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ЕФИ-6I4(4)-83

N.S.ANANIKIAN, G.L.BALAYAN

SUMMATION OF DIAGRAMS BY MEANS OF LEGENDRE TRANSFORMATION IN SCALAR PARAMETER. HIGHER ORDERS

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SUMMATION OF DIAGRAMS BY MEANS OF LEGENDRE TRANSFORMATION IN SCALAR PARAMETER. HIGHER ORDERS

Results of the investigation of higher orders of diagram summation mathod by means of the Legendre transformation in scalar parameter are presented. The investigations show a number of advantages of such summation as compared with the standard perturbation theory.

Yerevan Physics Institute

Yerevan 1983

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Н.С.АНАНИКЯН, Г.Л.БАЛАЯН

СУММИРОВАНИЕ ДИАГРАММ С ПОМОЩЬЮ ПРЕОБРАЗОВАНИЯ ЛЕЖАНДРА ПО СКАЛЯРНОМУ ПАРАМЕТРУ. ВЫСШИЕ ПОРЯДКИ

Приводятся результаты исследования высших порядков метода суммирования диаграмм при помощи преобразования Лежандра по скалярному параметру. Исследования показывают ряд преимуществ такого суммирования по сравнению со стандартной теорией возмущения.

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Ереван 1983

Y E R E V A N   P H Y S I C S   I N S T I T U T E

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The standard perturbation theory by the small coupling constant (λ) applied to the problem of anharmonic oscillator yields good results only at $\lambda \lesssim 0.01$. With the increase of the coupling constant at a transition to a higher order the accuracy deteriorates, which is due to the asymptotic property of perturbation theory series. Moreover, the energy curves $\mathcal{E}(\lambda)$ as functions of coupling constants begin to oscillate already at $\lambda \sim 0.1$, i.e. they are now higher, now lower the exact curve $\mathcal{E}^{exact}(\lambda)$, since the perturbation theory series is sign-changing.

In the present paper the method of resumming of series [1] allowing to eliminate the oscillations and obtain a good agreement with the exact data [2] in a wide range of values of λ is considered. The method is based on the Legendre transformation in scalar parameter.

1. The Legendre transformation in scalar parameter was considered in ref. [3] , where the complete set of equations for the effective potential Γ (φ, G, H, S) was derived, depending on one(φ)-, two(G)-, three(H)-point Green functions and

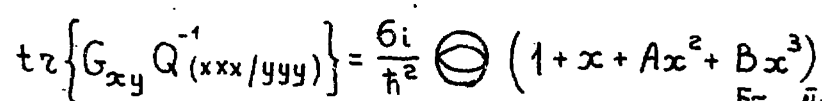
the scalar parameter $S = \langle S_{cl}(\varphi) \rangle$ (the vacuum expectation value of the classical action).

The potential $\Gamma(\varphi, G, H, S)$ may be represented in the form $\Gamma = S + F(\tilde{S}, G, H)$ [3] where

$$\tilde{S} = S - S(\varphi) - \frac{i\hbar}{2} \mathcal{D}^{-1} G - \frac{g\hbar}{4} G \varphi^2 - \frac{g\hbar^2}{8} G^2 - \frac{g\hbar^2}{3!} H \varphi \qquad (1.1)$$

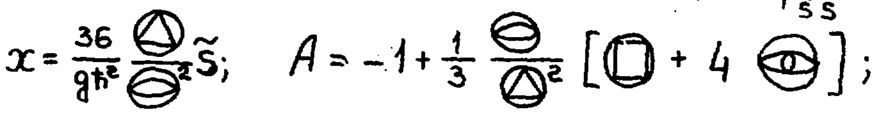
In (1.1) $S(\varphi)$ is the action functional of the theory (for $g\varphi^4$-theory it has the form $S(\varphi) = \frac{1}{2} i \mathcal{D}^{-1} \varphi\varphi + \frac{1}{4!} g \varphi^4$, and $i\mathcal{D}^{-1}_{xy} = -(\Box_x + m^2)\delta(x-y)$ is the "bare" propagator.

