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G.V.GRIGORYAN, R.P.GRIGORYAN, I.V.TYUTIN

ON ANOMALIES IN THE WARD IDENTITIES

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The question of the presence of anomalies in a field theoretical model is of primary importance. Anomalies appear in the theory after giving prescriptions for calculations of divergent integrals or values of the operators at the singularities. They were first discovered in the  $\sigma$ -model and in the theory with the axial current [1-3]. Subsequent investigations of anomalies resulted in the formulation of a model independent properties of anomalies, their connections with the algebraic properties of the theory, and also the group theoretical characterization of the axial current anomaly was established [4,5]. Considerations of other Ward identities (scale, superidentities, etc.) revealed new anomalies [6-8]. This makes the investigations of the general properties of anomalies worthwhile. In this paper the momentum dependence of anomalies is examined and anomalies are shown to be a local functionals of the fields.

Consider a theory described by the Lagrangian  $\mathcal{L}(\phi)$ , which is invariant under infinitesimal transformations

$$\phi \rightarrow \phi' = \phi + \varepsilon F(\phi) \quad (1)$$

where  $F(\phi)$  is a local functional of the fields  $\phi$  ( $\phi = \{\phi_i\}$  is the set of all fields of the theory; the derivatives over the fields are

right, over the sources - left). Under the transformations (1) the Green's function generating functional  $Z(J)$

$$Z(J) = \int d\phi \exp\left\{\frac{i}{2}(S + J\phi)\right\}$$

where  $\eta$  is a loop-decomposition parameter, transforms into

$$Z(J) = \int d\phi \exp\left\{\frac{i}{2}(S + J\phi) + \frac{i}{2}\epsilon JF(\phi)\right\} \quad (2)$$

(In deriving (2) we didn't take into account the Jacobian of the transformation).

From (2) follows the Ward identity

$$J \int d\phi \exp\left\{\frac{i}{2}(S + J\phi)\right\} \cdot F(\phi) = 0 \quad (3)$$

or

$$JF(\phi)Z(J) = 0$$

where in  $F(\phi)$  a substitution  $\phi \rightarrow \frac{\eta}{i} \frac{\delta}{\delta J}$  is supposed.

The identity (3) will be referred to as "naive Ward Identity".

Introducing in the usual way the generating functional of proper vertices.  $\Gamma(\phi) = -i\eta \ln Z(J) - J\phi$  we obtain from (3) the Ward identity for the  $\Gamma(\phi)$

$$\frac{\delta \Gamma}{\delta \phi} \langle F(\phi) \rangle = 0 \quad (4)$$

where  $\langle F(\phi) \rangle$  is the vacuum expectation which is a function of  $\phi$  and is obtained from  $F(\phi)$  via a substitution

$$\phi_i \rightarrow \phi_i + i\eta (-1)^{P_i(P_i+1)} (\Gamma'')^{-1}_{ij} \frac{\delta}{\delta \phi_j}$$

Here  $\delta_{\leftarrow} / \delta \phi$  is a left derivative,  $P_i$  is the Grassmann parity of the field  $\phi_i$ , and  $(\Gamma'')^{-1}_{ij}$  is the matrix, which is inverse to the matrix  $(\Gamma'')_{ij} \equiv \delta^2 \Gamma / \delta \phi_i \delta \phi_j$

The relation (4) contains divergences thus being senseless by itself. To make it meaningful, we must renormalize  $S$  and also the operator  $F(\phi)$ .

To investigate the structure of the Ward identities in a renormalized theory we'll proceed as follows.

Let us introduce the regularized action

$$S_\Lambda = S + \frac{1}{2} \phi \frac{1}{\Lambda^n} \partial^{n+2} \phi + \eta L_c$$

where  $L_c$  is the set of counterterms which ensure the finiteness of the theory after the removal of the regularization. The regularized transformations are given by a functional  $F(\tilde{\phi})$ , instead of  $F(\phi)$  in (1),

$$F(\tilde{\phi}) = F(\phi) + \Delta F(\phi), \quad \tilde{\phi} = \frac{1}{\Lambda} + \partial^k / \Lambda^k \cdot \phi$$

Here  $\tilde{\phi}$  was introduced to regularize the vertices in  $F(\phi)$ . Note that  $\Delta F(\phi)$  is proportional to  $\Lambda^{-k}$ .

