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M.A.AGINIAN, S.G.ARUTUNIAN

FORCE LINES OF ELECTRIC AND MAGNETIC FIELDS OF ARBITRARY MOVING CHARGE

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FORCE LINES OF ELECTRIC AND MAGNETIC FIELDS OF ARBITRARILY MOVING CHARGE

Usual differential equations which determine parametrical form of the force lines of the point charge electric and magnetic fields at any distances from the charge are found. It is proved in general form that for the fixed instant of observational time the force lines of magnetic field are circles formed by the intercept of the sphere of simultaneous arrival of signals and bundle of planes with the axis parallel to trajectory binormal at a relevant delaying instant of time. The explicit expressions for the force lines of electric field are derived in some particular cases. Also the neutral line of magnetic field is determined.

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СИЛОВЫЕ ЛИНИИ ЭЛЕКТРИЧЕСКОГО И МАГНИТНОГО ПОЛЕЙ ПРОИЗВОЛЬНО ДВИЖУЩЕГОСЯ ЗАРЯДА

Найдены обыкновенные дифференциальные уравнения, определяющие параметрический вид силовых линий электрического и магнитного полей точечного заряда на любых расстояниях от него. В общем виде доказано, что для фиксированного момента времени наблюдения силовые линии магнитного поля представляют собой окружности, образованные пересечением сферы одновременного прибытия сигналов и пучка плоскостей с осью, параллельной бинормали траектории в соответствующий запаздывающий момент времени. Явные выражения для силовых линий электрического поля получены в некоторых частных случаях. Определяется также нейтральная линия магнитного поля.

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In solving a series of problems in electrodynamics, the required space-time pattern of field of an arbitrarily moving in vacuum charge can be received by means of the Lienard-Vichert potentials. Such method of finding field reduces in general case to the solution of the transcendental algebraic delay equation [1] . In the alternative approach Fourier expansions of solutions of Maxwellian equations are used, however it is not convenient in all cases. First, in order to apply the spectral analysis, the motion trajectory should be prescribed over the whole time interval, whereas from the uniqueness of the delay equation it follows that the field behaviour within limited volume during finite time interval depends only on local properties of trajectory. Second, the frequency width of Fourier expansion of relativistic particles fields is great.

An additional visual information on field of the arbitrarily moving charge can be obtained not involving the spectral analysis but via construction of single-moment force lines of electric and magnetic fields. The delay equation in this case is satisfied by relevant parametrization of formulae which describe the force lines. In case of uniform and rectilinear motion of the charge, the shape of those lines is well known (see, e.g. [2]). In [3] , there is considered also the case of motion with a jump of velocity.

The space-time structure of Lienard-Vichert field of the rectilinearly moving uniformly accelerated charge is studied either (see, e.g.[4]). The present work deals with the force lines of field of the arbitrarily moving charge. Usual differential equations determining the parametrical form of the force lines of electric and magnetic fields of the charge at any distances from the latter are found. It is proved in general form that for the fixed instant of observational time the force lines of magnetic field represent circles formed by the intercept of the sphere of simultaneous arrival of signals and bundle of planes with the axis parallel to trajectory binormal at a relevant delaying instant of time. The position of the bundle axis in space is determined by the particle velocity and acceleration at the same instant of time.

The explicit expressions for the force lines of electric field are obtained in some particular cases: uniform plane motion along curvilinear trajectory, motion along normal with arbitrarily variable velocity, as well as uniform screw motion with twisting proportional to the trajectory curvature.

We have studied separately the structure of the force lines of synchrotron radiation. It is shown that the force lines of electric field at the observational moment t are given parametrically via the delaying time t' and at the change of the latter execute revolutions around the particle with frequency $\omega_0 \gamma$, where γ is the particle Lorentz factor, ω_0 is the frequency of the particle's orbital reversal.

The case of uniformly accelerated motion is considered.

The characteristic region of spatial localization of high-energetic part of field (γ -region) of the ultrarelativistic charge is shown to concentrate near the neutral line of magnetic field. The computed results of estimates of the neutral line for the charge moving along trajectories with sign-variable curvature are presented.

The force lines together with the neutral line of magnetic field produce a visual equal-time pattern of field of the arbitrarily moving charge. The results obtained can be applied both to find conditions for coherent addition of fields of many particles in local regions of space and to analyze collective effects in relativistic beams.

General Formulae for Fields. Nulls of Magnetic Field.

