\partial j_1} \ , \quad (4.12)$$

$$\dot{j}_2 = 0 \quad , \qquad \dot{\phi}_2 = \frac{\partial \bar{H}_1}{\partial j_2}$$

define the slow, "age" variation of the parameters j_2, ϕ_2 .

If the system with the Hamiltonian $H_o + \bar{H}_1$ is integrable, and in our case it is so, then at a proper selection of new variables $S_1, S_2, \Theta_1, \Theta_2$ the age part of H will be independent of angular variables. The periodical part of H leads merely to an additional vibration of the perturbed trajectory by the quasi-periodical motion with rapid $\xi_1 \sim 1/\mathcal{T}$ and slow $\xi_o \sim \omega$ frequencies

$$\dot{S}_1 = 0, \ \dot{S}_2 = 0 \ , \ \dot{\Theta}_1 = \xi_o , \ \dot{\Theta}_2 = \xi_1 \ . \quad (4.13)$$

The basic result obtained by Arnold [25,26] is that at a small enough perturbation H_1 for the majority of initial conditions the perturbed motion is indeed quasi-periodical and close to the above quasi-periodical motion (4.12-13) with the Hamiltonian $H_o + H_1$ at all t

4b. Quasi-Periodical Motion with the Hamiltonian

The system (4.10), (4.12) with the Hamiltonian $H_o + \bar{H}_1$ is integrable; along with the energy is conserved the quantity

$$J = J_1 + J_2 = j_1 \quad . \quad (4.14)$$

since $H_0 + \tilde{H}_1$ depends on the difference of angular variables $\varphi_1 - \varphi_2$ only.

Let us define the phase picture of this system in the plane $\alpha = (q_2, p_2)$ at $p_1(\alpha) > 0$ or in the variables $J_1, J_2, \varphi_1, \varphi_2$ (4.2) in the plane $\varphi = 0$. Let us substitute $J_1 = J - J_2$. from (4.14) into (4.10)

$$\mu^4 = J\omega + \frac{g^2 J_2 (J - J_2)}{2\omega^2}\left[1 + \frac{1}{2}\cos 2\varphi_2\right] \qquad (4.15)$$

This algebraic equation defines the lines of intersection of invariant tores with the plane α , since one may find from it J_2 as a function of φ_2 and then substitute it in (4.2). As a result the phase picture will look as shown in fig. 1.

To centers of two closed curves correspond stable periodical trajectories, and the two points of contact are met by unstable periodical trajectories, separatrices. We find the periodic trajectories from the condition [28]

$$\dot{J}_1 = \dot{J}_2 = \dot{\varphi}_1 - \dot{\varphi}_2 = 0 \qquad (4.16)$$

For the stable we have:

$$J_1 = J_2 , \quad \varphi_1 - \varphi_2 = \frac{\pi}{2}, \ \frac{3\pi}{2} \qquad (4.17)$$

and for the unstable

$$J_1 = J_2 , \quad \varphi_1 - \varphi_2 = 0, \ \pi \qquad (4.18)$$

It is seen from these relations that it is the vicinity of the resonance tore of the system with the Hamiltonian

$$h = (J_1 + J_2)\omega + \frac{g^2 J_1 J_2}{2\omega^2}, \tag{4.19}$$

$$\Omega_1 = \omega + \frac{g^2 J_2}{2\omega^2}, \quad \Omega_2 = \omega + \frac{g^2 J_1}{2\omega^2},$$

that is substantially distorted: (at $J_1 = J_2$ we have $\Omega_1 = \Omega_2$) the circle is replace by a zone containing crescent curves grouping near periodical trajectories (see fig. 1).