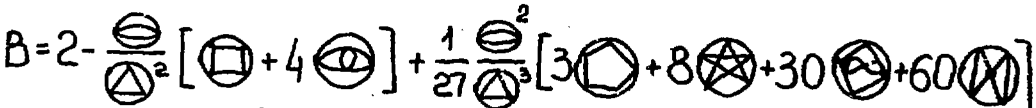
The functional F may be constructed by means of iterations over G^{-1}. To do so, one should derive an equation of the type $F_G = \ldots$ [4] where in the righthand side there are functional derivatives in G of a higher order. Such an equation for $F(\tilde{S}, G, H)$ was derived in [3]. Therefore, F may be constructed in an arbitrary order in G^{-1}. In the present paper the functional F is constructed up to the ninth order in G^{-1} inclusively. Due to the bulk, the complete form of F is not written. Let us however present the expression for the quantity that we shall later need.



(1.2)

where Q^{-1} is the operator inverse to $Q_{HH} = \frac{F_{\tilde{S}H} \, F_{\tilde{S}H}}{F_{\tilde{S}\tilde{S}}} - F_{HH}$





In diagrams, G corresponds to each line. Note also that in (1.2) $\varphi = H = 0$.

The introduction of an additional scalar parameter $\tilde{S}$ allows to derive an equation of the form [3]

$$F_{\tilde{S}} = -\frac{1}{\hbar}\left(\frac{4!}{g\hbar^2}\right)^2 \frac{\tilde{S}}{4\,tr\{G\,Q^{-1}\}} \qquad (1.3)$$

In terms of the variable x it reads

$$F = -\frac{1}{g\hbar}\frac{\ominus^4}{\triangle^2}\int \frac{x\,dx}{tr\{GQ^{-1}\}} \qquad (1.4)$$

The integral (1.4) allows to carry out an additional resummation, for which purpose one should calculate $tr\{GQ^{-1}\}$ (1.2). In [1] $tr\{GQ^{-1}\}$ was calculated in lower orders.

Such a summation may be useful in the field theory and statistical physics problems. The advantages of such a summation method are obvious on the example of anharmonic oscillator.

2. In the case of an anharmonic oscillator ($\mathcal{L} = \frac{m\dot{x}^2}{2} - \frac{m\omega^2 x^2}{2} + \frac{g x^4}{4!}$) which is a one-dimensional analogue of $g\varphi^4$ theory, one may calculate all the diagrams entering (1.4) in the approximation $G \equiv \mathcal{D}$. The difference of ground state energies of the full ($g \neq 0$) and free ($g = 0$) theory is determined by the formula [4]

$$-\mathcal{E}(g)\int dt = \Gamma/_{\tilde{S}=\tilde{S}_{vac}} - \frac{i\hbar}{2}\,tr\,\hat{1} - \frac{\hbar}{2i}\,tr\,\ln G \qquad (2.1)$$

where $\tilde{S}_{vac}$ is the solution of stationarity equation $\Gamma_S = 0$.

Substituting in (2.1) the expression for F (1.4) we shall obtain the ground state energy $\tilde{\mathcal{E}}^{(5)}(\lambda)$ of anharmonic oscillator as a function of dimensionless interaction constant $\lambda = -\frac{g\hbar}{4!\,m^2\omega^3}$ (the index $^{(5)}$ implies that diagrams of not higher than fifth order in λ are summed).

The form of the function $\tilde{\mathcal{E}}^{(5)}(\lambda)$ is shown in fig. (curve 6). Curve 5 corresponds to the case when diagrams of not higher than third order in λ are summed [1], which means A=B=0. Curves 2,3,4 correspond to $\mathcal{E}^{(3)}(\lambda)$, $\mathcal{E}^{(4)}(\lambda)$, $\mathcal{E}^{(5)}(\lambda)$ in the standard perturbation theory. Curve 1 is exact [2]. The function $\mathcal{E}^{(4)}(\lambda)$(A≠0; B=0) was investigated in 1 . Note that it lies between curves 5 and 6.

Numerical values of the ground state energies $\mathcal{E} = \frac{1}{2} + \frac{\mathcal{E}^{(i)}}{\hbar\omega}$ calculated in various approximations are presented in table 1.

The analysis of the described diagram summation method allows to come to the following conclusions (see fig. and table):

- unlike the standard perturbation theory the oscillations of curves at transitions to higher order have vanished;
- the application region has considerably increased in at least for the ground state energies;
- a monotonous increase of accuracy is observed together with approximation order.

