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S.G.ARUTUNYAN, H.R.AVAKYAN, H.Z.BASEYAN

HOMOGENEOUS MODELS OF YANG-MILLS CLASSICAL FIELDS WITH EXTERNAL SOURCES

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Homogeneous models of Yang-Mills classical fields with external static sources are considered. For equations derived numerical experiments on a computer are performed with the aim to clarify the influence of external sources on the homogeneous models stochasticity.

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ОДНОРОДНЫЕ МОДЕЛИ КЛАССИЧЕСКИХ ПОЛЕЙ ЯНГА-МИЛЛСА С ВНЕШНИМИ ИСТОЧНИКАМИ

Рассмотрены однородные модели классических полей Янга-Миллса с внешними статическими источниками. Для полученных уравнений проведены численные эксперименты на ЭВМ с целью выяснения влияния внешних источников на стохастичность однородных моделей.

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1. Introduction

It was shown in Ref. [1] that Yang-Mills classical fields contain an untrivial class of space-homogeneous fields with a zero Pointing vector. Such homogeneous models reduce to a study of relevant mechanical systems treated in Refs. [2-5] . In Refs. [2,3] , via numerical experiments with a computer, a stochastic nature of homogeneous models was shown and mechanisms of this stochasticity were analyzed [4] .

Homogeneous models are a rather particular case of Yang-Mills classical fields. Notwithstanding their apparent simplicity, such models have many interesting properties typical of non-abelian theories. The applicability region of homogeneous models is determined by a condition that time variations occur far more quicker than spatial ones.

One can say that this corresponds to long-wave spectrum region (infrared region) which is not described by a perturbation theory after quantization. For the infrared region the following condition holds:

$$\lambda^2 F \gg 1 \quad (1)$$

where λ is a wavelength, F is fields magnitude.

Hence one may think that the study of homogeneous models is interesting also in connection with the absence of a correct solution of the infrared catastrophe in non-abelian theories.

This paper deals with homogeneous models of Yang-Mills classical fields with external static sources.

2. Equations of Motion

Equations of motion for Yang-Mills fields with external current J_μ^a have the form:

$$D_\mu^{ab}(A)\, F_{\mu\nu}^{b}(A) = J_\nu^a \qquad (2)$$

where $D_\mu^{ab}(A) = \delta^{ab}\partial_\mu + \epsilon^{acb} A_\mu^c$ is a covariant derivative,
$F_{\mu\nu}^a(A) = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \epsilon^{abc} A_\mu^b A_\nu^c$, ϵ^{abc} are group structure constants.

A covariant analog of equations of continuity for Yang-Mills current in the static sources case, $J_\mu^a = (\rho^a, 0)$, reads:

$$\dot{\rho}^a + \epsilon^{abc} A_0^b \rho^a = 0$$

In a hamiltonian gauging $A_0 = 0$ external sources are constant.

Following Ref. [6] we cancel $A_3 = 0$. This particular case contains all interesting features of homogeneous models dynamics, being relatively simple for a numerical analysis.

It is shown in Ref. [6] that in this case system (2) may be reduced to a mechanical problem with two degrees of freedom whose Lagrangian

$$\mathcal{L} = \frac{1}{2}\left(\dot{\tau}^2 + \dot{R}^2\right) - \frac{1}{2}\left(\frac{1}{\tau^2} + \frac{\lambda^2}{R^2}\right) - \frac{1}{4}\left(\tau^2 - R^2\right)^2 \qquad (3)$$

$\lambda = \left(\frac{\mu - \rho}{\mu + \rho}\right)$, where μ is moment integral, ρ is vacuum charge density.

In Ref. 4 there was studied the system (3) with $\lambda^2 = 1$, which corresponds to the absence of external charges. A mechanism of the occurrence of stochasticity in this system was revealed.

In this work a behaviour of this system for arbitrary values of λ^2 is studied.

