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ON TIME EVOLUTION OF PARTICLE DISTRIBUTION FUNCTION IN HIGH-TEMPERATURE PLASMAS

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Ф.А.АГАРОНЯН, А.М.АТОЯН

К ВОПРОСУ О ВРЕМЕННОЙ ЭВОЛЮЦИИ ФУНКЦИИ РАСПРЕДЕЛЕНИЯ ЧАСТИЦ В ВЫСОКОТЕМПЕРАТУРНОЙ ПЛАЗМЕ

Рассмотрена эволюция во времени одночастичной функции распределения частиц в нерелятивистской плазме при отсутствии внешнего поля. Выведено линейное дифференциальное уравнение, описывающее высокоэнергичную часть функции распределения. На основании приближенного аналитического решения этого уравнения получено время термализации (максвеллизации) частиц в области энергий $\varepsilon \gg kT$: $t_\varepsilon \simeq 0{,}64(\varepsilon/kT)^{3/2} t_0$, где t_0 есть время релаксации в области характерных средних $(\varepsilon \sim kT)$. Значение полученных результатов обсуждается на примере γ - светимости аккреционной плазмы вокруг черной дыры.

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The evolution of one-particle distribution function in nonrelativistic plasma in the absence of external field is considered. A linear differential equation describing a high-energy tail of the distribution function is derived. The approximate analytical solution of this equation yields the thermalization (maxwellization) time of the particles in the energy region $\mathcal{E} \gg \kappa T$: $t_{\mathcal{E}} \simeq 0{,}64\left(\frac{\mathcal{E}}{\kappa T}\right)^{3/2} t_{o}$, where t_{o} is the relaxation time in the range of characteristic means ($\mathcal{E} \sim \kappa T$). The significance of the obtained results is discussed on the example of γ-ray luminosity of the accretion plasma around a black hole.

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1. Introduction.

The thermal equilibrium establishment times for different components of plasma is of great importance in the problems related to different physical processes in the nondegenerated hot plasma. The scales of the bulk thermalization times of the electron (t_{ee}) and ion (t_{zz}) components as well as the time of energy exchange between these components (t_{ez}) were studied pretty in detail (see, e.g. reviews [1-3]). However the nature and the time of the thermal equilibrium establishment of the high-energy (with respect to the means) tail of the Maxwell distribution remains so far vague in many respects, although it is evident that the establishment of such equilibrium is a process slower than thermalization in the range of typical means [4] . At the same time, in many dynamical problems the evolution of the very high-energetic tail of the distribution function plays the most important role.

This paper deals with the investigation of the problem for nonrelativistic systems.

2. Kinetic Equation for Homogeneous Systems in the Weak Interaction Approximation

As is known, a kinetic stage of the evolution in the nondegenerated entirely ionized plasma can be divided into three different time scales during which consequently the establishment of the thermal equilibrium of the electron component, ion component and further on between these two ones takes place. This circumstance allows one to reduce the investigation of the behaviour of the whole assembly to independent consideration of the evolution of each of the components.

Let us consider a nonrelativistic system of N identical particles described by the Hamiltonian

$$\hat{H} = \sum_{i=1}^{N} \left[\frac{\hat{p}_i^2}{2m} + \frac{1}{2} \sum_{i \neq j}^{N} \hat{V}(\vec{r}_i - \vec{r}_j) + \hat{U}(\vec{r}_i) \right] , \tag{1}$$

where m is mass, $\vec{r}$ is vector radius, $\hat{p}$ is momentum operator, $\hat{V}$ is the operator of interaction, $\hat{U}$ is external field operator. At the kinetic stage of evolution, when the system can be described by one-particle distribution function $f(\vec{p}, \vec{r}, t)$, a so-called Fokker-Planck integral equation holds (see, e.g. [5])

