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SYMMETRIES OF THE RENORMALIZED THEORIES WITH SYMMETRICAL CLASSICAL ACTION

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1. The question of whether symmetries of field theories are conserved after renormalization plays an important role in the investigations of the general properties of the quantized field theories. Numerous papers are devoted to the demonstration of the conservation of definite symmetries after renormalization. This paper deals with the investigation of the problem in general. Namely, the symmetries of renormalized action S_R and generating functional of proper vertices Γ_R of the theory with a given symmetry of the classical action S_0 are investigated, and both S_R and Γ_R are shown to have definite symmetries. In Sec,2 theories without gauge symmetries are discussed, while Secs.3,4 are devoted to the investigation of the same question in the gauge theories. In Sec.3 it is shown that a symmetry of the classical action results in the symmetries of the modified $S_M(\phi,K)$ and effective $S_\Psi(\phi,K)$ actions. In Sec.4 we demonstrate that under the same conditions $S_{\Psi R}$ and Γ_R also have certain symmetry properties.

Henceforth the notation $\approx$ will be used for the equality to the first order of the small parameter ε (see formula 1); the derivatives over the fields are always right, over the sources - left.

2. Consider, first, theories without gauge symmetries (i.e. theories

with nonsingular Lagrangians). Thus, suppose the theory, described by the action $S_0(\phi)$ (ϕ being the set of all the fields) is symmetric under the infinitesimal local transformations of variables.

$$\phi \to \phi' = \phi + \varepsilon\Delta\phi \qquad (1)$$

where $\Delta\phi(\phi)$ is a local functional of the fields ϕ . To find out the symmetry properties of the renormalized action $S_R(\phi)$ and the generating functional $\Gamma_R(\phi)$ of proper vertices we'll investigate how they transform under the transformations (1). Note that this question was discussed for final transformations in [1] . Here we repeat the derivation of necessary relations for infinitesimal transformations. To this end, let us consider the difference of the generating functionals of Green's functions $Z(J)$ and $\tilde{Z}(J)$ of theories described by the actions $S_0(\phi)$ and $\tilde{S}_0(\phi) = S_0(\phi + \varepsilon\Delta\phi)$, respectively. Under the change of variables $\phi \to \phi - \varepsilon\Delta\phi$ the generating functional

$$\tilde{Z}(J) = \int d\phi \, exp\left\{\frac{i}{\eta}\left[S_0(\phi + \varepsilon\Delta\phi) + J\phi\right]\right\}$$

(η is the loop-decomposition parameter) transforms into

$$\tilde{Z}(J) = \int d\phi \, exp\left\{\frac{i}{\eta}\left[S_0(\phi) + J(\phi - \varepsilon\Delta\phi)\right]\right\} \approx$$

$$\approx \int d\phi \, exp\left\{\frac{i}{\eta}\left[S_0(\phi) + J\phi\right]\right\}\left(1 - \frac{i\varepsilon}{\eta}J\Delta\phi\right) = Z(J) - \frac{i}{\eta}J\varepsilon\Delta\phi Z(J) \qquad (2)$$

In the last expression the substitution $\phi \to -i\eta\,\delta/\delta J$ is supposed. In deriving the formula (2) we omitted the Jacobian of the change of variables $\left|\delta(\phi - \varepsilon\Delta\phi)/\delta\phi\right|$ because it is equal to unity, since owing to the locality of $\Delta\phi$ the quantity $\delta(\Delta\phi)/\delta\phi \sim \delta(0)$ and under suitable regularization (e.g. dimensional regularization) is equal to zero.

From (2) we have

$$\Delta Z = \tilde{Z}(J) - Z(J) \approx -\frac{i}{\eta}J\varepsilon\Delta\phi Z(J) \qquad (3)$$

Introducing in a usual way the generating functional $\Gamma(\phi)$ of proper vertices

$$\Gamma(\phi) = -i\eta \ln Z(J) - J\phi\,, \quad \phi_i = -i\eta\frac{\delta}{\delta J_i}\ln Z(J)$$

we rewrite (3) in terms of $\Gamma(\phi)$

$$\Delta\Gamma = \tilde{\Gamma}(\phi) - \Gamma(\phi) \approx \varepsilon\frac{\delta\Gamma}{\delta\phi}\langle\Delta\phi\rangle \qquad (4)$$

where $\tilde{\Gamma}(\phi)$ and $\Gamma(\phi)$ are proper vertices generating functionals of the theories with actions $\tilde{S}_0(\phi)$ and $S_0(\phi)$, respectively, $\langle\Delta\phi\rangle$ is the vacuum expectation value of the operator $\Delta\phi$, which is a function of ϕ and which is obtained from the operator $\Delta\phi$ by a substitution

$$\phi_i \to \phi_i + i\eta(-1)^{P_j(P_i+1)}\left(\Gamma''\right)^{-1}_{ij}\frac{\delta_L}{\delta\phi_j}$$