Now under the change of variables

$$\phi \rightarrow \phi' \equiv \phi + \epsilon F(\tilde{\phi})$$

the generating functional

$$Z_\Lambda = \int d\phi \exp\left\{\frac{i}{2}(S_\Lambda + J\phi)\right\}$$

transforms into

$$Z_\Lambda = \int d\phi \exp\left\{\frac{i}{2}[S_\Lambda + J\phi + \epsilon \frac{\delta S_\Lambda(\phi)}{\delta \phi} F(\tilde{\phi}) + \epsilon JF(\tilde{\phi})]\right\} \times \left| \frac{\delta(\phi + \epsilon F(\tilde{\phi}))}{\delta \phi} \right|,$$

$$\frac{\delta S_\Lambda(\phi)}{\delta \phi} F(\tilde{\phi}) = \eta \frac{\delta L_c}{\delta \phi} F(\tilde{\phi}) + \frac{1}{\Lambda^n} (\partial^{n+2} \phi) F(\tilde{\phi}) + \frac{\delta S}{\delta \phi} \Delta F(\phi).$$

In writing down the last summand the relation  $\frac{\delta S}{\delta \phi} \Delta F(\phi) = 0$ , reflecting the invariance of  $S(\phi)$  under the transformations (1), was taken into account.

Using the relation

$$\left| \frac{\delta(\phi + \varepsilon F(\tilde{\phi}))}{\delta \phi} \right| = \exp \left\{ \int \delta p \ln \left( 1 + \frac{\varepsilon F(\tilde{\phi})}{\delta \phi} \right) \right\}$$

we get

$$Z_\Lambda = \int d\phi \exp \left\{ \frac{i}{\eta} \left[ (S_\Lambda + J\phi) + \varepsilon (JF(\tilde{\phi}) + \eta \frac{\delta L_c}{\delta \phi} F(\tilde{\phi}) + \frac{1}{\Lambda^n} (\partial^{n+2} \phi) F(\tilde{\phi}) + \frac{\delta S}{\delta \phi} \Delta F(\phi) + \frac{\eta}{i} \int \delta p \frac{\delta F(\tilde{\phi})}{\delta \phi}) \right] \right\}.$$

Following the derivation of (4), we obtain the Ward identity for the proper vertices generating functional  $\Gamma_\Lambda(\phi)$ .

$$\langle F(\tilde{\phi}) \rangle \frac{\delta \Gamma_\Lambda}{\delta \phi} = \eta \left\langle \frac{\delta L_c}{\delta \phi} F(\tilde{\phi}) \right\rangle + \left\langle \frac{1}{\Lambda^n} (\partial^{n+2} \phi) F(\tilde{\phi}) \right\rangle + \left\langle \frac{\delta S}{\delta \phi} \Delta F(\phi) \right\rangle + \frac{\eta}{i} \left\langle \int \delta p \frac{\delta F(\tilde{\phi})}{\delta \phi} \right\rangle. \quad (5)$$

To compare (4) and (5), let us write down the loop-decomposition of  $\Gamma_\Lambda$

$$\Gamma_\Lambda = S + \phi \frac{1}{\Lambda^n} \partial^{n+2} \phi + \eta \Gamma_{1\Lambda} + O(\eta^2) \equiv S^\Lambda + \eta \Gamma_{1\Lambda} + O(\eta^2).$$