It is shown that for a given energy $E \geqslant 1$ a value $\lambda^2_{c\tau}$ exists, beginning with which the system comes close to a regular one. The found approximate dependence of $\lambda^2_{c\tau}$ on E is confirmed by a numerical computer analysis.

3. Qualitative Analysis

The equations of motion that are relavant to a Lagrangian (3) have the form:

$$\begin{aligned} \ddot{R} &= \frac{\lambda^2}{R^3} + R\left(\tau^2 - R^2\right) \\ \ddot{\tau} &= \frac{1}{\tau^3} - \tau\left(\tau^2 - R^2\right) \end{aligned} \qquad (4)$$

The motion integral - the system (4) energy - reads:

$$E = \frac{1}{2}\left(\dot{\tau}^2 + \dot{R}^2\right) + \frac{1}{2}\left(\frac{1}{\tau^2} + \frac{\lambda^2}{R^2}\right) + \frac{1}{4}\left(\tau^2 - R^2\right)^2 \qquad (5)$$

Formula (5) determines the equipotential lines which can be parametrized in the form:

$$R^2(S) = -\frac{S}{2} + \frac{(1+\lambda^2)}{(4U-S^2)} + \sqrt{\frac{S^2}{4} + \frac{(1+\lambda^2)^2}{(4U-S^2)^2} + \frac{(1-\lambda^2)S}{(4U-S^2)}}$$

$$\tau^2(S) = \frac{S}{2} + \frac{(1+\lambda^2)}{(4U-S^2)} + \sqrt{\frac{S^2}{4} + \frac{(1+\lambda^2)^2}{(4U-S^2)^2} + \frac{(1-\lambda^2)S}{(4U-S^2)}} \tag{6}$$

where parameter S varies within $-2\sqrt{U} \leq S \leq 2\sqrt{U}$. The region of motion of the particle with energy E is limited by the equipotential line with $U = E$.

It is convenient to go to new coordinates:

$$x = \tau - R$$

$$y = \tau + R$$

for which system (4) and Hamiltonian (5) are:

$$\ddot{x} = \frac{8}{(y+x)^3} - \frac{8\lambda^2}{(y-x)^3} - xy^2$$

$$\ddot{y} = \frac{8}{(y+x)^3} + \frac{8\lambda^2}{(y-x)^3} - yx^2 \tag{7}$$

$$2E = \frac{1}{2}(\dot{x}^2+\dot{y}^2) + \frac{1}{2}x^2y^2 + \frac{4}{(y+x)^2} + \frac{4\lambda^2}{(y-x)^2} . \tag{8}$$

At $y \gg x$ the particle motion in the region restricted to asymptotes $x \approx \pm\frac{2\sqrt{E}}{y}$. Expanding Eqs.(7) in small ratio $(x/y) \ll 1$ we arrive at

$$\ddot{x} = -xy^2 + \frac{8}{y^3}(1-\lambda^2) \tag{9a}$$

$$\ddot{y} = -yx^2 + \frac{8(1+\lambda^2)}{y^3} - \frac{24x(1-\lambda^2)}{y^4} \tag{9b}$$

The Hamiltonian of this system reads:

$$2E = \frac{1}{2}(\dot{x}^2+\dot{y}^2) + \frac{1}{2}x^2y^2 + \frac{4(1+\lambda^2)}{y^2} + \frac{8x(\lambda^2-1)}{y^3} \tag{10}$$

Quick oscillations over X with a frequency $\sim y$ are due to the first term of Eq.(9a). The second term of the equation is responsible for the shift of the equilibrium position of the X -coordinate relative to zero value. Note that the y -coordinate is a slow-varying function as compared with the X -coordinate oscillations. This taken into account, the solution of Eq.(9a) with an accuracy up to terms $(X/y)^2$ can be written in the form:

$$x(t) = \frac{\alpha}{\sqrt{y}}\cos\left(\int y(t)\,dt + \beta\right) - \frac{8(\lambda^2-1)}{y^5} \tag{11}$$

where α and β are constant. Substituting (11) into (9b) and averaging over quick oscillations, we arrive at

$$\ddot{y} = \frac{8(\lambda^2+1)}{y^3} - \frac{\alpha}{2} - \frac{256(\lambda^2-1)^2}{y^9} \tag{12}$$