Here $\delta_L/\delta\phi$ is the left derivative, P_i is the Grassmann parity of the field ϕ_i , and $(\Gamma'')^{-1}_{ij}$ denotes the matrix reverse to the matrix $(\Gamma'')_{ij} \equiv \delta^2\Gamma/\delta\phi_i\delta\phi_j$

Writing down the loop-wise expansion of the quantities entering (4)

$$\Gamma = S_0 + \eta\left(\Gamma^{(1)}_{div} + \Gamma^{(1)}_{fin}\right) + O(\eta^2)$$

$$\tilde{\Gamma} = \tilde{S}_0 + \eta\left(\tilde{\Gamma}^{(1)}_{div} + \tilde{\Gamma}^{(1)}_{fin}\right) + O(\eta^2)$$

$$\langle\Delta\phi\rangle = \Delta\phi + \eta\left(\langle\Delta\phi\rangle^{(1)}_{div} + \langle\Delta\phi\rangle^{(1)}_{fin}\right) + O(\eta^2)$$

we obtain from (4) in the tree approximation

$$\Delta S(\phi) = \tilde{S}_0(\phi) - S_0(\phi) = S_0(\phi+\varepsilon\Delta\phi) - S_0(\phi) \approx \varepsilon \frac{\delta S_0}{\delta\phi}\Delta\phi,$$

while in one-loop approximation Eq.(4) can be divided into two equations for finite and divergent parts of $\Delta\Gamma$

$$\Delta\Gamma^{(1)}_{div} = \tilde{\Gamma}^{(1)}_{div} - \Gamma^{(1)}_{div} \approx \varepsilon \frac{\delta S_0}{\delta\phi}\langle\Delta\phi\rangle^{(1)}_{div} + \varepsilon \frac{\delta\Gamma^{(1)}_{div}}{\delta\phi}\Delta\phi \tag{5}$$

$$\Delta\Gamma^{(1)}_{fin} = \tilde{\Gamma}^{(1)}_{fin} - \Gamma^{(1)}_{fin} \approx \varepsilon \frac{\delta S_0}{\delta\phi}\langle\Delta\phi\rangle^{(1)}_{fin} + \varepsilon \frac{\delta\Gamma^{(1)}_{fin}}{\delta\phi}\Delta\phi \ .$$

Now we consider a new action

$$S_{1R}(\phi) = S_0(\phi) - \eta\,\Gamma^{(1)}_{div}(\phi)$$

Then, taking into account (5) we find for the renormalized action (which we'll denote $\tilde{S}_1(\phi)$) of the theory with initial action $\tilde{S}_0(\phi)$ the following:

$$\tilde{S}_1(\phi) = \tilde{S}_0(\phi) - \eta\,\tilde{\Gamma}^{(1)}_{div}(\phi) \approx$$

$$\approx S_1(\phi) + \varepsilon\frac{\delta S_0}{\delta\phi}\Delta\phi - \varepsilon\eta\frac{\delta S_0}{\delta\phi}\langle\Delta\phi\rangle^{(1)}_{div} - \varepsilon\eta\frac{\delta\Gamma^{(1)}_{div}}{\delta\phi}\Delta\phi$$

It's easy to see that $\tilde{S}_1(\phi)$ coincides with the one-loop approximation of action

$$\tilde{S}_{1R}(\phi) \approx S_{1R}\left(\phi + \varepsilon\Delta\phi - \eta\varepsilon\langle\Delta\phi\rangle^{(1)}_{div}\right)$$