Let $\vec{r}_0(t)$ be vector-radius of moving point charge, and $\vec{r}$ vector-radius directed to the point of observation. It is convenient to introduce the natural coordinate system, where $\vec{e}_1{}'$ and $\vec{e}_2{}'$ are unit vectors directed along basic normal of trajectory and particle velocity $\vec{\beta}c$ at a delaying instant of time t'. We take $\vec{e}_3{}'=[\vec{e}_1{}'\times\vec{e}_2{}']$ Then, for electric $\vec{E}$ and magnetic $\vec{H}$ fields we shall get the expressions

$$\vec{E}=\frac{e}{D^2(1-\beta\nu_2)^3}\Big\{\vec{e}_1{}'\big[\gamma^{-2}\nu_1+D\varkappa\beta^2(1-\beta\nu_2-\nu_1^2)+\nu_1\nu_2\beta D/c\big]+ \qquad (1)$$

$$+\vec{e}_2{}'\big[(\nu_2-\beta)(\gamma^{-2}-D\varkappa\beta^2\nu_1)-(1-\nu_2^2)\dot{\beta}D/c\big]+\vec{e}_3{}'\nu_3\big[\gamma^{-2}-D\varkappa\beta^2\nu_1+\nu_2\dot{\beta}D/c\big]\Big\},$$

$$\vec{H}=\frac{e}{D^2(1-\beta\nu_2)^3}\Big\{\vec{e}_1{}'\nu_3\big[\beta(\gamma^{-2}-D\varkappa\beta^2\nu_1)+\dot{\beta}D/c\big]+ \qquad (2)$$

$$+\vec{e}_2{}'\nu_3 D\varkappa\beta^2(1-\beta\nu_2)+$$

$$+\vec{e}_3{}'\big[-\beta\nu_1\gamma^{-2}+D\varkappa\beta^2(\beta(\nu_1^2+\nu_2^2)-\nu_2)-\nu_1\beta D/c\big]\Big\},$$

where $\vec{D}=D(\nu_1\vec{e}_1{}'+\nu_2\vec{e}_2{}'+\nu_3\vec{e}_3{}')$ is the vector from the delay point $\vec{r}_0(t')$ to the point of observation $\vec{r}$; $D=|\vec{D}|=c(t-t')$ t' is the instant of observation time, $\gamma=(1-\beta^2)^{-1/2}$ is the particle Lorentz factor, $\varkappa$ is the trajectory curvature at the point $\vec{r}_0(t')$, $\dot{\beta}=d\beta/dt'$

At variation of quantities ν_1, ν_2, ν_3, coupled by the relation $\nu_1^2+\nu_2^2+\nu_3^2=1$, the end of vector $\vec{D}$ describes a sphere with radius $c(t-t')$ and centre at point $\vec{r}_0(t')$ which we shall call light sphere.

For fixed point of observation $\vec{r}$ the instant of time is derived from the equation

$$c(t-t')=|\vec{r}-\vec{r}_0(t')|, \qquad (3)$$

which being substituted into (1) and (2) yields explicit dependence of the field on $\vec{r}$ and t . Eq.(3) is solved correctly for rectilinear uniformly accelerated motion of the particle. In case of uniform circular motion, when the point of observation is on orbit, the solution of (3) is analogous to the Kepler problem [5] and is to be expanded in a power series in Bessel functions. At large γ the number of main terms of this series $\sim\gamma^3$ coincides with the number of fundamental harmonics of synchrotron radiation.

In the ultrarelativistic case, (3) can be solved near the particle by expanding in γ^{-2} and in small ratio X/R , where X is a distance from the point of observation to the charge moving along trajectory with curvature radius R . The delay equation here reduces to algebraic fourth-order equation to be solved in radicals [6] , which allows, in particular, to clarify geometrical characteristics of γ -region, i.e. spatial localization of high-energetic part of synchrotron radiation (here field is proportional to γ^4). Near trajectory of motion, γ -region is elongated along radius and has small transverse size in plane of orbit $\sim R/\gamma^3$. It is shown that outside γ -region as well as outside Coulomb vicinity the field is essentially asymmetric and γ -independent [7].

The field pattern found in this approximation has mosaic structure - closed formulae of fields take place separately for γ-region and outside of it at large and small angular deviations. Besides, presented in [6-7] formulae do not define fields at distances comparable with curvature radius R.

A similar pattern can be built up using one-momentum lines of field: force lines for electric and magnetic fields, neutral lines for magnetic field. Note that for electric field there are no neutral points and lines at any motion of charge. It turns out that under proper parametrization, there is no need to solve the delay equation in order to find out these lines.