Eq.(12) describes the particle motion in the potential well $U_{eff} = \frac{\alpha y}{2} + \frac{4(1+\lambda^2)}{y^2} - \frac{32(1-\lambda^2)^2}{y^8}$ (see Fig.1). The potential U_{eff} has a local maximum $U_{cr} = \frac{3\alpha^{2/3}}{2^{2/3}}(1+\lambda^2)^{1/3}$; $y_{cr} = \frac{2^{5/6}(\lambda^2-1)^{1/3}}{(1+\lambda^2)^{1/6}}$

When condition

$$2E < U_{cr} \tag{13}$$

holds, the particle executes periodical oscillations in the region $y < y_{cr}$. As far as the approximation $x << y$ is not violated in this case, such oscillations in the mean potential U_{eff} corresnond to a nearly regular motion of the initial system (7). Note that the region of finite motion $y < y_{cr}$, when (13) holds, is physically meaningless since the condition $x << y$ is here broken. The above-found stability of y-oscillations for low energies is determined by quick oscillations of the x-coordinate with small amplitudes by analogy with the Kapitsa pendulum effect [7].

Consider the motion of particle with energy $2E > U_{cr}$. The region of motion along y is divided into two segments: $y < y_{cr}$ ("root" region) and $y >> y_{cr}$ (asymptotical region, "channel"), $x \sim y$ in the "root" region.

In this region all the terms of Eq.(7) are of the same order, therefore motion here has a random character. However in the "root" region the particle spends an insignificant fraction of time, since it is affected by the force with a positive y-component that pushes it out into the asymptotic region. The further motion is described with a sufficient accuracy by the effective potential U_{eff}, the return point over y being defined by initial conditions generated in the "root" region.

$$y_{max} \sim \frac{2E}{\alpha}, \tag{14}$$

where α implies an angle of incidence into the "channel".

Hence the motion in the asymptotic region essentially depends on that in the "root" region, where a practically random re-definition of parameter α which is responsible for the depth of the particle penetration into the "channel" takes place.

This reiterated process, i.e. the re-determination in the "root" region and the escape to the asymptotics, can be considered as an original mechanism of occurrence of stochasticity at an energy above a critical one.

Since the value of critical energy depends on λ^2 ($2E_{cr} = 0.94\alpha^{2/3}\frac{(\lambda^2+1)^{4/3}}{(\lambda^2-1)^{2/3}}$), for the given energy of the particle the increase of λ^2 results in that the system motion becomes close to a regular one. The threshold over λ^2, dividing energy region into regular and stochastic sectors, has a small width that decreases with increasing λ^2.

It is essential that the region of regular motion does not arise for systems with zero moment in the absence of external sources. It should be noted that the suggested in Ref.[8] comparison method which can be used to determine a critical value of the stochasticity parameter for homogeneous models of Yang-Mills fields with Higgs vacuum with zero moment [9] is inapplicable here.

4. Numerical Analysis

The set of equations (7) was analyzed numerically on a computer. It was found out that the condition (13) being fulfilled, the system under study stands close to the integrated one - during the experiment the particle executed $\sim 10^3$ cycles around one and the same orbit. This allows one to suppose the existence of a sufficiently well conserved value for the system (7) if the condition (13) holds. At $2E > U_{cr}$ the particle after each cycle moved along a new orbit, the orbits sizes being generated in a random way. Thus the numerical analysis confirms the above-stated qualitative consideration for homogeneous models of Yang-Mills fields.

The numerical analysis was carried out for fixed values of energy

$\mathcal{E} = 2E$ with variation of parameter λ^2 above and below the stochasticity level (13) for a few initial values of coordinates and momenta of particle.