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \frac{\partial f}{\partial \vec{r}} - \frac{\partial U}{\partial \vec{r}} \frac{\partial f}{\partial \vec{p}} = - \frac{\partial J_i}{\partial p_i} , \tag{2}$$

where J is the functional vector with the components

$$J_i = J_i(\vec{p}, \vec{r}, t) = C \int d\vec{p}_1 \, |\vec{p} - \vec{p}_1|^{-3} \left[(\vec{p} - \vec{p}_1)^2 \delta_{ik} - (\vec{p} - \vec{p}_1)_i (\vec{p} - \vec{p}_1)_k \right] \left(f \frac{\partial f_1}{\partial p_{1k}} - f_1 \frac{\partial f}{\partial p_k} \right)_{\vec{r}_1 \to \vec{r}} ; \tag{3}$$

$$C = \frac{m}{8\pi}\int_0^\infty q^3 V_q^2\, dq\ , \qquad V_q = \int V(\vec{r})\, e^{-i\vec{q}\vec{r}}\, d\vec{r}\ . \tag{3a}$$

In Eqs.(2) and (3) the summation is implied under repeated Latin indices. Besides, for briefness we introduced the designations $f \equiv f(\vec{p},\vec{r},t)$ and $f_1 \equiv f(\vec{p}_1,\vec{r}_1,\ t)$. In a particular case, when mean changes of physical values in each elementary act are small as compared with their initial values, Eq.(2) can be reduced to the Fokker-Planck linear differential equation. Note also that for the collision integral $L^{(2)} = -\frac{\partial \vec{J}}{\partial \vec{p}}$ Eq.(3) holds when:

a) typical inhomogeneity dimensions of the system are considerably larger than the radius of the interaction;

b) the mean energy of the interaction $\langle V \rangle$ is much less than the mean kinetic energy $\sim \kappa T$; thus, for the Coulomb interaction this means that $e^2 n^{1/3} << \kappa T$, i.e. $n << 5 \cdot 10^{37}(\kappa T / m_e c^2)^3 cm^{-3}$.

Consider now a particular case of a homogeneous and isotropic system. Then the distribution function depends on the absolute value of momentum p and time t : $f = f(p,t)$. Besides, it is evident that the vector $\vec{J} = \vec{J}(\vec{p},t)$ should be parallel to the vector $\vec{p}$, whose direction is the only one distinguished in space. The latter circumstance permits to simplify significantly the collision integral $L^{(2)} = -\mathrm{div}_{\vec{p}}\vec{J} = -\frac{\partial}{p^2\partial p} J_p p^2$ when integrating it over directions $\vec{p}_1$, yielding

$$L^{(2)} = \frac{8\pi C}{3p^2}\frac{\partial}{\partial p}\left(\int_0^p \frac{p_1^3}{p} + \int_p^\infty p^2\right)\left(f_1\frac{\partial f}{\partial p} - f\frac{\partial f_1}{\partial p_1}\frac{p}{p_1}\right)p_1\, dp_1\ . \tag{4}$$

After integrating over p_1 Eq.(4) comes to the form

$$L^{(2)}(p,f) = \frac{8\pi C n}{3p^2}\Big[\frac{m\varepsilon(<p)}{2\pi}\frac{\partial}{\partial p}\Big(p^{-1}\frac{\partial f}{\partial p}\Big) + \frac{3W(<p)}{4\pi}\frac{\partial f}{\partial p} + \tag{5}$$

$$+ \frac{3p^2}{n} f + \Big(\frac{\partial}{\partial p} p^2 \frac{\partial f}{\partial p}\Big)\int_p^\infty \frac{p_1 f_1}{n} dp_1\Big] ,$$

where $n = 4\pi\int_0^\infty p_1^2 f_1 dp_1$ is particle number density;

$$W(<p,t) = \frac{4\pi}{n}\int_0^p p_1^2 f_1 dp_1 \tag{6}$$

is the probability to find the particles with momentum $< p$ at an instant of time t ;

$$\varepsilon(<p,t) = \frac{4\pi}{n}\int_0^p \frac{p_1^4}{2m} f_1 dp_1 \tag{7}$$

is the mean energy of particles having momentum $< p$ at an instant of time t .

Thus, for spatially isotropic and homogeneous ($u = 0$) system we obtain the equation for the one-particle distribution function

$$\frac{\partial f}{\partial t} = L^{(2)}(p,f) , \tag{8}$$

where $L^{(2)}$ is defined by relations (3a), (5)-(7).