Let us study the possibility of turning the magnetic field (2) to zero on arbitrary light sphere ($D>0$). Excluding the trivial case of the rest charge, note firstly that at $\varkappa=0$ the field is zero over the whole sphere if the relation

$$\beta\gamma^{-2}+\dot{\beta}D/c=0 \tag{4}$$

holds. It is interesting to find motion trajectory for which (4) holds at all values of $t'<t$. Treating (4) as differential equation with respect to β we find

$$\beta(t')=\frac{t-t'}{\sqrt{w^2+(t-t')^2}}, \tag{5}$$

where w^{-1} is integration constant with the meaning of acceleration. It is seen that (5) corresponds to rectilinear uniformly accelerated motion of charge whose field was considered in [3]. Further, at $\nu_3 = 0$ and $\varkappa \neq 0$ magnetic field turns to zero only at two points of the light sphere. The cosine guides of these points are defined by the equation

$$D\varkappa\beta^2(\beta-\nu_2)-\nu_1(\beta\gamma^{-2}+\dot{\beta}D/c)=0. \qquad (6)$$

Assuming $\delta = D\varkappa\beta^2/(\beta\gamma^{-2}+\dot{\beta}D/c)$ we shall obtain the solution of (6) in the form

$$\nu_1=\frac{\delta(\beta\mp\sqrt{1+\delta^2/\gamma^2})}{1+\delta^2}, \quad \nu_2=\frac{\beta\delta^2\pm\sqrt{1+\delta^2/\gamma^2}}{1+\delta^2}, \qquad (7)$$

in which to different signs there correspond two branches of magnetic field neutral line. Varying parameter D within the limits $[0,+\infty]$ with respect to (3) we can construct a neutral line for any pre-set position of charge on the trajectory.

When the particle is moving on the section with $\dot{\beta}=0$ and curvature $\sim R^{-1}$, the distance between the branches of the neutral line is $\sim R/\gamma^3$ in the ultrarelativistic case, which coincides with the characteristic wavelength of synchrotron radiation (curvature is assumed constant on wavelength $\gtrsim R/\gamma$). In the case $\delta \gg \gamma \gg 1$, from (7) we have $\nu_2 \approx \beta$, $\nu_1 \approx \pm\gamma^{-1}$, i.e. signals forming neutral line are in the cone of synchrotron radiation with the expansion angle $\sim\gamma^{-1}$, directed along the particle velocity at a delay point. As is mentioned in [9], here the magnetic field neutral line marks high-energetic localizations of field in space.

Figs. 1-4 present calculations performed on a computer for the neutral line of magnetic field of the charge uniformly moving along trajectory given on plane (x, y) by the equation $y = \sin x$. The following values of Lorentz factor were chosen: γ = 5; 2; 1.5 ; 1.06. For $\gamma \gtrsim 5$ graphical presentation of neutral line on scale comparable with characteristic sizes of trajectory is somewhat complicated because of the appearance of small geometrical quantity $\sim R/\gamma^3$. In Fig.1 the branches of the neutral line for γ =5 are indistinguishable. At γ=2 (Fig.2) there arises small-scale structure of magnetic field of undulator radiation - the relativistic γ-regions are concentrated in the direction of motion (the charge moves from the right to the left and at a given instant of time is near the origin of coordinates). The case γ= 1.5 (Fig.3) is analogous to the case γ = 2. For the nonrelativistic motion γ = 1.06 (Fig.4), the structure of neutral line essentially changes, diffusion of small-scale structure takes place and asymmetry of lines density in the drawing before and behind the particle disappears.

Determining succession of zeros along tangents to trajectory one can, in principle, obtain information on spectral composition of field.

It is interesting to consider field of the charge moving along trajectory of "an eight" type, when the inflection points of trajectory coincide at prolonged motion. Fig.5 shows neutral line of magnetic field of the charge uniformly moving along trajectory parametrized on plane (x, y) by the equations: $x = \sin\alpha$, $y = \sin 2\alpha$, $\gamma = 5$ The circles corresponding to the turn of one of the neutral line branches around the inflection point are connected by two indistinguishable at the given scale branches lying in γ -regions.

The correspondence of the neutral line points to the trajectory sections can be obtained by drawing tangents through the inflection points. At these points signals forming the neutral line are emitted along the tangent forward and backward in the direction of motion.

"Tied" to the charge, pictures of the neutral line of magnetic field are of interest when solving the problems of coherent addition of fields of many particles in limited volumes [10] as well as when determining optimal directions and local sites for devices to observe fields of beams with given configuration of particles.

Force Lines of Magnetic Field.

Magnetic field of the charge moving in vacuum is expressed through electric one

$$\vec{H} = [\vec{D} \times \vec{E}]/|\vec{D}| \tag{8}$$

via vector $\vec{D} = D(\nu_1 \vec{e}_1' + \nu_2 \vec{e}_2' + \nu_3 \vec{e}_3')$ describing a light sphere with radius D . It follows from formula (8) that the magnetic field vector is tangential to this sphere for arbitrary motion of the charge (see also (2)).