Finally, it is useful to write this integrable system in the old variables (q_1, q_2, q_1, q_2)

$$H_0 + \tilde{H}_1 = \frac{1}{2}\left(p_1^2 + \omega^2 q_1^2\right) + \frac{1}{2}\left(p_2^2 + \omega^2 q_2^2\right) + \frac{g^2}{8\omega^4}\left(p_1^2 + \omega^2 q_1^2\right) \cdot$$

$$\left(p_2^2 + \omega^2 q_2^2\right) + \frac{g^2}{16\omega^4}\left[\left(p_1 p_2 + \omega^2 q_1 q_2\right)^2 - \omega^2\left(p_1 q_2 - p_2 q_1\right)^2\right] \cdot \tag{4.20}$$

whose Lagrangian is an infinite series in $1/\omega^2$, and the quantity

$$\omega J = \frac{1}{2}\left(p_1^2 + \omega^2 q_1^2\right) + \frac{1}{2}\left(p_2^2 + \omega^2 q_2^2\right) \tag{4.21}$$

is the second integral. The periodical part of $\tilde{H}_1$ is

$$\tilde{H}_1 = -\frac{g^2}{8\omega^4}\left(p_1^2 + \omega^2 q_1^2\right)\left(p_2^2 - \omega^2 q_2^2\right) - \frac{g^2}{8\omega^4}\left(p_1^2 - \omega^2 q_1^2\right)\left(p_2^2 + \omega^2 q_2^2\right) \tag{4.22}$$

$$+ \frac{g^2}{16\omega^4}\left[\left(p_1 p_2 - \omega^2 q_1 q_2\right)^2 - \omega^2\left(p_1 q_2 + p_2 q_1\right)^2\right]$$

Thus at small perturbation the system (4.1) has a phase picture close to that of the integrable system (4.20) in terms of the above KAM-theory[22].

It should be now clarified what will happen to the phase picture when the perturbation increases ? The KAM-theory gives no answer to this question.

4c. Amplitude Instability

The answer to this question is in principle known. It has been shown in the third section that the classical gauge system without spontaneous symmetry breaking is a K-system and has strong statistical properties because at $\mathfrak{N} = 0$ ($\eta = 0$) we have a K-system, and at $\mathfrak{N} = \infty$ an integrable system (4.20). What happens in the intermediate region ? Is there any critical value $\mathfrak{N}_c$ at which most KAM-tores will be destroyed ?

There are no satisfactory answers to such questions yet but it has been ascertained by means of numerical experiments that isolated resonances have no serious effect on the motion, but as soon as resonance zones are overlapped, there appears an instability, and relatively many trajectories leave their tores too [31].

One may estimate the critical value $\mathfrak{N}_c$ at which the overlapping of resonance zones occurs. To do so, one should, following Walker and Ford [28], turn to the variables $S_1, S_2, \theta_1, \theta_2$ (4.13), where the Hamiltonian $H_0 + \bar{H}_1$ is diagonal

$$H = H_0(S_1, S_2) + \bar{H}_1(S_1 S_2) + \tilde{H}_1(S_1 S_2 \theta_1, \theta_2) \qquad (4.23)$$

Expanding $\tilde{H}_1$ in Fourier series in θ_1 and θ_2

$$\tilde{H}_1 = \sum_{n,m} \tilde{H}_{1mn}(S_1 S_2)\, e^{i(n\theta_1 + m\theta_2)} \qquad (4.24)$$

we shall obtain new, the so-called secondary ($n \cdot m$) resonances, whose position on the plane α is estimable, since at each fixed (n , m) the system is integrable. The origin of secondary resonances is well illustrated in fig. 2. $\mathcal{T}_c$ may now be approximately estimated by means of the overlapping condition of the primary 2-2 and these secondary resonances. The "phase" transition detected and discussed in [1] is due to this very bifurcation.

Consider now the three-dimensional case

$$H = \frac{1}{2}\sum_{i=1}^{3}\left(p_i^2 + \omega^2 q_i^2\right) + \frac{g^2}{2}\left[(q_1 q_2)^2 + (q_2 q_3)^2 + (q_3 q_1)^2\right] \tag{4.25}$$

which also is characterized by one parameter $\mathcal{T}$. Let's intoduce the variables J_i, φ_i (4.2)

$$H = H_0 + H_1 , \qquad H_0 = (J_1 + J_2 + J_3)\,\omega$$

$$H_1 = \frac{g^2}{2\omega^2}\sum_{i<k} J_i J_k\left[1 - \cos 2\varphi_i - \cos 2\varphi_k + \frac{1}{2}\cos(2\varphi_i + 2\varphi_k) + \frac{1}{2}\cos(2\varphi_i - 2\varphi_k)\right]. \tag{4.26}$$