The integral operator $L^{(2)}$ (4) can be reduced to the linear differential form for the range of variable p greater than the characteristic mean $\tilde{p}$. Here, strictly speaking, the fulfilment of only the condition $\exp\left(\frac{\tilde{p}^2 - p^2}{\tilde{p}^2}\right) \ll 1$ is required, which means that the fulfilment of inequality $p^2 \gtrsim 3\tilde{p}^2$ is quite sufficient for the validity of the equation given below.

Indeed, for the values of $p^2 > 3\tilde{p}^2$ the main contribution to the collision integral $L^{(2)}$ (4) is determined by the integration region $p_1 \lesssim \tilde{p}$ since the distribution function $f(p,t)$ is rapidly decreasing for large p (in the limit case of $t \to \infty$ it is evident that $f(p) \sim e^{-p^2/2m\kappa T}$). Then in the right-hand side of (4) one can omit the second integral ($\int_p^\infty \cdots dp_1$) substituting ∞ for p in the integration limits. Finally, we obtain

$$L^{(2)}(p,f) \approx \frac{2C_n}{3p^2}\left[2m\tilde{\mathcal{E}}\frac{\partial}{\partial p}\left(p^{-1}\frac{\partial f}{\partial p}\right) + 3\frac{\partial f}{\partial p}\right], \tag{9}$$

where $\tilde{\mathcal{E}} = \mathcal{E}(<\infty)$ is mean energy of single particle. The condition $W(<\infty, t) = 1$ is also used here. As far as in the conservative system the mean energy $\tilde{\mathcal{E}} = \frac{4\pi}{n}\int_0^\infty \frac{p^4}{2m} f\, dp$ is constant, one may pass over to new independent dimensionless variables

$$z = \frac{p^2}{2m\Theta}, \qquad \tau = \frac{t}{t_0}, \tag{10}$$

where

$$\Theta = \frac{2}{3}\tilde{\mathcal{E}}, \qquad t_0 = \frac{(m\Theta)^{3/2}}{\sqrt{2}\,C_n} \tag{11}$$

Thus, Eq.(8) is reduced to the form of

$$\frac{\partial f}{\partial \tau} = \frac{1}{\sqrt{z}}\frac{\partial}{\partial z}\left(\frac{\partial f}{\partial z} + f\right). \tag{12}$$

It is evident that the Maxwell distribution $f = e^{-z}$ satisfies the stationary equation (12) ($\partial f/\partial \tau = 0$). The quantity Θ entering inexplicitly Eq.(12) is a temperature established in the system at $\tau \to \infty$ Just this is implied further on when talking of plasma temperature even in the case when the plasma (namely its high-energetic tail) is far from the equilibrium state.

In order to clarify the time evolution of the system energy distribution, it is convenient to make the substitution $f = g e^{-z}$ after which one comes to the equation

$$\frac{\partial g}{\partial \tau} = \frac{1}{\sqrt{z}} \left(\frac{\partial^2 g}{\partial z^2} - \frac{\partial g}{\partial z} \right). \tag{13}$$

The function

$$g(z, \tau) = e^{z} f(z, \tau) \tag{14}$$

determines the thermalization degree at the point z at an instant τ of time, and it tends to unity at $\tau \to \infty$.

Eq.(12) for the high-energy tail of distribution function $f(p, t)$ has been obtained earlier by Gould [3] in the "Fokker-Planck limit". However, Eq.(12) in [3] has been obtained under non-obvious assumption of validity of expanding the function f in the power series by parameter $\Delta p/p$, whereas in this paper it is obtained in a more consistent way. Particularly, we can now define correctly the applicability range of this equation, namely $z \geqslant 3$.

