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E.SH.EGORIAN, S.KALITSIN

SUPERFIELD FORMULATION OF STOCHASTIC QUANTIZATION
WITH ADDITIONAL TIME

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Эд.Ш.ЕГОРЯН, С.КАЛИЦИН

СУПЕРПОЛЕВАЯ ФОРМУЛИРОВКА СТОХАСТИЧЕСКОГО
КВАНТОВАНИЯ С ДОПОЛНИТЕЛЬНЫМ ВРЕМЕНЕМ

В настоящей работе найдена $D+1$ - мерная суперполевая теория, которая редуцируется к обычной D - мерной скалярной теории. Приведено также другое доказательство справедливости стохастического квантования с одним дополнительным временем D - мерной скалярной теории поля.

Ереванский физический институт

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E. SH. EGORIAN, S. KALITSIN*

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WITH ADDITIONAL TIME

The $D+1$ -dimensional superfield theory, which is reduced to a usual D -dimensional scalar theory, is found. Another proof for the validity of the stochastic quantization with one additional time of the D -dimensional scalar field theory is presented as well.

Yerevan Physics Institute

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* Institute of Nuclear Investigations and Nuclear Energetics
of Bulgarian Academy of Sciences.

Y E R E V A N P H Y S I C S I N S T I T U T E

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1. Introduction

In ref. [1] a method of stochastic quantization of a D-dimensional scalar theory is proposed, based on the Langevin equation with the additional time t :

$$\frac{\partial \varphi(x,t)}{\partial t} + \frac{\delta S}{\delta \varphi(x,t)} = \eta(x,t) \quad (1.1)$$

where $\eta(x,t)$ is the external random Gaussian source with the correlation function

$$\langle \eta(x,t) \eta(x',t') \rangle = 2 \delta(x-x') \delta(t-t') \quad (1.2)$$

Then the usual quantum averages are determined by the following equation [1]

$$\langle \varphi(x_1) \dots \varphi(x_n) \rangle = \lim_{t \rightarrow \infty} \langle \varphi_\eta(x_1, t) \dots \varphi_\eta(x_n, t) \rangle_\eta \quad (1.3)$$

where $\varphi_\eta(x,t)$ is the solution of eq. (1.1).

In ref. [2] the stochastic Langevin equation is written already in the D+2-dimensional space. For scalar theory this formulation proves to be equivalent to the D+2-dimensional su-

persymmetric theory, which is reduced to the D-dimensional usual scalar field theory.

In the present paper a similar attempt is made as applied to the Langevin equation with one additional time t . It turns out that in the scalar case this scheme is equivalent to some D+1-dimensional superfield (but, unlike 2, not supersymmetric) theory, which is reduced to the D-dimensional scalar theory.

The symmetry, responsible for this reduction, hasn't been found.

2. Superfield Formulation of the Langevin Equation

Let us introduce a production potential

$$Z(h) = \int \mathcal{D}\eta \exp \int d^p x dt \left(-\frac{1}{4} \eta^2 + h \varphi_\eta \right) \quad (2.1)$$

We shall then have the relation

$$\langle \varphi_\eta(x_1, t) \cdots \varphi_\eta(x_n, t) \rangle_\eta = \frac{\delta}{\delta h(x_1, t)} \cdots \frac{\delta}{\delta h(x_n, t)} Z(h) \quad (2.2)$$

Let us replace the variable η by ψ_η in (2.1). It is accomplished by substituting a unit in (2.1)

$$1 = \int \delta \left(\dot{\psi} + \frac{\delta S}{\delta \psi} - \eta \right) \det \left(\partial_t + \frac{\delta^2 S}{\delta \psi^2} \right) \mathcal{D}\psi$$

Using the standard technique of determinant recording by means of anticommuting scalars in the exponent and introducing the additional scalar field Λ , we shall obtain for $Z(h)$

$$Z(h) = \int \exp \int d^p x dt \left\{ \Lambda^2 - \Lambda \left(\dot{\psi} + \frac{\delta S}{\delta \psi} \right) + \bar{\psi} \left(\partial_t + \frac{\delta^2 S}{\delta \psi^2} \right) \psi + h \varphi \right\} \quad (2.3)$$