The one-dimensional magnetic field force line which lies on the light sphere we shall describe by functions $\nu_1(\sigma)$, $\nu_2(\sigma)$, $\nu_3(\sigma)$ dependent on the formal parameter σ . It follows from (2) that functions $\nu_i(\sigma)$, i = 1, 2, 3 are solutions of the system of equations:

$$\begin{aligned}
\frac{d\nu_1}{d\sigma} &= \nu_3(\beta(1-\beta^2 D\varkappa\gamma^2\nu_1) + \dot{\beta}D\gamma^2/c), \\
\frac{d\nu_2}{d\sigma} &= \nu_3 D\varkappa\gamma^2\beta^2(1-\beta\nu_2), \\
\frac{d\nu_3}{d\sigma} &= -\beta\nu_1 + D\varkappa\gamma^2\beta^2(\beta(\nu_1^2+\nu_2^2)-\nu_2) - \nu_1\dot{\beta}D\gamma^2/c .
\end{aligned} \tag{9}$$

The quantities β , γ , $\dot{\beta}$, æ and D are fixed on the light sphere, i.e. the Eq.(9) coefficients are constant with respect to the independent variable σ . We have the obvious integral of motion $\nu_1^2+\nu_2^2+\nu_3^2=1$. Besides, from the first two equations one more integral comes out:

$$\beta+\dot{\beta}D\gamma^2/c-\beta^3D\text{æ}\gamma^2\nu_1=\mu(1-\beta\nu_2), \tag{10}$$

where μ is the integration constant. This expression in space of directions (ν_1 , ν_2 , ν_3) is given by a bundle of planes normal to the plane (ν_1 , ν_2) and passing through the point (A , β^{-1} , 0), where $A=(\beta+\dot{\beta}D\gamma^2/c)/(\beta^3D\text{æ}\gamma^2)$. The intercepts of those planes with the unit sphere - a variable-radius circle - determine force lines in space (ν_1 , ν_2 , ν_3). Increasing the radius of the unit sphere in this space up to D and matching it with the corresponding light sphere we shall get the form of magnetic field force lines in real space.

It is essential that the point (A , β^{-1}, 0) in the space (ν_1 , ν_2 , ν_3) always lies outside the unit sphere, i.e. intercepts o the plane bundle with it take place for the limited interval of the values of μ :

$$D\text{æ}\beta^3\gamma^3(A\gamma-\sqrt{1+(A\beta\gamma)^2})\leq\mu\leq D\text{æ}\beta^3\gamma^3(A\gamma+\sqrt{1+(A\beta\gamma)^2})$$

For the values of μ , at the ends of this interval the planes of the bundle touch the sphere. As it should be expected, magnetic field is zero at relevant points of space.

In Fig.6 the intercepts by the bundle planes of the unit sphere at $\nu_3=0$ for two values of A ($A\ll 1$, $A\gg 1$) are shown. The case

$A \gg 1$ corresponds to field observation near the charge (small D) or to rectilinear motion of particle ($\varkappa = 0$, $A \to \infty$). The case $A \ll 1$ corresponds to noticeable distortion of the field pattern. Note that for any arbitrarily slower motion along curvilinear trajectory ($\beta \to 0$, $\varkappa \neq 0$) there may always happen such distances $D \gg 1/(\varkappa\beta)$ for which $A \ll 1$, i.e. curvilinear structure of field will be noticeable.

Force Lines of Electric Field.

To construct force lines of electric field, note that they pass through all light spheres that correspond to the given trajectory of motion. Indeed, the scalar product of vector $\vec{D}$ and the electric field vector $\vec{E}$

$$\vec{E}\cdot\vec{D} = \frac{e\gamma^{-2}}{D(1-\beta\nu_2)^2} \tag{11}$$

never yields zero. This means that as a parameter describing the force line of electric field one may choose a radius of light spheres D . The vector-radius $\vec{Q}$, connected with the force line, we look for in the form

$$\vec{Q} = \vec{r}_0(D) + D(\nu_1\vec{e}_1{}' + \nu_2\vec{e}_2{}' + \nu_3\vec{e}_3{}'), \tag{12}$$

where $\nu_i = \nu_i(D)$, vector $\vec{r}_0(D)$ describes the charge motion in the opposite direction. Differentiating expression (12) with respect to we find

$$\frac{d\vec{Q}}{dD} = \vec{e}_1'(\nu_1 + D(\frac{d\nu_1}{dD} + \nu_2\beta\varkappa - \nu_3\beta\tau)) + \vec{e}_2'(\nu_2 - \beta + D(\frac{d\nu_2}{dD} - \nu_1\beta\varkappa)) + \vec{e}_3'(\nu_3 + D(\frac{d\nu_3}{dD} + \nu_1\beta\tau)), \tag{13}$$

where τ is orbit twisting at the delay point.