3. The Solution of Kinetic Equation for the High-Energy Tail of the Distribution Function

Let at the initial instant of time the assembly consists of particles with the energy $\mathcal{E}$. Define now the character of the time evolution of high-energy tail of the distribution function $f = g e^{-z}$. Then it is necessary to solve Eq.(13) in the range $z_0 \leqslant z < \infty$ with the initial condition

$$g(z, 0) = 0 \tag{15a}$$

and boundary condition

$$g(z_o, \tau) = \mu(\tau). \tag{15b}$$

Turning to the variable $u = \frac{4}{5} z^{5/4}$ we arrive at

$$\frac{\partial g}{\partial \tau} = \frac{\partial^2 g}{\partial u^2} - b \frac{\partial g}{\partial u}, \tag{16}$$

where

$$b \simeq \left(u^{-1/5} - \frac{1}{5} u^{-1}\right)$$

Since $u \geq u_o = \frac{4}{5} z_o^{5/4} \gg 1$, and the function $u^{1/5}$ is a slow one, then, considering the solution of Eq.(16) with initial and boundary conditions (15) in the range $u_o \leq u \leq \gamma u_o$, where $\gamma \sim 10$, we can, to the first approximation, take the function b constant, replacing it by some average value in the range under consideration. Then Eq.(16) is the thermoconductivity equation with constant coefficients, whose solution (for the semi-infinite straight line) is familiar (see, e.g. [6]). Indeed, substituting the function $g = \varphi(x, \tau) \exp\left(\frac{bx}{2} - \frac{b^2}{4}\tau\right)$, where $x = u - u_o$ Eq.(16) reduces to the form

$$\frac{\partial \varphi}{\partial \tau} = \frac{\partial^2 \varphi}{\partial x^2} \tag{17}$$

with initial and boundary conditions

$$\varphi(x, 0) = 0,$$
$$\varphi(0, \tau) = \mu(\tau)\, e^{b^2/4\tau}. \tag{18}$$

The solution of this equation is written down in the form:

$$\varphi(x,\tau) = \frac{1}{2\sqrt{\pi}} \int_0^\tau \frac{x}{(\tau-y)^{3/2}} e^{\frac{b^2}{4y} - \frac{x^2}{4(\tau-y)}} \mu(y)\, dy . \tag{19}$$

To find out a particular solution $\varphi(x,\tau)$ one should know the boundary function $\mu(\tau)$, which, strictly speaking, should be determined from the solution of kinetic equation in the general case of $z \sim 1$ however, if assuming that the thermalization at the point z_0 takes place directly at the initial instant of time, that won't lead to a considerable distortion of the distribution function at $z \sim \gamma z_0 >> z_0$, $(u >> u_0)$ Therefore, taking $\mu(\tau) = 1$, after simple transformations in Eq.(19), one finds for the function $g(x,\tau)$:

$$g(x,\tau) = \frac{1}{\sqrt{\pi}} \left(\int_{\frac{x-b\tau}{2\sqrt{\tau}}}^{\infty} e^{-v^2} dv + e^{bx} \int_{\frac{x+b\tau}{2\sqrt{\tau}}}^{\infty} e^{-v^2} dv \right) , \tag{20}$$

where $x = \frac{4}{5}(z^{5/4} - z_0^{5/4})$, $b \simeq \langle U^{-1/5} - \frac{1}{5} U^{-1} \rangle$. Noticing that at $x >> 1$ the lower limit in the second integral in Eq.(20) is greater than unity, and error function $\Phi(s)$ decreases very rapidly for large S one may neglect the second term in Eq.(20) for any τ, if compared with the first one. Substituting for the average in b the corresponding value of the main term $U^{-1/5}$ at the point $z^* = z/2$, we obtain $b \to \frac{1.24}{z^{1/4}}$. Noticing also that $x = \frac{4}{5}(z^{5/4} - z_0^{5/4}) \simeq \frac{4}{5} z^{5/4}$ for $z >> 1$, one finds

$$g(z,\tau) \simeq \frac{1}{\sqrt{\pi}} \int_{a(z,\tau)}^{\infty} e^{-v^2} dv , \tag{21}$$

where

$$a(z,\tau) \simeq \frac{2}{5} \frac{z^{3/2} - 1.55\tau}{\sqrt{\tau}\; z^{1/4}} . \tag{22}$$

Since at $\tau \to \infty$ the quantity $\alpha(z,\tau)$ tends to $-\infty$, from (21) we directly obtain that the function g responsible for the thermalization degree of the particles in the region z tends to unity, which just should be expected.