The age part can be distinguished by introducing the variables j_i, ϕ_i by means of canonical transformation

$$F = j_1\varphi_1 + j_2(\varphi_2 - \varphi_1) + j_3(\varphi_3 - \varphi_1) \tag{4.27}$$

whence

$$\phi_i = \frac{\partial F}{\partial j_i} , \qquad J_i = \frac{\partial F}{d\varphi_i} . \tag{4.28}$$

For the Hamiltonian H_0 the variable ϕ_1 is rapid, and ϕ_2 and ϕ_3 are slow. Averaging the perturbation H_1 over the rapid variable ϕ_1 we obtain

$$\bar{H}_1 = \frac{g^2}{2\omega^2}\sum_{i<k} J_i J_k\left[1 - \frac{1}{2}\cos(2\varphi_i - 2\varphi_k)\right] \tag{4.29}$$

In the system with the Hamiltonian $H_0 + \bar{H}_1$ one may immediately find the second integral

$$\mathcal{J} = \mathcal{J}_1 + \mathcal{J}_2 + \mathcal{J}_3 = J_1 \tag{4.30}$$

Has it a third integral ? The answer to this question is unknown to **us**.

What will happen at various $\mathfrak{N}$ in the system (4.25) ? As in the two-dimensional case, at $\mathfrak{N} \to \infty$ we have three noninteracting oscillators. With the decrease in $\mathfrak{N}$ (i.e. with the increase in perturbation) the system (4.25) comes closer to the system with the Hamiltonian

$$h = (\mathcal{J}_1 + \mathcal{J}_2 + \mathcal{J}_3)\omega + \frac{g^2}{2\omega^2}(\mathcal{J}_1\mathcal{J}_2 + \mathcal{J}_2\mathcal{J}_3 + \mathcal{J}_3\mathcal{J}_1) \tag{4.31}$$

which in initial coordinates has the form

$$h = \frac{1}{2}\sum_{i=1}^{3}(p_i^2 + \omega^2 q_i^2) + \frac{g^2}{8\omega^4}\sum_{i<k}(p_i^2 + \omega^2 q_i^2)(p_k^2 + \omega^2 q_k^2) \tag{4.32}$$

Finally, at some $\mathfrak{N}$ the system will be stochastic.

5. YM and K-System Quantum Mechanics

In previous sections properties of classical solutions of Yang-Mills equations have been studied. It has been shown that in the phase space of the system the energe surface H = const is chaotically filled with trajectories, and that this flow is a K-flow close to that of the billiards system with hyperbolic walls (fig. 4). Therefore, the phase flow of the Yang-Mills classical mechanics possesses all the properties of K-flow: mixing of any multiplicity, the positive Kolmogorov entropy, the spectrum of the unitary operator U_T conjugated to T_{YM} , the countably multiple Lebesgue one.

The natural question arising now is what quantum-mechanical properties the system with the Hamiltonian H_{YM} possesses if in the classical limit $\hbar \to 0$ it is a K-system?

The significance of the answer to this question consists in the following. In the field theory, e.g. in QED, the electromagnetic field is represented in the form of a set of harmonic oscillators whose quantum-mechanical properties (as of an integrable system) are well known, and the interaction between them is taken into account by the perturbation theory. Such an approach excellently describes the experimental situation. In QCD the state of things is quite different. The properties of the YMCM as of a K-system can't be established in any finite order of perturbation theory to a harmonic oscillator * (see secs. 3 and 4). Therefore, to understand the QCD, it seems important to investigate the quantum-mechanical properties of the systems which in the classical limit are K-systems.

Historically the problem of the quantization of nonintegrable systems, i.e. systems which have no conserving integrals except for energy, goes back to the dawn of quantum mechanics after the definition of the Bohr-Sommerfeld quantization rules [20, 33, 34]. It has been proven by Poincare that the problem of three bodies is not integrable, and, therefore, the

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* The classical perturbation theory is implied, not the quantum-mechanical [16].

Bohr-Sommerfeld quantization rules cannot be applied to this system [20, 33, 34].