Then (5) takes the form

$$\begin{aligned} & \langle F(\tilde{\phi}) \rangle_0 \frac{\delta S^\Lambda}{\delta \phi} + \langle F(\tilde{\phi}) \rangle_1 \eta \frac{\delta S^\Lambda}{\delta \phi} + \eta \langle F(\tilde{\phi}) \rangle_0 \frac{\delta \Gamma_{1\Lambda}}{\delta \phi} = \\ & = \eta \frac{\delta L_c}{\delta \phi} F(\tilde{\phi}) + \frac{1}{\Lambda^n} (\partial^{n+2} \phi) F(\tilde{\phi}) + \eta \frac{1}{\Lambda^n} \langle (\partial^{n+2} \phi) F(\tilde{\phi}) \rangle_1 + \\ & + \left\langle \frac{\delta S}{\delta \phi} \Delta F(\phi) \right\rangle_0 + \eta \left\langle \frac{\delta S}{\delta \phi} \Delta F(\phi) \right\rangle_1 + \frac{\eta}{i} \int \delta p \frac{\delta F(\tilde{\phi})}{\delta \phi} + O(\eta^2), \end{aligned} \quad (6)$$

where the subscript "0" is for the tree approximation and "1" for one-loop approximation. The tree approximation of (6) after the removal of regularization reflects the invariance of  $S$  under the transformations (1). In one-loop approximation we have

$$\begin{aligned} & \frac{\delta \Gamma_{1\Lambda}}{\delta \phi} F(\tilde{\phi}) + \frac{\delta S^\Lambda}{\delta \phi} \langle F(\tilde{\phi}) \rangle_1 = \frac{\delta L_c}{\delta \phi} F(\tilde{\phi}) + \\ & + \frac{1}{\Lambda^n} \langle (\partial^{n+2} \phi) F(\tilde{\phi}) \rangle_1 + \left\langle \frac{\delta S}{\delta \phi} \Delta F(\phi) \right\rangle_1 + \frac{1}{i} \int \delta p \frac{\delta F(\phi)}{\delta \phi} \end{aligned} \quad (7)$$

Let us now rewrite (7) in the form

$$\begin{aligned} & \frac{\delta \Gamma_{1\Lambda}}{\delta \phi} F(\phi) + \frac{\delta S}{\delta \phi} \langle F_R(\phi) \rangle_1 = \\ & = \frac{\delta \Gamma_{1\Lambda}}{\delta \phi} (F(\phi) - F(\tilde{\phi})) + \frac{\delta S}{\delta \phi} (\langle F_R(\phi) \rangle_1 - \langle F(\tilde{\phi}) \rangle_1) + \\ & + \frac{1}{\Lambda^n} \langle (\partial^{n+2} \phi) F(\tilde{\phi}) \rangle_1 + \frac{\delta L_c}{\delta \phi} F(\tilde{\phi}) - \frac{1}{\Lambda^n} (\partial^{n+2} \phi) \langle F(\tilde{\phi}) \rangle_1 + \\ & + \left\langle \frac{\delta S}{\delta \phi} \Delta F(\phi) \right\rangle_1 + \frac{1}{i} \int \delta p \frac{\delta F(\phi)}{\delta \phi}, \end{aligned} \quad (8)$$

where  $F_R = F - F_c$ ,  $F_c$  is a local operator which eliminates the divergences of  $F$  as the regularization is removed.

After the removal of the regularization the left-hand side of eq.(8) takes the form

$$\frac{\delta \Gamma_1}{\delta \phi} F(\phi) + \frac{\delta S}{\delta \phi} \langle F_R(\phi) \rangle_1$$

while the right-hand side of (8) is a local functional  $P(\phi)$  of the fields and their derivatives.

Thus the Ward identity (8) after the removal of the regularization transforms into

$$\frac{\delta \Gamma_1}{\delta \phi} F(\phi) + \frac{\delta S}{\delta \phi} \langle F_R(\phi) \rangle_1 = P(\phi) \quad (9)$$

$P(\phi)$  is the anomaly which makes the identity (9) different from the naive one (4).

Hence we see that for any set of local counterterms of  $S$  and  $F$  the one-loop anomalies are local functionals of the fields and their derivatives.

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Г.В.ГРИГОРЯН, Р.П.ГРИГОРЯН, И.В.ТЮТИН  
ОБ АНОМАЛИЯХ В ТОЖДЕСТВАХ УОРДА

(на английском языке, перевод З.Н.Асланян)  
Ереванский физический институт

Редактор Л.П.Мукаян  
Тех.редактор А.С.Абрамян

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