Going through the steps analogous to those above with the actions $S_{1R}(\phi)$ and $\tilde{S}_{1R}(\phi)$, we obtain the equation

$$\Delta\Gamma_1 \approx \varepsilon\frac{\delta\Gamma_1}{\delta\phi}\langle\Delta_1\phi\rangle \tag{6}$$

where Γ_1 (the generating functional of proper vertices corresponding to the action $S_{1R}(\phi)$) is one-loop finite;

$$\Delta_1\phi = \Delta\phi - \eta\langle\Delta\phi\rangle^{(1)}_{div}$$

Then

$$\langle\Delta_1\phi\rangle = \Delta\phi - \eta\langle\Delta\phi\rangle^{(1)}_{div} + \eta\langle\Delta\phi\rangle^{(1)} + O(\eta^2) =$$

$$= \Delta\phi + \eta\langle\Delta\phi\rangle^{(1)}_{fin} + O(\eta^2) \ .$$

Also from (6) it follows that

$$\tilde{\Gamma}_1(\phi) \approx \Gamma_1\left(\phi + \varepsilon\Delta\phi + \varepsilon\eta\langle\Delta\phi\rangle^{(1)}_{fin}\right),$$

i.e. $\tilde{\Gamma}_1(\phi)$ is obtained from $\Gamma_1(\phi)$ by the change of variables

$$\phi \to \phi + \varepsilon\Delta\phi + \varepsilon\eta\langle\Delta\phi\rangle^{(1)}_{fin}$$

Using now the mathematical induction we obtain the final relations

$$\tilde{S}_R(\phi) \approx S_R(\phi + \varepsilon\Delta_R\phi) \tag{7}$$

$$\tilde{\Gamma}_R(\phi) \approx \Gamma_R(\phi + \varepsilon\langle\Delta_R\phi\rangle), \tag{8}$$

where

$$\Delta_R\phi = \Delta\phi - \sum_{n=1}^{\infty}\eta^n\langle\Delta\phi\rangle^{(n)}_{div}$$

and $\langle\Delta\phi\rangle^{(n)}_{div}$ are divergent parts of the n -loop approximation vacuum expectation values of the variation of ϕ with subtracted divergences of the subdiagrams.

Suppose now that the initial action is invariant under the transformations $\phi \to \phi + \varepsilon\Delta\phi$. Then $S_0(\phi) = \tilde{S}_0(\phi)$ and hence $S_R(\phi) = \tilde{S}_R(\phi)$, $\Gamma_R(\phi) = \tilde{\Gamma}_R(\phi)$, so that the Eq.(7) is just an expression of the renormalized action symmetry under the transformations

$$\phi \to \phi' = \phi + \varepsilon\Delta_R\phi \tag{9}$$

while (8), in turn, reveals the invariance of the generating functional $\Gamma_R(\phi)$ of the proper vertices under transformations

$$\phi \to \phi'' = \phi + \varepsilon\langle\Delta_R\phi\rangle \tag{10}$$

It's important to note that the very structure of the transformations (9) and (10) discloses that they are not identical transformations.

Thus, if the initial classical action $S_0(\phi)$ possesses some symmetry properties, then the renormalized action $S_R(\phi)$ and the generating functional $\Gamma_R(\phi)$ also have certain symmetry properties.

3. Let us turn now to the investigation of general gauge theories, the classical action of which $S_0(\varphi)$ is symmetric under the infinitesimal local transformations of variables.

Consider, first, the symmetry properties of the effective action. It's worth reminding [2] that the effective action $S_\psi(\phi, K)$ is obtained from the modified action $S_M(\phi, K)$ via the exchange $K \to \delta\psi(\phi,K)/\delta\phi$. Here ϕ symbolizes all fields $\phi = \{\varphi, c, \pi, \bar{c}\}$, where $\varphi = \{\varphi^i\}$ are the fields of the initial classical theory with the action $S_0(\varphi)$, $c = \{c^\alpha\}$

$\bar{c} = \{\bar{c}^\alpha\}$ are the ghost fields, $\pi = \{\pi^\alpha\}$ are auxiliary gauge fixing fields; $K = \{k, \ell, n, m\}$, where $k = \{k_i\}$, $\ell = \{\ell_\alpha\}$, $m = \{m_\alpha\}$, $n = \{n_\alpha\}$ are additional sources corresponding to the fields φ, c, $\bar{c}$, π, respectively; $\psi(\phi, K)$ is a degeneration eliminating gauge fermion.

The modified action $S_M(\phi, K)$ satisfies the Zinn-Justin equation [3]

$$(S_M, S_M) = 2\frac{\delta S_M}{\delta\phi}\frac{\delta S_M}{\delta K} = 0$$

with the initial condition

$$S_M(\phi, 0) = S_0(\varphi)$$

Suppose now that $S_0(\varphi)$ is invariant under transformations

$$\varphi \to \varphi' = \varphi + \varepsilon\Delta\varphi \tag{11}$$

$$\tilde{S}_0(\varphi) = S_0(\varphi + \varepsilon\Delta\varphi) = S_0(\varphi) \tag{12}$$