Consider the stationary equation of Schrodinger in the YM theory in the Hamiltonian gauge $A_o^a = 0$

$$\frac{1}{2}\int d^3x \left[-\frac{\delta^2}{(\delta A_i^a)^2} + H_i^a H_i^a \right] \psi[A] = E\psi[A]$$

with the coupling equations

$$[\delta^{ab}\partial_j + g\varepsilon^{acb} A_j^c] \frac{\delta}{\delta A_j^b} \psi[A] = 0$$

where $H_i^a = \frac{1}{2}\varepsilon_{ijk} G_{jk}^a$. Let the fields A be again time-dependent only. Then (5.1-2) may be rewritten as

$$\frac{1}{2}\left[-\frac{\delta^2}{(\delta A_i^a)^2} + \frac{g^2}{2}\left((A_i^a A_i^a)^2 - (A_i^a A_j^a)^2 \right) \right] \psi[A] = \varepsilon\psi[A]$$

$$\varepsilon^{abc} A_j^c \frac{\delta\psi}{\delta A_j^b} = 0$$

where ε - is the energy density. Let us call the system (5.3-4) the YM quantum mechanics. In the case when the matrice of the fields A_i^a is diagonal (see sec. 1), we shall have

$$-\frac{1}{2}\Delta\psi + \left[\frac{g^2}{2}(x^2y^2 + y^2z^2 + z^2x^2) - \varepsilon \right]\psi = 0$$

Eq. (5.5) is a quantum analogy of the FS (2.12). Since the FS is nonintegrable (see secs. 3 and 4) the Schrödinger equation (5.5) cannot be solved by the separation of variables. The only quantum number is the energy, since there are no other integrals.

The stationary wave functions of the quantum system depend quasi-periodically on time, therefore, there can be no question of the properties of mixing and instability like those of classical systems. The complexity of the classical system trajectories turns now into the peculiarities of the energy level spectrum and wave function of the corresponding quantum system.

It now seems reasonable to define more exactly the question raised at the beginning of the section in the following way: what is the structure of the energy level spectrum and wave functions of quantized gauge systems, if in the classical limit they are nonintegrable and are K-systems ?

Consider first integrable systems. The peculiarity of energy spectra of integrable systems with N degrees of freedom is the <u>degenracy</u> of their energy spectra, since in that case the Schrödinger equation is factorized into N one-dimensional equations with N various quantum numbers.

In contrast, nonintegrable systems <u>have no degeneracy</u> at all [35,36,37,13] . The energy levels are at finite distance from each other, and their relative arrangement is like that arising in the systems defined by the Hamiltonian matrice with randomly distributed elements [38, 39, 40]. There occurs the so-called "level repulsion" expressed in the fact that the probability of finding two neighbouring levels at a distance Δ smaller than the mean value $< \Delta >$ tends to zero as

$$\Delta^{\beta} \xrightarrow[\Delta \to 0]{} 0$$

where β is the critical exponent. This implies the existence of a strong correlation between near levels. For the typical — integrable systems this quantity behaves as

$$\exp\{-\Delta\} \underset{\Delta\to 0}{\longrightarrow} 1$$

These conclusions are confirmed at analyzing energy levels of concrete K-systems: square billiards with a scattering circle of the radius R in the middle [41] (see fig. 10), and the billiards with a stadium-like boundary - two circle halves of the radius r are connected by straight lines of the length a [42] (see fig. 11).

Both systems have integrable limits: the first at $R = 0$ - is an infinite potential well, the second at $a = 0$ is a round well of the radius r. As is known, the energy spectrum in this limit is strongly degenerate. At R and a being different from zero they are K-systems, and the behaviour of their energy levels is shown in fig. 12 from ref. [41]. The degeneracy is eliminated. Distributions of energy level spacing are shown in fig. 13 [41, 42], which is consistent with (5.6).

Of special interest is the structure of wave functions. In fig. 11 a typical wave function is presented. The nodal curves are seen to be irregular in direction and their separation is roughly regular verifying the prediction about the random orientation of the wave vector at different poritions.

Let us come back to the gauge system (5.5). Let's apply to it the regularization method applied to the FS (2.12), i.e. introduce a spontaneous symmetry breaking. The Hamiltonian will then have the form of (4.25), where p and q are the operators.

In the integrable limit ($\omega^2 \to \infty$) the spectrum of this system coincides with the spectrum of the sum of three harmonic oscillators. It is degenerate, as for the billiards systems, at $R = a = 0$. With the increase in perturbation ($\omega^2 < \infty$) the potential energy gains channels along the axes x, y, z. On eliminating the regularization $\omega^2 = 0(\eta = 0)$ the channels lengthen and tend to infinity.