Introducing the superfield $\Phi(x, t, \theta)$ and supersource $H(x, t, \theta)$

$$\Phi = \varphi + \bar{\theta}\psi + \bar{\psi}\theta + \bar{\theta}\theta A, \quad H = \bar{\theta}\theta h(x, t) \quad (2.4)$$

let us rewrite eq. (2.3) as.

$$\tilde{Z}(h) = \int \mathcal{D}\Phi \exp - \int [\mathcal{L}(\Phi) + \Phi \left(\frac{\partial^2}{\partial\theta\partial\bar{\theta}} + \theta \frac{\partial}{\partial\bar{\theta}} \partial_t - \frac{1}{2} \partial_t \right) \Phi - H\Phi] d^4x \quad (2.5)$$

The term $-\frac{1}{2} \Phi \partial_t \Phi$, a total derivative in t , is introduced in (2.5) for symmetrization of the operator in parentheses. It is necessary for the correct definition of perturbation theory.

The translation invariance of eq. (2.5) leads to the eq. (2.2) independence of t . Hence, to obtain averages of the type (2.2) in the expression (2.5), the source H may be taken in the form

$$H(x, t) = \bar{\theta}\theta h(x) \delta(t) \quad (2.6)$$

Let us now explain, how the reduction to the D -dimensional scalar theory occurs in (2.5), i.e. we shall prove that $Z(h)$ is equal to the production functional of the usual scalar field theory $\tilde{Z}(h)$

$$Z(h) = \tilde{Z}(h) \quad (2.7)$$

where

$$\tilde{Z}(h) = \int \mathcal{D}\varphi \exp \int d^D x [- \mathcal{L}(\varphi) + h\varphi]$$

Let us carry out the proof by means of perturbation theory.

The field propagator Φ has the form

$$(\square^2 - \partial_t^2)^{-1} (\square^2 - 2 \frac{\partial^2}{\partial\theta\partial\bar{\theta}} + 2\theta \frac{\partial}{\partial\bar{\theta}} \partial_t - \partial_t^2) \quad (2.8)$$

or, passing to the Fourier transform in x and t , we shall have

$$\langle \phi(p, \omega, \theta) \phi(p', \omega', \theta') \rangle = - \frac{2 + (p^2 - i\omega) \overline{\theta - \theta'} + (p'^2 + i\omega') \overline{\theta' - \theta}}{p^2 + \omega^2} \times \delta(p + p') \delta(\omega + \omega') \quad (2.9)$$

For the coefficient h^2 in the expansion Z we obtain

$$\frac{\delta^2 Z}{\delta h(p_1) \delta h(p_2)} = \int \bar{\theta}_1, \theta_1, \bar{\theta}_2, \theta_2 \langle \phi(p_1, \omega_1, \theta_1) \phi(p_2, \omega_2, \theta_2) \rangle \frac{d\omega}{2\pi} d\theta_1 d\bar{\theta}_1 \times$$

$$\times d\theta_2 d\bar{\theta}_2 = \frac{1}{p_1^2} \delta(p_1 + p_2) = \frac{\delta^2 \tilde{Z}}{\delta h(p_1) \delta h(p_2)}$$

i.e. the equality (2.7) in free h^2 approximation is valid.

Let's prove this equality for an arbitrary diagram. Let's single out some vertex in the arbitrary diagram (for simplicity consider a vertex of type ϕ^3)

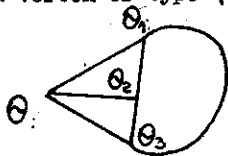


Fig. 1

The following expression corresponds to this diagram

$$\int \prod_{i=1}^3 \frac{2 + (p_i^2 - i\omega_i) \overline{\theta_i - \theta_i} + (p_i^2 + i\omega_i) \overline{\theta_i - \theta_i}}{p_i^2 + \omega_i^2} \delta\left(\sum_{i=1}^3 \omega_i\right) F \frac{d\omega_i}{2\pi} d\theta_i d\bar{\theta}_i \quad (2.10)$$

where F is the expression corresponding to an unknown block of the diagram in fig. 1; it is independent of $\theta, \bar{\theta}$ and has no poles in $\omega_1, \omega_2, \omega_3$.