A characteristic thermalization time at the given point z can be determined from the condition $g(z,\tau) = 0{,}5$; as a result, one arrives at the condition $\alpha(z,\tau) = 0$, i.e.

$$\tau_z \simeq 0{,}64\, z^{3/2} \tag{23}$$

It should be noted that the obtained approximate solution of (21), (22) is actually independent of z_o . Besides, the thermalization time τ_z for the region $z \sim \gamma z_o$ ($\gamma \sim 10$) turns out much greater than that for z_o ($\sim z^{3/2}$). Therefore, the assumption above $g(z_o,\tau) = \mu(\tau) = 1$ is justified for any z_o used to find out the approximate solution $g(z,\tau)$ in the range of $z \sim 10\, z_o$. Then it follows that the above-implied limitation $\gamma \sim 10$ turnes out non-critical.

In Fig.1 thermalization degree $g(z,\tau)$ is presented for several values of τ . It is seen from the Figure that the high-energy part ($z \gg 1$) of the distribution function at $\tau < \tau_z$ is substantially suppressed as compared with the Maxwell distribution. It is evident therefore that the fulfilment of condition $\Delta t \sim t_o$, where Δt is a characteristic life-time of plasma, is yet insufficient to use the Maxwell distribution for calculating the rates of physical processes in plasma.

So far we have been discussing the evolution of the distribution function not specifying physical processes leading to plasma thermalization. The only parameter dependent on the interaction process and entering inexplicitly into Eq.(12), is a scale factor t_o having the bulk-re[illegible]ation time meaning. Its value is defined by Eqs.(11) and (3a). In the particular case of the Coulomb interaction ($V(r) = e^2/r$) we obtain the well-known result for the relaxation time (see, e.g. [3])

$$t_0 = \frac{(\kappa T)^{3/2} m^{1/2}}{2\sqrt{2}\,\pi e^4 \Lambda n} \, , \tag{24}$$

where Λ is the Coulomb logarithm, and m is the mass of colliding particles.

In proton-proton collisions, besides the Coulomb forces, nuclear forces may affect as well. Moreover, they become essential beginning with $\kappa T \gtrsim 1$ MeV. A total cross section of the inelastic proton-proton scattering in the energy range of 10-300 MeV is well approximated by the formula [7]

$$\sigma_{pp}^N = \sigma_{nn}^N = \left[10.63/\beta^2 - 29.92/\beta + 42.4\right] \mathrm{mb} \, , \tag{25}$$

where $\beta = v/c$; v is a relative velocity of colliding particles.

To estimate the relaxation time due to nuclear forces, one may take advantage of the relation

$$t_0 \simeq \frac{1}{\langle \sigma_t \cdot v \rangle n} \, , \tag{26}$$

where

$$\sigma_t = \int (1 - \cos\theta)\, d\sigma \tag{27}$$

is a so-called transport cross section that determines the energy exchange between colliding particles.

In the c.m.s.-frame the angular distribution of scattered nucleons is almost isotropic [7] , and hence $\sigma_t = \frac{1}{3}\sigma_{pp}^N$. Respectively,

$$t_0^N \equiv \frac{q(T)}{n} \simeq \frac{10^{16}}{n}\left[0.61\left(\frac{m_p c^2}{\kappa T}\right)^{1/2} + 7.34\left(\frac{\kappa T}{m_p c^2}\right)^{1/2} - 3\right]^{-1} c . \tag{28}$$

It follows from the comparison of Eqs.(28) and (24) that at $\kappa T \simeq 10$ MeV the nuclear relaxation time is about 1.5 orders as less as that due to the Coulomb collisions. When increasing the temperature, the ratio $t_0^N : t_0^c$ becomes even smaller. Thus, contrary to Gould's statements [8,9], the nuclear elastic collisions result in the thermalization of plasma with temperature $\kappa T \gtrsim 1$ MeV much more efficiently than the Coulomb interactions, despite the long-range nature of the latter. Hence for the high-temperature ion plasma, nuclear forces are dominating. This leads to the closening of the relaxation times τ_{pp} and τ_{ee}, while τ_{pe} remains substantially larger than τ_{pp} and τ_{ee}. Therefore the independent treatment of time evolution of distribution functions of electrons and protons becomes even more justified.