For the transformation (11) a canonical transformation with a generating functional X can be constructed:

$$X = K'(\varphi + \varepsilon\Delta\varphi) + \ell'c + n'\pi + m'\bar{c}$$

Using this transformation the action $S_M(\phi, K)$ can be found

$$\tilde{S}_M(\phi, K) = S_M(\phi_X, K_X) \tag{13}$$

where $\phi_X\ (K_X)$ are fields (sources) which are obtained from $\phi(K)$

as a result of the canonical transformation X . On the other hand, due to (12) the initial condition for $\tilde{S}_M(\phi, K)$ is

$$\tilde{S}_M(\phi, 0) = \tilde{S}_0(\varphi) = S_0(\varphi)$$

and therefore $\tilde{S}_M(\phi, K)$ and $S_M(\phi, K)$ are two solutions of the Zinn-Justin equation with the same initial condition, and they must be connected with each other by a canonical transformation Y (2) [3] (which adds to the fields $\varphi = \{\varphi_i\}$ only summands containing ghost fields)

$$\tilde{S}_M(\phi, K) = S_M(\phi_Y, K_Y) \tag{14}$$

Comparing (13) and (14), we find

$$S_M(\phi, K) = S_M(\phi_{Y^{-1}X}, K_{Y^{-1}X}) \tag{15}$$

i.e. $S_M(\phi, K)$ is invariant under canonical change of variables with the generating functional $Y^{-1}X$. Note that $Y^{-1}X$ isn't identity transformation, since the transformation X changes the fields φ^i by $\varphi^i + \Delta\varphi^i$, while the Y^{-1} doesn't change the fields φ^i (up to the ghost fields).

Taking into account that the transition from $S_M(\phi, K)$ to the effective action S_Ψ is realized through the canonical transformation with a generating functional $Z = K'\phi + \Psi(\phi)$ [4] and using (15) we find that

$$S_\Psi(\phi, K) \approx S_M(\phi_Z, K_Z) \approx S_M(\phi_{Y^{-1}XZ}, K_{Y^{-1}XZ}) \approx$$
$$\approx S_\Psi(\phi_{Z^{-1}Y^{-1}XZ}, K_{Z^{-1}Y^{-1}XZ}) \tag{16}$$

The equation (16) means that the effective action S_Ψ is symmetric under local canonical transformations with a generating functional

$$T \approx Z^{-1}Y^{-1}XZ \approx K\phi + \varepsilon\Delta T(\phi, K) \tag{17}$$

where $\Delta T(\phi, K)$ is a local functional of the fields ϕ and sources K . Thus we have found that if the classical action $S_0(\varphi)$ of the gauge theory has a symmetry properties, then the modified action $S_M(\phi, K)$ and the effective action $S_\Psi(\phi, K)$ also have certain symmetries.

4. Let us consider now how the renormalized actions $S_{\Psi R}$, $\tilde{S}_{\Psi R}$ and generating functionals Γ_R , $\tilde{\Gamma}_R$ of proper vertices of the theories with effective actions S_Ψ and $\tilde{S}_\Psi$ are connected, provided the latters are connected with each other by a canonical transformation (17).

In accordance with the results of [4] , the variation of the renormalized action under the canonical transformation (17) can be presented in the form

$$\delta S_{\Psi R}(\eta) = (\Delta_R T, S_{\Psi R}(\eta)) \tag{18}$$

which corresponds to a canonical transformation with a generating functional $T_1 = K\phi + \Delta_R T$, where instead of ΔT in (17) we have

$$\Delta_R T \approx \Delta T - \sum_{n=1}^{\infty} \eta^n \langle \Delta T \rangle_{div}^{(n)}$$

The relevant variation $\delta\Gamma_R$ is given by

$$\delta\Gamma_R = (\langle \Delta_R T \rangle, \Gamma_R) \tag{19}$$

which corresponds to a canonical transformation with a generating functional

$$T_2 \approx K\phi + \langle \Delta_R T \rangle$$

As a result we have

$$S_{\psi R}(\phi, K) = S_{\psi R}(\phi_{T_1}, K_{T_1}) \quad (20)$$

and

$$\Gamma_R(\phi, K) = \Gamma_R(\phi_{T_2}, K_{T_2}) \quad (21)$$

The relations (20) and (21) disclose the symmetry of $S_{\psi R}$ and Γ_R under the canonical transformations with generating functionals T_1 and T_2, respectively.

Thus in the case of general gauge theories to any symmetry of the initial action $S_0(\varphi)$ corresponds a certain symmetry of the renormalized action and renormalized generating functional Γ_R of proper vertices.

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