The components of vector $d\vec{Q}/dD$ are proportional to those of electric field (curly bracket in (1)) if the following relations hold:

$$\frac{d\nu_1}{dD} = -\beta\varkappa\gamma^2\nu_2 + \beta\tau\nu_3 + \beta^2\varkappa\gamma^2(1-\nu_1^2) - \frac{\nu_1\nu_2}{\beta\gamma}\frac{d\gamma}{dD},$$

$$\frac{d\nu_2}{dD} = \beta\varkappa\gamma^2\nu_1 - \beta^2\varkappa\gamma^2\nu_1\nu_2 + \frac{1-\nu_2^2}{\beta\gamma}\frac{d\gamma}{dD}, \tag{14}$$

$$\frac{d\nu_3}{dD} = -\beta\tau\nu_1 - \beta^2\varkappa\gamma^2\nu_1\nu_3 - \frac{\nu_2\nu_3}{\beta\gamma}\frac{d\gamma}{dD},$$

where β , γ , $\varkappa$, τ are in the general case functions of D. For quantities ν_1 , ν_2 , ν_3 again the relation $\nu_1^2 + \nu_2^2 + \nu_3^2 = 1$ holds.

In some cases Eqs.(14) determining together with (12) the force lines of electric field of arbitrarily moving charge can be solved explicitly.

A. At uniform motion of charge along arbitrary plane trajectory ($\tau = 0$, $d\gamma/dD = 0$, $\varkappa = \varkappa(D)$), the dependence of coefficients in Eqs.(14) on D can be eliminated replacing the independent variable of the form

$$\alpha = \beta\gamma^2 \int_0^D \varkappa(D)\,dD . \tag{15}$$

Such replacement, generally speaking, holds on a sign-definite section of trajectory, therefore further we suppose $æ > 0$. With respect to (15) we have one more integral of motion:

$$\nu_3 = \mu(1-\beta\nu_2), \tag{16}$$

where μ is integration constant. The latter relation determines the form of the force lines in space of directions (ν_1 , ν_2 , ν_3). Just as for magnetic field, these circles are formed in this case by the intercepts of unit sphere by the bundle of planes passing through the point ($0, \beta^{-1}, 0$) with the axis directed along the axis ν_i (see Fig.7). In case of electric field, cosine guides are functions of D , hence the shape of the force lines in real space depends on the velocity of motion of vector ($\nu_1(D)$, $\nu_2(D)$, $\nu_3(D)$) along the circles of the unit sphere in space (ν_1 , ν_2 , ν_3). With respect to (16) we find functions $\nu_i(D)$ in explicit form

$$\nu_1 = \sqrt{\frac{1-\mu^2\gamma^{-2}}{1+\beta^2\mu^2}}\ \frac{2\xi}{1+\xi^2},$$

$$\nu_2 = \frac{1}{1+\beta^2\mu^2}\left(\beta\mu^2 + \sqrt{1+\mu^2\gamma^{-2}}\ \frac{1-\xi^2}{1+\xi^2}\right), \tag{17}$$

$$\nu_3 = \frac{\mu}{1+\beta^2\mu^2}\left(1-\beta\sqrt{1-\mu^2\gamma^{-2}}\ \frac{1-\xi^2}{1+\xi^2}\right),$$

where

$$\xi=-\frac{\sqrt{1+\beta^2\mu^2}}{\gamma(1+\beta\sqrt{1-\mu^2\gamma^{-2}})}tg\left[\frac{\beta\gamma}{2}\int_0^D \varkappa(D)dD+\varphi_0\right], \tag{18}$$

Here φ_0 is a constant characterizing the phase at moving along circles in space (ν_1,ν_2,ν_3); $-\gamma\leq\mu\leq\gamma$. Formulae (17, 18) being substituted into (12) yield the shape of force lines in real space. The case $\mu=\pm\gamma$ is distinguished, corresponding to touching of the bundle planes with unit sphere, when $\nu_1=0,\ \nu_2=\beta,\ \nu_3=\pm\gamma^{-1}$. Excluding this case, vector $(\nu_1(D),\nu_2(D),\nu_3(D))$ moves along the circles in space (ν_1 , ν_2 , ν_3) executing a complete revolution at changing the quantity $\beta\gamma\int_0^D\varkappa(D)dD$ by 2π . The latter is zero at $\beta=0$ or at $\varkappa=0$ (resting or rectilinearly and uniformly moving charge). The force lines in this case are straight, following from the charge.

In the ultrarelativistic case for trajectories with nonzero lower limit $\varkappa$ the quantity ξ is a fast-oscillating function of D , i.e. the force lines acquire small-scale (as compared with $\varkappa^{-1}$) structure.