The time delay of thermalization of the high-energy tail of the distribution function affects the formation of the plasma radiation spectrum connected both with the electron and ion components. Let us consider this question on the particular example of γ-ray luminosity of plasma as a result of the decay of secondary π^0-mesons.

4. High-Temperature Nucleon Plasma Luminosity Due to Production and Decay of π^0-Mesons

Starting with the proton energy $\gtrsim 150$ MeV in c.m.s.-frame an inelastic channel of π-meson production takes place. Since the cross section of π-meson production up to ~ 500 MeV is noticeably less than that of the elastic proton-proton scattering, the inelastic collisions will not substantially affect the evolution of particle distribution function in plasma with $\kappa T \lesssim$ 100 MeV. Nevertheless, the process of the production and decay ($\sim 10^{-16}$s) of π°-mesons is of certain interest from the viewpoint of plasma γ-ray luminosity. Thus, for the Maxwell plasma this mechanism of the radiative cooling of nucleons becomes dominating with the temperatures of $\kappa T \gtrsim$ 20 MeV [10].

The rate of π°-meson production in plasma with the Maxwell distribution of particles was calculated by many authors [11-14]. The radiation spectrum has a characteristic maximum at $E_{\gamma} \sim$ 70 MeV. Its shape weakly depends on the proton distribution function, bei due mainly to the kinematics of $\pi^{\circ} \to 2\gamma$ decay. Hence the plasma luminosity L_{γ} is determined directly by the π°-meson production rate which depends essentially on the proton distribution function. Therefore at the times

$$\Delta t < t_{\pi} \simeq \left(\frac{\varepsilon_{\pi}}{\kappa T}\right)^{3/2} t_{\circ}\,; \qquad \varepsilon_{\pi} \simeq 150\,\mathrm{MeV}, \tag{29}$$

the luminosity L_{γ} will be suppressed comparing with that of plasma with the Maxwell distribution. This can be seen from Fig.2, where the ratio of the plasma luminosities

$$\varkappa = \frac{R_{\pi}(T,\tau)}{R_{\pi}(T,\infty)} \simeq \frac{L_{\gamma}(T,\tau)}{L_{\gamma}(T,\infty)}$$

is shown versus the time $\tau = t/t_{\circ}$. The calculations of $R_{\pi}(T,\tau)$ were carried out for the distribution $f = g e^{-z}$ on the basis of the π°-meson production cross section compiled by Stecker [15]. The calculations of $R_{\pi}(T,\infty)$ corresponding to the Maxwell distribution coincide with the earlier obtained results of Weaver [13].

High-temperature plasma ($T \gtrsim 10^{11}$ K) can be formed near compact relativistic objects such as neutron stars and black holes [11,12,16-20].

To estimate γ -ray luminosity of the accretion plasma, one usually determines the mean energy of hot nucleons in the radiation generation region, assuming then that the nucleons have the established Maxwell distribution with $\kappa T = \frac{2}{3} \langle \varepsilon \rangle$, one calculates $R_{\pi}(T) \equiv R_{\pi}(T, \infty)$. However the obtained in this way results will be valid only if the condition

$$\Delta t \geq t_{\pi} \tag{30}$$

holds, where t_{π} is defined from Eq.(29), Δt is the lifetime of hot nucleon plasma. Let us discuss the fulfilment of this condition for two models of the high-temperature accretion plasma around a black hole.

a) Spherically-Symmetric Accretion.