Making use of the results obtained at analyzing the billiards systems [41, 42] , one may qualitatively describe the gauge spectrum. It consists of the nonzero energy ground state and perturbed states where degeneracy is lacking. There occurs a level repulsion (5.6) with the critical exponent β so far unknown. The wave function relief is like the one depicted in fig. 11. With the decrease in ω^2 the ground state energy tends to zero together with the average distance between levels. This fact is easy to understand taking into account that there are no dimensional parameters at $\omega^2 = 0$. Note that the dimensional parameter may arise only when allowing for space degrees of freedom that we have neglected.

In conclusion consider the connection of the discussed problems with confinement. In papers by Nielsen, Olesen, Parisi et al. [43-45] a simple argument has been proposed why one has confinement in a random vacuum. The assumption that in vacuum there exist random fields with nontrivial topology has been based on the vacuum instability [43, 46] . A number of questions the answers to which should be given by the complete dynamical theory of vacuum, remained unsolved. The results obtained at the investigation of a one-dimensional model of gauge fields (YMCM) allows to hope that the assumption of randomness has a deep dynamical origin.

6. Conclusion

The basic purpose of this paper is to investigate the Yang-Mills classical equations in the case when the vector potential depends on the time only. The FS of the YMCM is shown to be a Kolmogorov K-system and, hence, possess strong statistical properties. Phase trajectories of this system by their structure are similar to the trajectories of the Krylov-Sinai billiards system. Like every K-system the FS of the YMCM has a positive Kolmogorov entropy, a mixing of any multiplicity, and the spectrum of one-parametric group of unitary operators conjugated to the flaw T_{YM} is a countably-multiple Lebesgue one.

Considered are also the systems with spontaneous symmetry breaking. It is shown that in the "deep" Higgs region $\mathfrak{N} \to \infty$ the system is close to the integrable one, and the phase space is in the main composed of invariant tores. Then, with the decrease in $\mathfrak{N}$, by means of the infinite sequence of rapidly growing bifurcations the system becomes stochastic. All these results are of a theoretically-nonperturbative character and cannot be obtained in any finite order of classical perturbation theory.

Ergodic properties of classical gauge systems affect the structure of energy levels of appropriate quantum systems. The energy spectrum of the systems with a spontaneously broken symmetry in the region $\Pi \to \infty$, where one may still make use of conserving integrals, is close to that of harmonic oscillator. With the decrease in Π there occurs a relevelling and the degenerate spectrum transforms into the nondegenerate spectrum of a quantum K-system, whose basic property is the level repulsion, and their wave fucntions are isotropic.

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Appendix A

In section 3 the mapping of the phase space M on itself depending on the time T^t was considered. It is natural to consider also discrete mappings which merely repeatedly act on M. These mappings are automorphisms [7]. Through $T^n X$ the result of n-fold mapping is denoted.

In the proposed example [11-13] as a phase space we have taken the surface of the tore with geographic coordinates: the longitude φ_1 and the latitude φ_2. The square $0 \leq \varphi_{1,2} \leq 2\pi$ on the plane φ_1, φ_2 serves as the tore map. One may as well consider the whole plane (φ_1, φ_2) without distinguishing the points with the coordinates multiple to 2π.

The tore automorphism is determined by the integer number unimodular linear transformation of the plane (φ_1, φ_2) with the matrix [11-13]

$$\begin{pmatrix} \varphi_1' \\ \varphi_2' \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \quad (A.1)$$

The action of the transformations T^n, $n > 0$ on the plane consists in a rapid, at an exponential speed, compression in one direction and extension in another one. These directions are determined by eigenvectors of the transformation (A.1).

The properties of automorphism (A.1) are well investigated in [11-13]. It has a countable number of cycles, is coarse (see the final paragraphs of sec. 3) and possesses compressing and extending fibers and, hence, is a K-system.

To make this example look more "physical" let us draw a square with a black blot in the centre. Under the effect of the transformation (A.1), which is not difficult to construct, the blot, already after several first iterations will uniformly spread over the square (fig. 8).

