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STOCHASTIC QUANTIZATION OF N=1 SUPERSYMMETRIC THEORIES

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Э.Ш.ЕГОРЯН

СТОХАСТИЧЕСКОЕ КВАНТОВАНИЕ $N = 1$ СУПЕРСИММЕТРИЧНЫХ ТЕОРИЙ

В работе приводится схема стохастического квантования суперсимметричной модели Весса-Зумино и Абелевой $N = 1$ суперкалибровочной теории в четырехмерном пространстве.

Ереванский физический институт
Ереван 1983

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The scheme of stochastic quantization of supersymmetric Wess-Zumino model and N=1 Abelian supergauge theory in the four-dimensional space is proposed.

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The method of stochastic quantization of D-dimensional scalar theory, based on the Langevin equation with additional time, is proposed in Ref./1/.

In the work /2/ it is proved that the stochastic quantization is equivalent to reduction on D-dimensional space of D+1 dimensional superfield theory and a different proof of stochastic quantization is suggested.

The scheme of stochastic quantization of supersymmetric Wess-Zumino model and N=1 Abelian supergauge theory in the four-dimensional space is proposed in this work. The proof of the validity is based on the method suggested in the work /2/ .

Consider supersymmetric theories with the action

$$S = \int d^4x\, d^4\theta \{ \tfrac{1}{2} \Phi A \Phi + L_{int} (\Phi) \} \qquad (1)$$

where Φ is general scalar superfield and A is an operator formed by derivatives ∂ , D , $\bar{D}$. Theories of the type of (1) are stochastically quantized, if real parts of A -operator eigenvalues are positive.

Now let us formulate the rules of stochastic quantization for the theories of the type of (1). Later, we shall suppose that the transitions $p_0 \to i p_0$, $\vec{p} \to \vec{p}$ to Euclidean space are made, and θ structure of the theory is unchanged.

Let us assume that Φ depends on additional parameter t and satisfies Langevin equation:

$$\frac{\partial\Phi(x,\theta,t)}{\partial t} + \frac{\delta S(\Phi)}{\delta\Phi} = \mathfrak{J}(x,\theta,t) \tag{2}$$

where $\mathfrak{J}(x,\theta,t)$ is the external random Gaussian source, i.e. the mean value of functional $F(\mathfrak{J})$ is determined by the formula:

$$\langle F\rangle_{\mathfrak{J}} = \int F(\mathfrak{J})(\exp-\int\mathfrak{J}^2 d^4x\, d^4\theta\, dt)\,\mathcal{D}\mathfrak{J} \tag{3}$$

Then, the usual quantum Green functions are determined by the following equation:

$$\langle\Phi(x_1,\theta_1)\cdots\Phi(x_n,\theta_n)\rangle = \lim_{t\to\infty}\langle\Phi_{\mathfrak{J}}(x_1,\theta_1,t)\cdots\Phi_{\mathfrak{J}}(x_n,\theta_n,t)\rangle_{\mathfrak{J}} \tag{4}$$

where $\Phi_{\mathfrak{J}}$ is the solution to equation (2).

Let us demonstrate all aforesaid in the free case (I_{int} = 0). In this case Eq.(2) takes the form:

$$\left(\frac{\partial}{\partial t} + A\right)\Phi(x,\theta,t) = \mathfrak{J}(x,\theta,t) \tag{5}$$

with the general solution

$$\Phi(x,\theta,t) = (\exp-At) + (\exp-At)\int_{-\infty}^{t} d\tau\, \mathfrak{J}(\tau)\exp A\tau \tag{6}$$

The expression (6) is determined if the real parts of A -operator eigenvalues are positive. Using the relation

$$\langle\mathfrak{J}(x_1,\theta_1,t)\mathfrak{J}(x_2,\theta_2,t)\rangle_{\mathfrak{J}} = 2\,\delta(x_1-x_2)\delta(t_1-t_2)\delta(\theta_1-\theta_2) \tag{7}$$

which is a consequence of Eq.(1.3) and via Eqs.(4), (6) we can determine the two-point Φ field Green-function:

$$\langle \Phi(x_1,\theta_1)\Phi(x_2,\theta_2)\rangle = A^{-1}\delta(x_1-x_2)\delta(\theta_1-\theta_2) \tag{8}$$

Now we shall explain how in all orders of perturbation theory Eqs.(2)-(4) give usual Green functions of the theory (1). Our explanation is based on the method developed in Ref./2/ . Just like in /2/ , Eqs.(2)-(4) are equivalent to the theory described by the action

$$Z(h) = \int \mathcal{D}\Upsilon \exp - \int [\mathcal{L}(\Upsilon) + \Upsilon(\frac{\partial^2}{\partial\alpha\partial\bar{\alpha}} + \alpha\frac{\partial}{\partial\alpha}\partial_t - \frac{1}{2}\partial_t)\Upsilon - H\Upsilon] d^4x\, d^4\theta\, dt\, d\alpha\, d\bar{\alpha} \tag{9}$$

where
$$H = \bar{\alpha}\alpha\, h(x,\theta)\delta(t)$$

$$\Upsilon(x,\theta,t,\alpha) = \Phi(x,\theta,t) + \bar{\alpha}\psi(x,\theta,t) + \bar{\psi}(x,\theta,t)\alpha + \bar{\alpha}\alpha A(x,\theta,\alpha) \tag{10}$$

and α, $\bar{\alpha}$ are additional Grassmann parameters.