For uniform motion of particle along circle (synchrotron radiation), functions $\nu_i(D)$ are periodic over D with a period $D_0=2\pi\beta/(\varkappa\gamma)$ It is interesting to consider the behaviour of force lines in the orbital plane. Taking in formulae (17, 18) $\mu=0$, $\varkappa^{-1}=R$ we find

$$\nu_1=\frac{2\xi}{1+\xi^2},\ \nu_2=\frac{1-\xi^2}{1+\xi^2},\ \xi=-\frac{1}{\gamma(1+\beta)}tg\left(\frac{\gamma s}{2R}+\varphi_0\right) \tag{19}$$

Here for obviousness we have turned from the radius of light sphere to the length of trajectory $s=\beta D$; R is the circle radius. The quantity ξ in the ultrarelativistic case is small as compared with unity almost during the whole interval of periodicity $s_0=2\pi R/\gamma$ except its ends, where $tg(\gamma s/2R+\varphi_0)$ turns to infinity. The interval of negative values $\Delta s\approx R/\gamma^2$, corresponding to the unlimited increase of ξ is far less than the length of periodicity s_0. This means that the force line is formed basically by signals with ν_2 close to unity, i.e. the given part of the force line represents involute of trajectory. On the ends of the interval of periodicity the force line executes a revolution around the particle and again closes itself on the trajectory involute.

Note that the quantity s_0 is of the order of the synchrotron radiation formation length R/γ [11], i.e. the latter acquires obvious geometrical meaning. It is essential that R/γ contains smaller scale of $\Delta s_0\sim R/\gamma^2$.

B. Eqs.(14) can be solved also for arbitrary rectilinear motion ($\varkappa=\tau=0$). The force lines lie in the planes passing through the motion axis, so the pattern is azimuthally symmetric. Having chosen for definiteness a semi-plane $\nu_3=0$, $\nu_1\geqslant 0$ we shall derive for the electric field force line an equation

$$\frac{d\nu_2}{dD}=\frac{1-\nu_2^2}{\beta\gamma}\frac{d\gamma}{dD} \tag{20}$$

whose solution yields

$$\nu_1=\frac{\sqrt{1-\nu_0^2}}{\gamma(1-\beta\nu_0)},\quad \nu_2=\frac{\beta-\nu_0}{1-\beta\nu_0} \tag{21}$$

where ν_0 = const. Expressions (21) are written in such a form that for the resting charge a parameter ν_0 determines inclination of the force lines relative to the fixed axis.

For the uniform rectilinear motion (γ = const) the force lines represent straight lines with the slope tangent of Φ to the axis $\vec{e}_2'$:

$$tg\phi = \gamma tg\phi_0 , \tag{22}$$

where

$$tg\phi_0 = -\sqrt{1-\nu_0^2}/\nu_0$$

Note that the general expressions (21) agree with the particular case of uniform motion with velocity jump considered earlier [3].

Write down the expression for the vector-radius $\vec{Q}$ describing the force line of the rectilinearly moving uniformly accelerated charge

$$\vec{Q} = D_0 \left\{ \frac{\sqrt{1-\nu_0^2}\,(\beta\gamma - \beta_0\gamma_0)}{\gamma(1-\beta\nu_0)}\vec{e}_1' - \left(\frac{(\nu_0-\beta)(\beta\gamma-\beta_0\gamma_0)}{1-\beta\nu_0} + \gamma \right)\vec{e}_2' \right\} \tag{23}$$

where β_0 , γ_0 are velocity and Lorentz factor of the charge at the instant of observation τ , D_0 = const. As a parameter giving the force line we have chosen the particle velocity β at a delay instant of time: $\beta_0 \leq \beta < 1$. The coordinate along the axis $\vec{e}_1'$ at $\beta \to 1$ comes out to asymptote $D_0 = \sqrt{(1+\nu_0)/(1-\nu_0)}$, the coordinate along the $\vec{e}_2'$ axis tending to $-\beta_0\gamma_0 D_0$, i.e. the force lines do not escape out of the plane normal to $\vec{e}_2'$ and located at a distance of $D_0\gamma_0(1-\beta_0)$ from the charge.

C. Eqs.(14) can be solved also in the case of uniform screw motion with twisting τ proportional to trajectory curvature $\varkappa$. Without writing out formulae for force lines in real space, note that in the space λ.

of directions (ν_1, ν_2, ν_3) the latters are determined by intersections of the bundle of planes of the unit sphere. The bundle is formed by planes normal to the plane (ν_2, ν_3) and passing through the point with coordinates ($0, \beta^{-1}, -k\beta^{-1}$), where $k\gamma^2$ is the proportionality constant between τ and $\varkappa$: $\tau = k\gamma^2\varkappa$.

Conclusion.