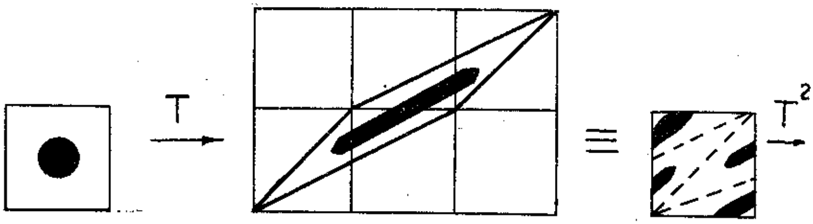


Fig. 8

This is a good model of "irreversible" solutions of black ink in a glass of transparent water, with the transformation (A.1) corresponding to the mixing.

Appendix B

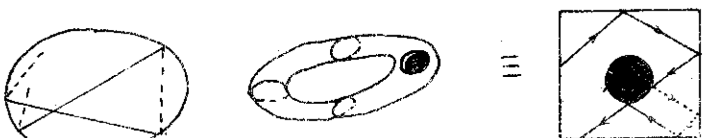
The example proposed by Arnold [16, 30] illustrates how the mixing (K-mixing) appears in the Krylov-Sinai billiards system, and how this property of being a K-system relates to the behaviour of geodesic lines on surfaces of negative curvature.

Consider simultaneously two systems. In the first the point moves by the geodesical on the surface of a triaxial ellipsoid of fig. 9a, in the second by the surface of a pretzel from fig. 9b. The ellipsoid has a positive curvature whose integral is 4π, the pretzel has a negative curvature whose integral is -4π, therefore, the geodesic flux on the pretzel surface is a K-flux, and on the ellipsoid surface it is quite integrable [13] and, therefore, non-mixing.

On the other hand, if the small axis of the ellipsoid is reduced to zero, the ellipsoid turns into an ellipse and the geodesical flux in the limit transforms into the billiards system in the region limited by the ellipse of fig. 10a, with

a) Fig. 9 b)

the trajectories within the ellipse never being everywhere dense. Let us turn the pretzel in the second system along the circumpherence C: the pretzel degenerates into a two-sided tore with a hole (fig. 10b), and the geodesic flow into the billiards system on the tore with a hole (fig. 10b) (the dotted



a) Fig. 10 b)

line denotes the motion in an internal surface of tore).

Thus the toric billiards with a dissipating round wall of the convex inside region of motion has a strong mixing and is a K-system. The instability develops on the round wall - the negative curvature of the pretzel is gathered along the circumpherence edge during the flattening.

Here we see a good analogy with gauge systems, (2.12) whose motion is stable within the region $U_{YM} < E$ and the instability develops at scattering on the boundary formed by the equipotential surface $U_{YM} = E$ of the convex inside region of motion.