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Table

$\varepsilon(\lambda)$ \ λ	0.1	0.2	0.3	0.4	0.6	1
ε^{exact}	0.5591	0.6024	0.6380	0.6688	0.7210	0.8038
$\varepsilon^{(3)}_{p.t}$	0.5696	0.7115	1.0507	1.712	4.5005	19.4375
$\tilde{\varepsilon}^{(3)}$	0.5724	0.6418	0.7099	0.7771	0.9100	1.1726
$\varepsilon^{(5)}_{p.t}$	0.5812	1.4504	7.7980	32.2042	251.6865	3359.1289
$\tilde{\varepsilon}^{(5)}$	0.5692	0.6363	0.7019	0.7672	0.8972	1.1569

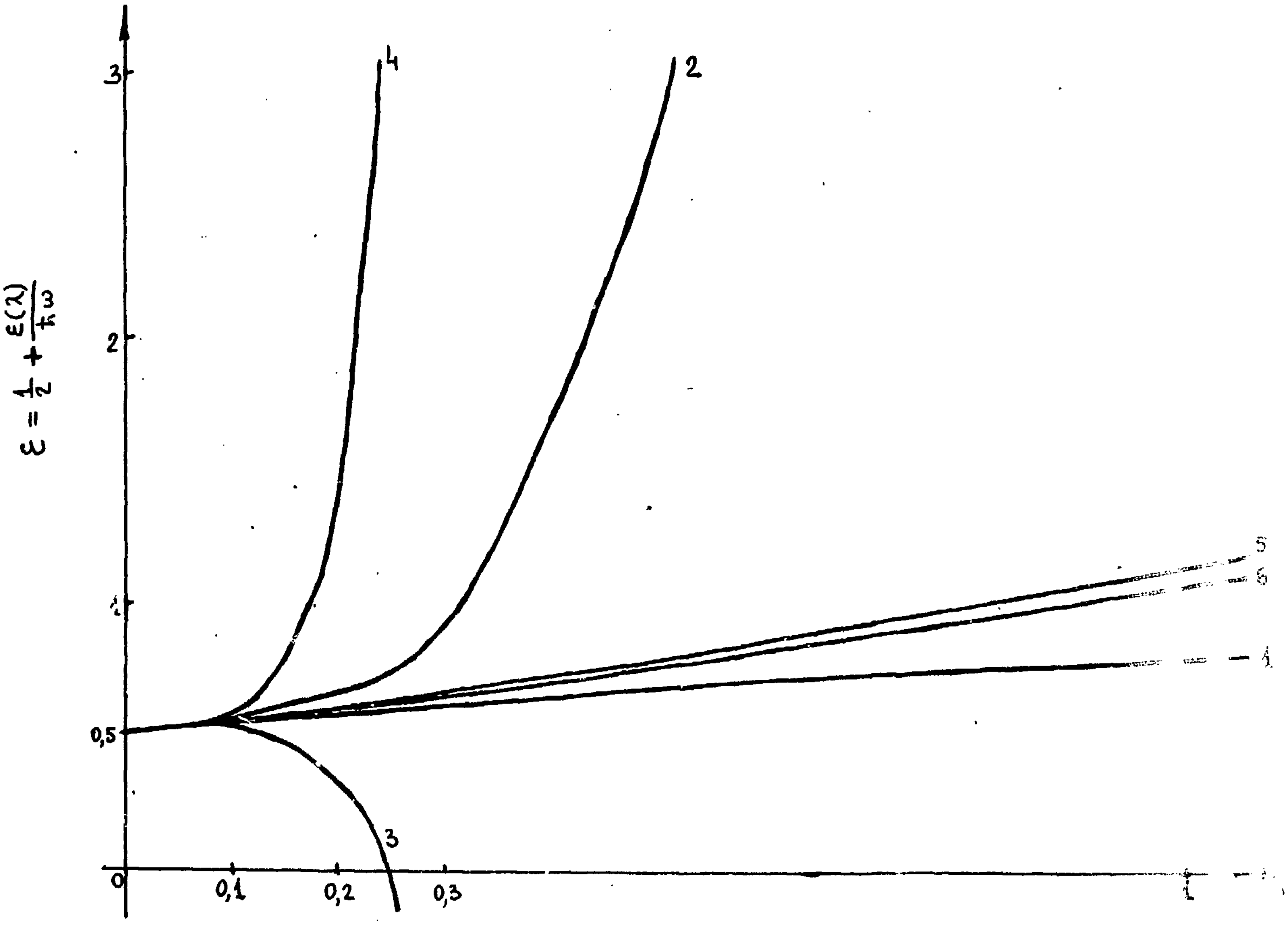
$\varepsilon = \frac{1}{2} + \frac{\varepsilon(\lambda)}{\hbar\omega}$
3
2
0,5
0
0,1
0,2
0,3
4
2
3
5
6

Fig.

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