Integrating over $\theta, \bar{\theta}$ we obtain the following expression

$$\int \frac{4 \sum_i p_i^2 + 2 \prod_{i,j} (p_i^2 - i\omega_i)(p_j^2 + i\omega_j) \bar{\theta}_i \theta_j}{\prod_i (p_i^2 + \omega_i^2)} \delta(\sum_i \omega_i) F \frac{d\omega_i}{2\pi} \quad (2.11)$$

The second component in (2.11) turns to zero. To make sure in this, one should merely have to present the delta-function in the form

$$\delta(\sum_i \omega_i) = \int_{-\infty}^{+\infty} e^{it(\sum_i \omega_i)} dt \quad (2.12)$$

and properly choose the integration contours over ω_i at positive and negative t (see Appendix).

The first component in (2.11) is

$$\frac{1}{\prod_i p_i^2} \tilde{F}$$

where \tilde{F} is the contribution by the unknown block of the fig. 1 diagram after all ω , θ integrations, i.e. the integration over ω_i and θ results in the fact that in the fig. 1 diagram the propagator connected with the vertex is equal to $1/p^2$. Thus, after all ω , θ integrations we shall obtain a usual D-dimensional scalar theory.

3. Conclusion

Thus a D+1-dimensional superfield theory, which is reduced to a usual D-dimensional scalar theory, is found in the present paper. Another proof for the validity of the stochastic quantization with one additional time of the D-dimensional scalar field theory is presented as well.

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Appendix

A. Let's prove the equality

$$I_1 \equiv \int \frac{(P_1^2 - i\omega_1)(P_2^2 + i\omega_2)}{\prod_i (P_i^2 + \omega_i^2)} \delta(\sum_i \omega_i) F(P_i, \omega_i) \frac{d\omega_i}{2\pi} = 0 \quad (A.1)$$

where F is the smooth function in ω_i . Let's substitute (2.12) in (A.1). At $t > 0$ contours of the integration over all ω_i should be taken in the upper halfplane. Then the pole in ω_2 reduces with the numerator $P_2^2 + i\omega_2$, and the integral in ω_2 zeros I_1 . At $t < 0$ the contours are taken in the lower halfplane and the integral in ω_1 turns to zero.

Thus (A.1) is proven.

B. Let's prove the equality

$$I_2 \equiv \int \frac{4(\sum_i P_i^2) \delta(\sum_i \omega_i)}{\prod_i (P_i^2 + \omega_i^2)} F(P_i, \omega_i) \frac{d\omega_i}{2\pi} = \frac{1}{\prod_i P_i^2} \tilde{F}(P_i) \quad (A.2)$$

where \tilde{F} is given below. As in point A, substitute (2.12) in (A.2) and the integral in t into two integrals with positive and negative t . At $t > 0$, choosing the contours in ω_i in the upper halfplane, integrating over all ω_i , and then over t in the range $(0, \infty)$, we shall obtain

$$I_2^+ = \frac{1}{\prod_i P_i^2} \cdot \frac{1}{2} F(P_i, iP_i^2)$$

Similarly, for $t < 0$ we shall obtain

$$I_2^- = \frac{1}{\prod_i P_i^2} \cdot \frac{1}{2} F(P_i, -iP_i^2)$$

Then, for $I_2 = I_2^+ + I_2^-$ we shall obtain the equality (A.2)

$$\tilde{F} = \frac{1}{2} [F(P_i, iP_i^2) + F(P_i, -iP_i^2)]$$

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Эд. Ш. КЮРЯН, С. КАМИНИ

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