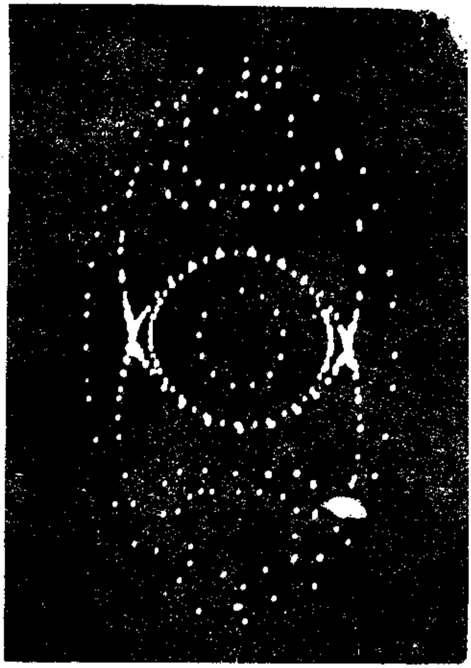


Fig. 1

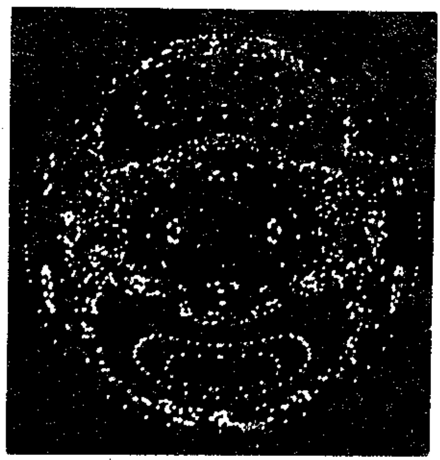


Fig. 2

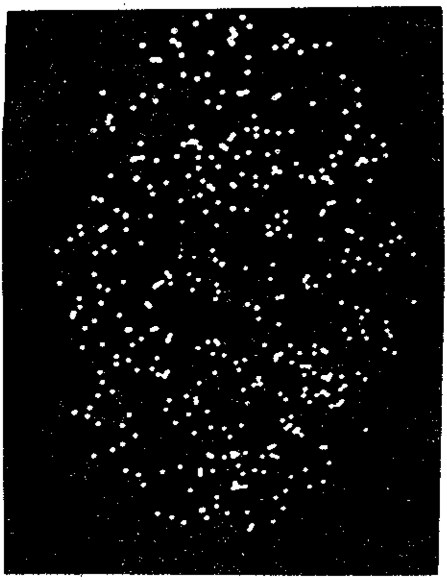


Fig. 3

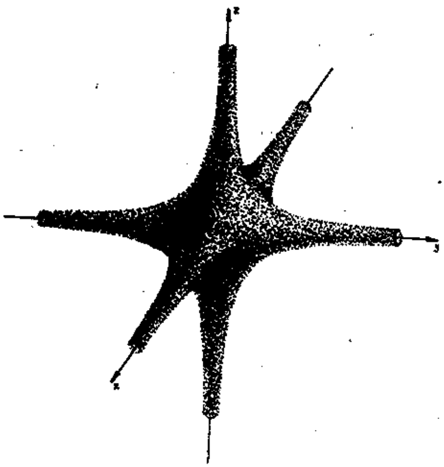


Fig. 4

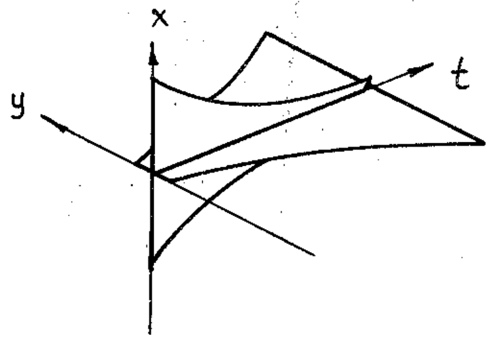


Fig. 6

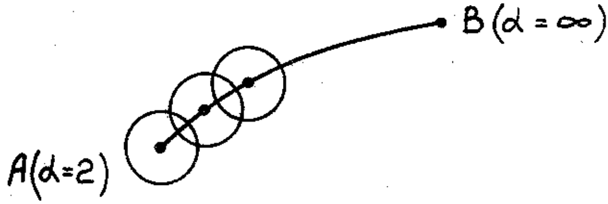


Fig. 7

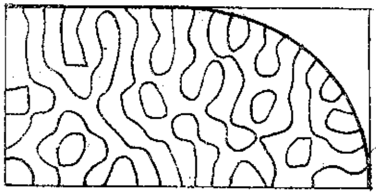


Fig. 11

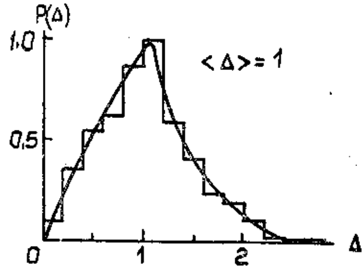


Fig. 13

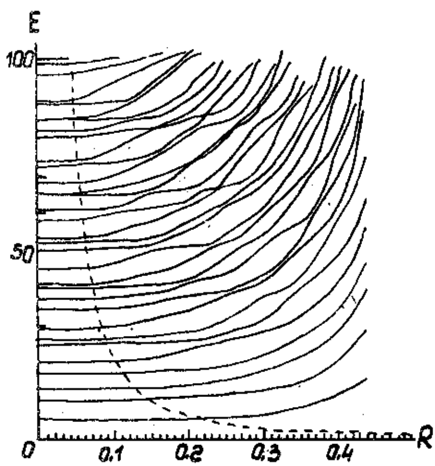


Fig. 12

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