An obvious pattern of field of moving in vacuum charge given by force lines is a valuable addition to the electrodynamical problems where the spectral or Lienard-Vichert approach for the definition of field is used. Apart from that, the derived in the work formulae for force lines of the arbitrarily moving charge may be of self-interest in a number of problems.

The preliminary analysis of field geometry for separate charges is essential in determining collective forces of interaction in relativistic beams of accelerators and storage rings. It is well known that the eigen field near a particle may be approximated by the field of the uniformly and rectilinearly moving charge. The region of such Coulomb fields is essentially anisotropic - its form depends on characteristics of external fields and particle energy. As a criterion of distortion of Coulomb fields may serve in this case the deviation of electric field force lines from the straight lines starting from the charge. Characteristic ratio of dimensions of Coulomb regions of separate particles to the average distance between the particles determines the influence of trajectory curvature on summary Lorentz forces in the beam. Curvature of trajectories is essential if the given ratio is much less than unity [12,13]. Thus, for the electron storage ring PETRA [14] characteristic dimensions of Coulomb fields $\sim R\gamma^{-2} \approx$ $\approx 2.6 \cdot 10^{-5}$ cm, whereas an average distance between particles $\approx 3 \cdot 10^{-4}$ cm.

It is also essential that outside Coulomb regions the Lorentz two-particle force does not contain inverse powers of γ occurring in interaction of particles in rectilinear beams [15].

By means of constructing force lines of a single charge one can find trajectories of particles yielding interference maxima of fields in the given regions. Such problem for obtaining the coherent synchrotron radiation is considered in [10].

From the foregoing analysis one may determine conditions under which a distinguished in space circle of arbitrary radius would be force line of magnetic field of all particles of the beam at some instant of time. Such interference maximum of field of toroidal type can be obtained by modulation of monoenergetic beam in uniform magnetic field.

Note that the obvious pattern of field might be helpful also in solving electrodynamic inverse problems, i.e. when according to the given configuration of field one should find the current which produces that configuration.

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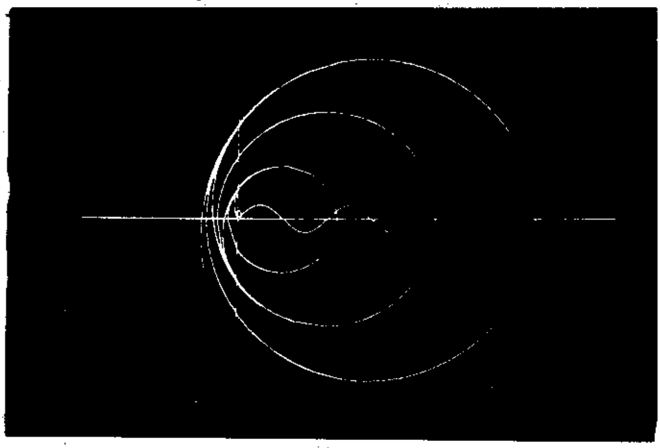

Fig.1. Neutral line of magnetic field of the charge moving along "sinusoid" $\gamma = 5$. The particle is moving from the right to the left and at the instant of observation is being near the origin of coordinates. The branches of neutral line are indistinguishable within the limits of γ -regions.

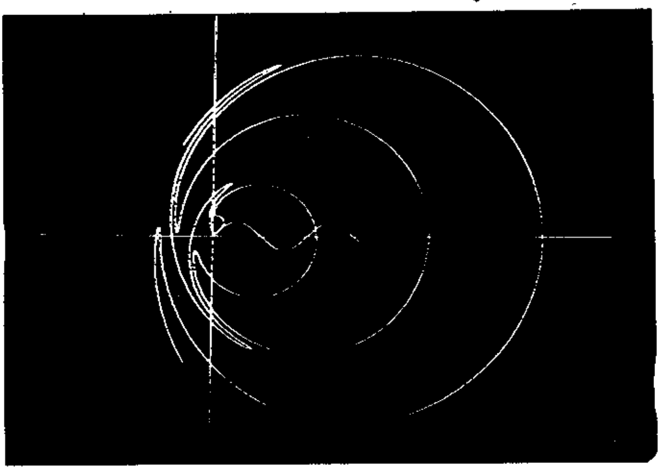

Fig.2. The same for $\gamma = 2$. The relativistic γ-regions concentrated in the direction of motion are seen.