The quantities $U = xy$, $V = \frac{1}{2}(x^2 + y^2)$, $\dot{V}$ are chosen as independent coordinates. The intersections of the particle phase trajectory with the $U = 0$ plane are shown in the Figures. Figs.2 and 3 correspond to a value $\mathcal{E} = 3$; $\lambda = 4$; 4.5. One can see that in the first case the motion is stochastic, in the second case it is close to a regular one. The orbits width is conditioned both by roughness of the computer drawing and by the circumstance that the potential well U_{eff} is preserved in the mean only. The value of λ_{cr} calculated by formula (13) is equal to 4.05. In more details the transition from a regular sector to a stochastic one is presented in Figs.4,5,6, for $\mathcal{E} = 10$, $\lambda = 10$; 12; 15 ($\lambda_{cr} = 9.87$). A phase picture of stochastic motion for $\mathcal{E} = 100$; $\lambda = 61.55$ ($\lambda_{cr} = 55.49$) is shown in Fig.7. It should be noted that the calculated by formula (13) systematical value of λ_{cr} as compared with the experimental threshold is due to that the qualitative considerations in the basis of formula (13) imply the particle motion in the depth of the potential well U_{eff}. Nevertheless formula (13) is quite applicable for rough estimations of the stochasticity threshold.

5. Discussion of the Results

It follows from the qualitative analysis and numerical experiments that the stochasticity occurrence mechanism suggested in Ref.[4] for $\lambda^2 = 1$ holds for arbitrary λ^2

Note that the division of phase space into regular and stochastic sectors is due to the presence of nonzero moment in the system. In a particular case (μ = 0) [2] the region of regular motion does not exist. It should be noted that the motion of a charged particle in electromagnetic fields is in some cases an analog of homogeneous models of Yang-Mills fields. Thus, the system of homogeneous fields with μ = 0 is equivalent to the problem on the charged particle motion in magnetic fields with longitudinal component gradient [11] . The existence of stochastic classes of solutions in homogeneous models [2-5] allows one to assume that Yang-Mills classical equations are nonintegrable. A possibility of solving the infrared problem by attracting the properties of homogeneous models will be discussed elsewhere.

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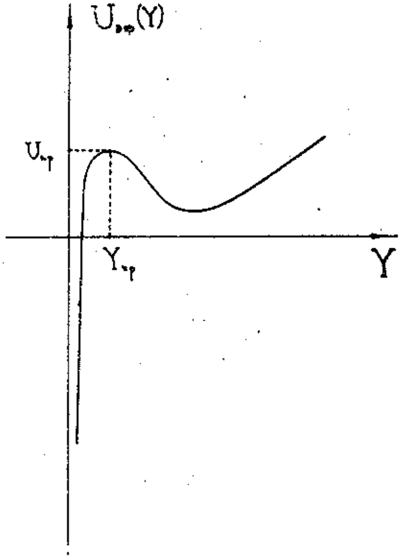