In the standard spherical accretion the ion temperature at a distance r from the gravitational centre is

$$\frac{\kappa T}{m_p c^2} \simeq 0.1 \frac{r_g}{r} \left(1 + \frac{\mu^2}{3}\right)^{-1}, \tag{31}$$

where $r_g = 2\,GM/c^2 \simeq 3 \cdot 10^6\,(M/10M_\odot)$ cm is the gravitational radius, and $\mu \gtrsim 1$ is the Mach number. It follows then that at $r \lesssim 10 r_g$ the ion temperature T_p ($\simeq \frac{2}{3} \langle \varepsilon \rangle$) achieves the value $\gtrsim$ 10 MeV. At such temperatures, due to nuclear spallation, plasma consists mainly of protons and neutrons with negligible amount of nuclei [10]. Hence π -mesons are produced only as a result of nucleon-nucleon collisions.

The plasma falling time $\Delta t \sim t_{ff}$ and number density n_p at a distance r are (see, e.g. [21]):

$$\Delta t \sim t_{ff} = \frac{(\mu^2+3)^{1/2}}{\mu} \frac{r_g}{c} \left(\frac{r}{r_g}\right)^{3/2}, \tag{32a}$$

$$n_p \simeq \frac{8}{3\sqrt{3}} \frac{\dot{m}_o}{\sigma_T \eta r_g} \left(\frac{r_g}{r}\right)^{3/2} \left(1 + \frac{\mu^2}{3}\right)^{-3/2}, \tag{32b}$$

where σ_T is the Thomson cross section, $\eta = L/\dot{M}c^2$ characterizes the efficiency of energy release due to accretion, $\dot{m}_0 = \dot{M}/M_{cr}$, $\dot{M}_{cr} = 2\pi r_g m_p c/\eta\sigma_T$ is the accretion critical rate. The ratio of the relaxation time t_0^N (28) to the plasma falling time is equal to

$$\frac{t_0^N}{t_{ff}} \simeq \frac{10^{16} q(T)}{n_p t_{ff}} \simeq 10\, q(T)\, \frac{\eta}{\dot{m}_0} . \tag{33}$$

Substituting the characteristic values $q \simeq 5$ (10 MeV $\leq \kappa T \leq$ 100 MeV), $\eta \simeq 0.1$, $\dot{m}_0 \simeq 0.1$, we arrive at $t_{ff} \ll t_0^N$, and hence, the meson production in plasma under spherical accretion is strongly suppressed.

b) Disk Accretion

Shapiro, Lightman and Eardley [17] have shown that the regime of disk accretion onto the black hole is possible, where the rapid cooling of the plasma electrons due to unsaturated Comptonization leads to the formation of the two-temperature plasma with $T_e \sim 10^9$ K and $T_i \gtrsim 10^{11}$ K.

At the disk accretion the plasma radial falling time is about

$$\Delta t \sim \frac{R}{V_R} \simeq 2\left(\frac{r}{r_g}\right)^2 \frac{r_g \tau_T \eta}{c\,\dot{m}_0} , \tag{34}$$

where $\tau_T = nh\sigma_T$ is electron scattering optical depth, being of the order of unity for the model under discussion, $h \simeq 0.1\, r$ is the disk half-thickness. Using the characteristic value of $\eta \simeq 0.06$ in the disk inner region $r \lesssim 10\, r_g$, where high-temperature ion plasma is formed, we obtain

$$\frac{t_0^N}{\Delta t} \simeq 2\dot{m}_0 \left(\frac{2}{\tau_T}\right)^2 \frac{10 r_g}{r}$$

where $\dot{M}/\dot{M}_{cr} = \dot{m}_0$ as before. To reach the γ-ray luminosity of the Maxwell plasma, the condition (29) should hold, i.e.

$$\left(\frac{10\,\mathrm{MeV}}{\kappa T}\right)^{-3/2}\left(\frac{2}{\tau_T}\right)^{2}\left(\frac{10\,r_g}{r}\right)^{2}\dot{m}_0 \geqslant 1. \qquad (35)$$

Unfortunately, because of still existing great uncertainties in the values of parameters describing the disk accretion (see, e.g. [22]), it seems impossible at present to answer unambiguously the question on the fulfilment of the condition (35), and consequently on the effective π - meson production in accretion disks around Schwarzschild black holes.