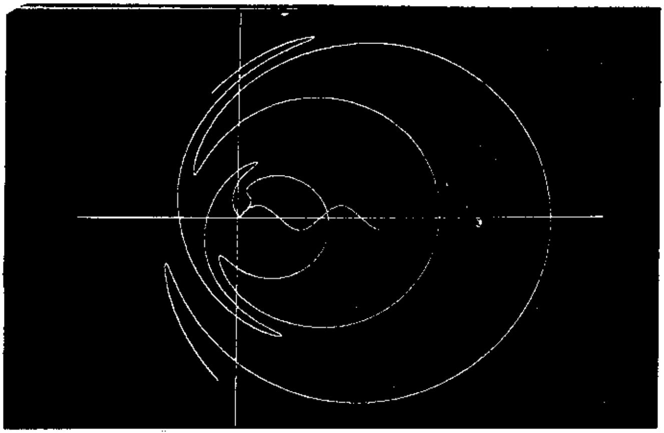

Fig.3. The same for γ = 1.5. Small-scale structure of undulator radiation is conserved.

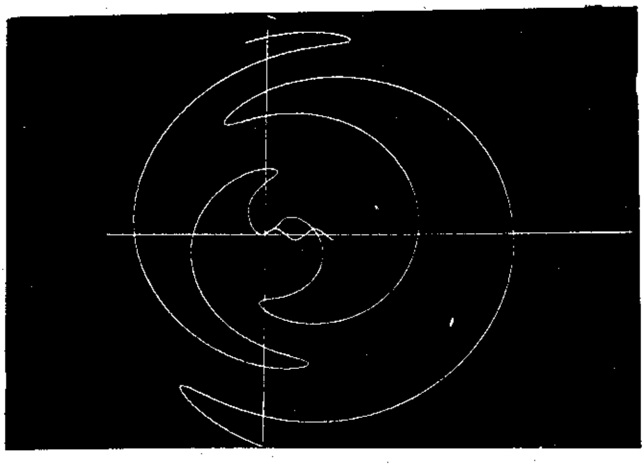

Fig.4. The same for γ = 1.06. Nonrelativistic motion. Diffusion of small scales takes place.

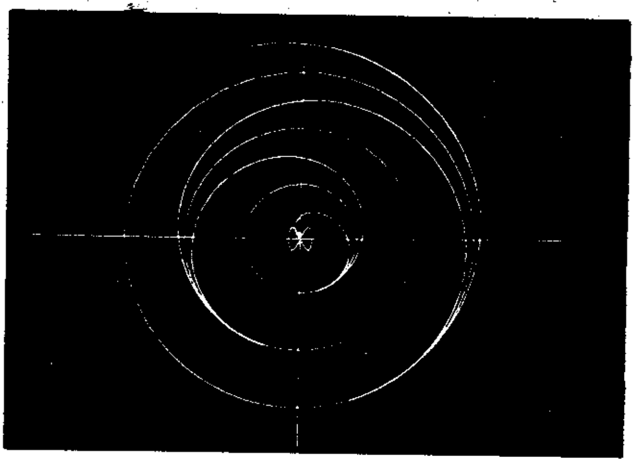

Fig.5. Neutral line of magnetic field of the charge moving along trajectory of "an eight" type. The circles corresponding to the turn of one of the neutral line branches around the inflection point are connected by two branches indistinguishable at the given scale. $\gamma = 5$

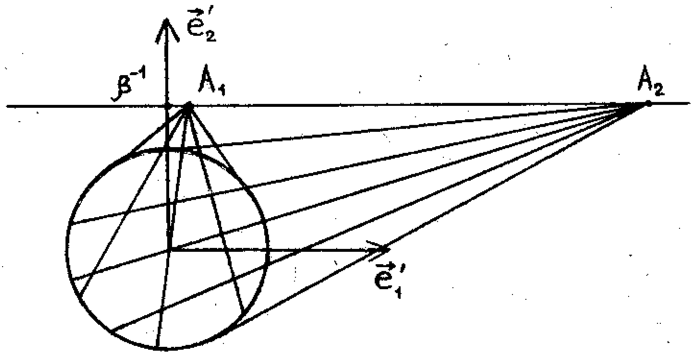

Fig.6. Force lines of magnetic field of arbitrarily moving charge in space of directions (ν_1 , ν_2 , ν_3). $A \ll 1$ corresponds to essential distortion of the field pattern, $A_2 \gg 1$ corresponds to observation of field near the charge or to rectilinear motion of the particle.

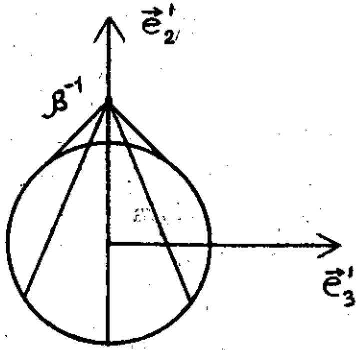

Fig.7. Force lines of electric field in space of directions (ν_1 , ν_2 , ν_3) of the charge uniformly moving along plane trajectory.

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