The equivalence of Eqs.(2)-(4) and the theory (9)-(10) means that Green functions determined from (1.4) are the variational derivatives of the production functional $Z(h)$. Then we shall show the validity of the equation

$$Z(h) = \tilde{Z}(h) \tag{11}$$

where
$$\tilde{Z}(h) = \int \mathcal{D}\Phi \exp - \int [\mathcal{L}(\Phi) - h\Phi] d^4x\, d^4\theta$$

This equation completes the proof of validity of stochastic quantization (i.e. Eqs.(2)-(4)).

The propagator of the theory (9) is

$$\langle \Psi(x,\theta,\omega,\alpha)\Psi(x',\theta',\omega',\alpha')\rangle = -(A^2+\omega^2)^{-1}[2+(A-i\omega)\overline{(\alpha-\alpha')}\alpha$$
$$+(A+i\omega)\overline{(\alpha'-\alpha)}\alpha']\delta(x+x')\delta(\omega+\omega')\delta(\theta-\theta') \tag{12}$$

where Fourier transformation over t is made. Let us consider a diagram with external lines $h(x,\theta)$ which is given by production functional (9). Integration over all parameters $\alpha, \bar{\alpha}, \omega$ is made in any such diagram and the answer is independent of them. Let us prove that the integration over $\alpha, \bar{\alpha}, \omega$ restores the standard diagram technique given by $\tilde{Z}(h)$. Let us single out some vertex from a diagram:

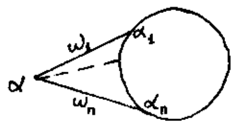

Fig. 1

The following expression corresponds to this diagram:

$$\int \prod_i (A_i^2+\omega_i^2)^{-1}[2+(A_i-i\omega_i)\overline{(\alpha_i-\alpha)}\alpha_i+(A_i+i\omega_i)\overline{(\alpha-\alpha_i)}\alpha]$$
$$\delta(\sum_i \omega_i)\, F\, \frac{d\omega_i}{2\pi}\, d\alpha\, d\bar{\alpha}\, d\alpha_i\, d\bar{\alpha}_i \tag{13}$$

where F is the expression corresponding to the unknown block of the diagram in Fig.1; it does not depend on $\alpha, \bar{\alpha}$. Under the condition that the real parts of eigenvalues of A operator are positive, one can easily prove, as in /2/ , that the $\omega, \alpha, \bar{\alpha}$ integration in the formula (13) results in

$$\prod_{i=1}^{n} A_i^{-1}\tilde{F} \tag{14}$$

where $\tilde{F}$ is the contribution by the unknown block after all integrations over ω, α . Thus the integration over ω_i, α results in the fact that the propagators connected with a vertex become A_i^{-1}, i.e. the standard perturbation theory given by the production functional $\tilde{Z}(h)$ is restored.

Now let us discuss some well-known supersymmetric models.

Consider the Wess-Zumino model described by Lagrangian

$$\mathcal{L} = \frac{1}{8}(\bar{D}D)^2(\Phi_+\Phi_-) - \frac{m}{4}(\bar{D}D)(\Phi_+^2+\Phi_-^2) - \frac{g}{2}(\bar{D}D)\frac{1}{n!}(\Phi_+^n+\Phi_-^n) \tag{15}$$

where $\Phi_\pm$ are chiral superfields ($\Phi_+ = \Phi_-^+$). It is easy to show that this theory may be described in terms of the general scalar superfield with the following action:

$$S = \int d^4x\, d^4\theta \{ \frac{1}{2}\Phi^2 + \frac{m}{2}\Phi(\frac{1}{2\Box}\bar{D}D)\Phi + V(\Phi) \} \tag{16}$$

$$V(\Phi) = \frac{g}{n!}\{ \Phi \frac{1}{2\Box}\bar{D}D[(E_+\Phi)^{n-1} + (E_-\Phi)^{n-1}] \}$$

where $E_\pm$ are projection operators. Green functions of chiral superfields $\Phi_\pm$ are obtained by the action of operators $E_\pm$ of the Φ fields Green functions of the theory (16). But the theory (16) is a particular case of the theory (1). The A-operator in this case has the form:

$$A = 1 + \frac{m}{2\Box}\bar{D}D \tag{17}$$

One can prove that the eigenvalues λ of the operator $\frac{m}{2\Box}\bar{D}D$ satisfy the equation

$$\lambda(\lambda^2 - \frac{m^2}{p^2}) = 0 \tag{18}$$

If one proceeds to Euclidean space according to the rule ($p_0 \to ip_0$, $\vec{p} \to \vec{p}$), then the numbers λ are zero or pure imaginary, i.e. the real parts of A-operator eigenvalues are positive and the theory can be stochastically quantized.

The N=1 Abelian supergauge theory is described by the action of the type of (1), with the following A-operator:

$$A = \Box + \frac{1}{4}(1+\alpha)(\bar{D}D)^2 \tag{19}$$

The eigenvalues of the operator $\frac{1}{4}(\bar{D}D)^2$ satisfy the equation

$$\lambda(\lambda+\Box)=0$$

Thus eigenvalues of the operator (19) coincide with those of the operator $\Box$ or $-\alpha\Box$, i.e. under the above-mentioned analytical continuation and under the condition $\alpha<0$ the eigenvalues of the operator (19) are positive and the condition of the stochastic quantization holds.

Thus the models of the Wess-Zumino type and the N=1 Abelian supergauge theories are stochastically quantized according to Eqs.(2)-(4). In the case of N=1 non-Abelian supergauge theories, there exist ghost terms, and such theories may be stochastically quantized by adding Langevin equations for these ghost fields.

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