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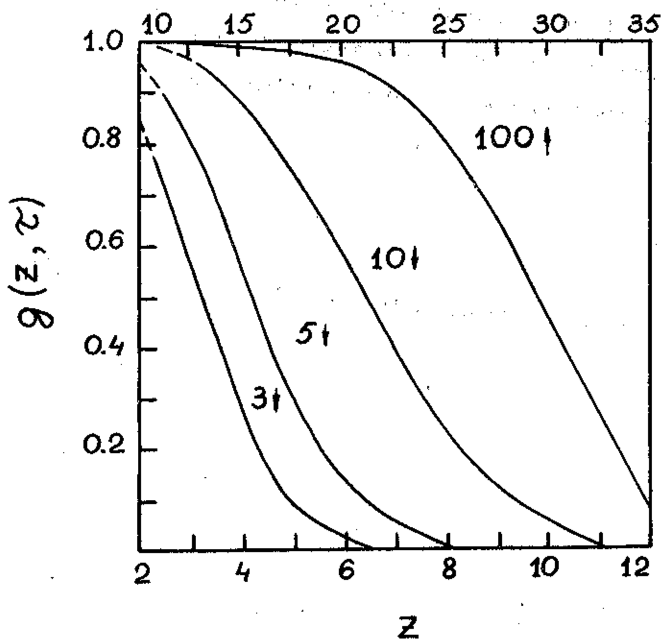

Fig. 1. Thermalization degree $g(z,\tau)$ as a function of $\tau = t/t_o'$ and $z = E/\kappa T$. The values of τ and the corresponding Z-axis are indicated near the curves.

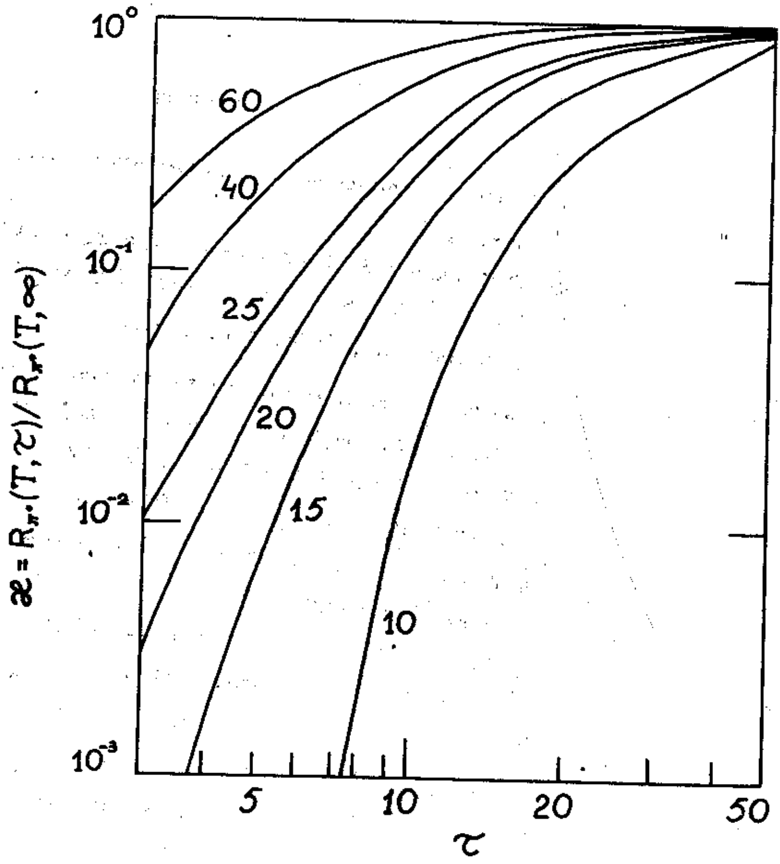

Fig. 2. $\varkappa = \dfrac{R_{\pi}(T,\tau)}{R_{\pi}(T,\infty)}$ as a function of time $\tau = t/t_0$ and plasma temperature κT. The values of temperatures κT (MeV) are indicated near the curves.

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