Fig.1

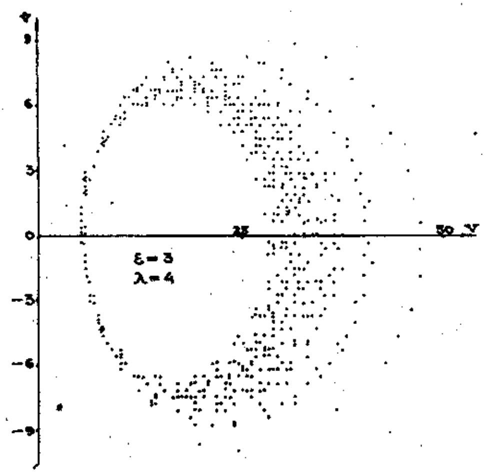


Fig.2

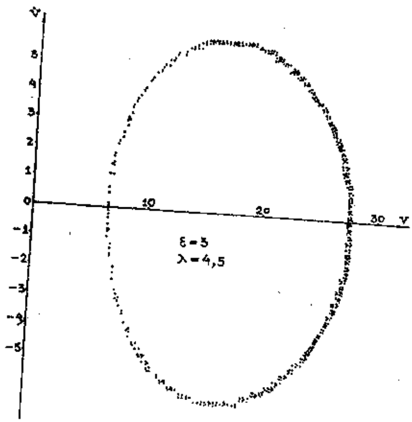


Fig.3

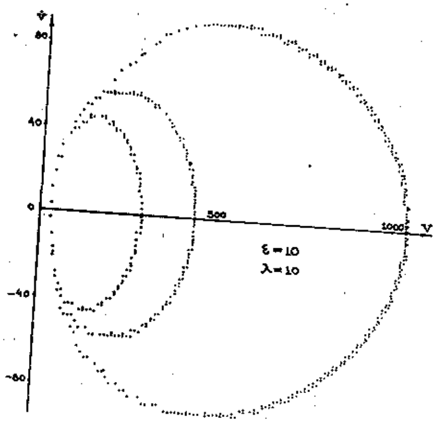
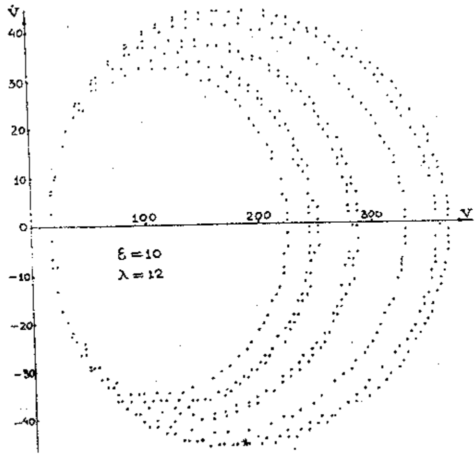
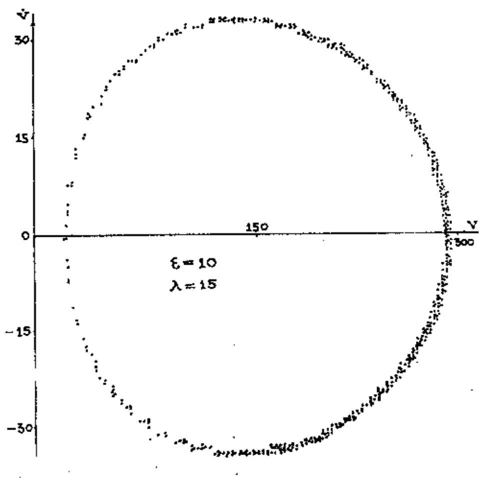


Fig.4



$\dot{V}$
40
30
20
10
0
-10
-20
-30
-40
100
200
300
V
$\varepsilon = 10$
$\lambda = 12$

Fig.5



$\dot{V}$
30
15
0
-15
-30
150
300
V
$\varepsilon = 10$
$\lambda = 15$

Fig.6

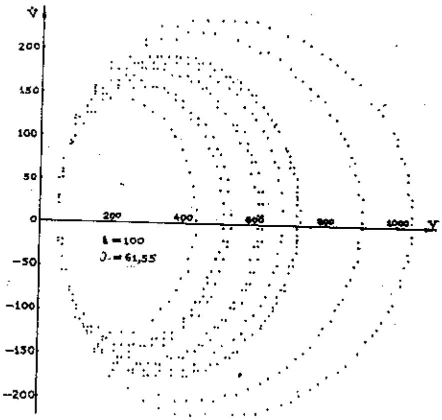


Fig.7

А.Р.АВАКЯН, С.Г.АРУТЮНЯН, Г.З.БАСЕЯН

ОДНОРОДНЫЕ МОДЕЛИ КЛАССИЧЕСКИХ ПОЛЕЙ
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