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ЦЕНТРАЛЬНЫЙ НАУЧНО-ИССЛЕДОВАТЕЛЬСКИЙ ИНСТИТУ
ИНФОРМАЦИИ И ТЕХНИКО-ЭКОНОМИЧЕСКИХ ИССЛЕДОВАНИ
ПО АТОМНОЙ НАУКЕ И ТЕХНИКЕ

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INTERACTION OF ULTRASHORT LIGHT PULSES IN A MEDIUM WITH TWO-PHOTON TRANSITIONS IN THE PRESENCE OF RESONANCE FIELD

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1. Introduction

For the last 20 years there appeared a large number of both theoretical and experimental works on resonance interactions (RI) of ultrashort pulses (USP) of light with matter (or coherent interactions) (see e.g. refs. [1-29]). The simplest are one-photon resonance coherent interactions. The investigation of these interactions was initiated by Kopvilem and Nagibarov, who have predicted in 1962 the phenomenon of photon echo [1] , as well as by Kurnit et al. 2 , who experimentally found it in 1964. Later on a number of new coherent phenomena were discovered at one-photon resonance: the self-induced transparency (SIT) [3-5], optical nutation [7,8], free induction decay [9], adiabatic passage [10] . The interest to coherent processes is first of all determined by a large amount of spectroscopic information on matter (see ref. [24] and references therein). Still greater are the possibilities of two-photon coherent processes. First, one may obtain with their help new information on matter (e.g. coherent spectroscopy free of the Doppler broadening [25,26] or three-level stimulated echo [27-29]). Second, the two-photon RI of USP may be utilized in problems on the short pulse frequency transformation as well as the shortening of their duration [16, 21-23, 30-32]. Despite this fact, experimentally two-photon coherent interactions are still insufficiently investigated (one may note refs. [33-36] where two-photon SIT and the Raman beating were investigated [37,38]). This is, apparently, due to the fact that for the observation of two-photon coherent processes the laser pulse should satisfy stricter requirements. In particular, the pulse power should be much more than at one-photon processes, and the phase modulation should be smaller. Quite often the available sources, retuned to the USP frequency , do not satisfy either the first or the second requirement. Besides if coherent interaction (e.g. two-photon SIT) at large distances are investigated, the inhomogeneity in transverse cross section of USP [39] as well as the medium relaxation time finiteness acquire special significance. Both these factors (and the phase modulation as well [40]) lead to the breaking of the USP propagation coherence and pulse damping [23]. On the other hand, if an additional field, parametrically connected to USP fields, is supplied to the medium, then along with the pulse energy loss (as a result of diffraction, relaxation etc.) there may occur the enhancement of USP due to the energy pumping from the additional field. Thus, the existence of self-sustaining pulses turns in principle possible.

In this paper we consider the coherent interaction of USP in the medium with two-photon absorption (enhancement) in the presence of the field of the frequency resonant to the transi-

tion frequency. The purpose of our paper is the investigation of the following possibilities: 1). coherent propagation of USP, inhomogeneous in transverse cross section, at large distances without loss in media with finite relaxation times as well as the self-compression of USP in such media; 2). observation of the two-photon SIT (and other related coherent phenomena) from pulses having small power and substantial phase modulation.

2. Initial Equations

Let us assume that both one-photon and two-photon transitions are allowed in dipole approximation between a pair of I and m levels of the matter atoms (molecules). As is known, this occurs in media lacking symmetry centers, such as, e.g. solid bodies with resonant similarly oriented molecules of admixture [41], or gases located in an electrostatic field. Consider the interaction of the short pulse of fields

$$E_{1,2}=C_{1,2}\exp[i(\omega_{1,2}t-\kappa_{1,2}z)]+K.C.\,,\quad C_{1,2}=A_{1,2}\exp(-i\varphi_{1,2})$$

with the medium, where before the USP arrival a non-zero polarization is established by means of the resonance field

$$E_0=[C_{01}\exp(-i\kappa_0 z)+C_{02}\exp(i\kappa_0 z)]\exp(i\omega_0 t)+K.C.$$

where C_{01}, C_{02} – are the amplitudes of forward and backward resonance waves, respectively. The time scheme of interactions is shown in fig. 1. At first, the medium is excited by a resonance field of the frequency ω_0 , beginning with the time moment $t=t_0$. Then at the moment $t=0$ the USP of the fields $C_{1,2}$ of the duration τ_n are supplied to the medium (the resonance field may act at $t>0$ as well). Resonance conditions have the form (see also fig. 2)

$$\omega_1+\omega_2=\omega_{m1}+\nu_1\,,\quad \omega_0=\omega_{m1}+\nu_0 \tag{1}$$

where ω_{m1} is the frequency of the operating transition between I and m levels; ν_0, ν_1 are the small detunings from resonance. The change in the fields $C_{1,2}$ of USP (we shall later on call them transformable fields) and resonance fields C_{01}, C_{02} are described by eqs.

$$\left(\frac{\partial}{\partial z}+\frac{1}{v}\frac{\partial}{\partial t}\right)C_{1,2}+\frac{i}{2K_{1,2}}\Delta_\perp C_{1,2}=-\frac{2\pi i\omega_{1,2}N}{n_{1,2}c}\Big[\frac{1}{2}(x^{mm}_{1,2}-x^{11}_{1,2})C_{1,2}\langle\eta\rangle + x_{12}\exp\{i(K_1+K_2)z\}C^*_{2,1}\langle 6\rangle\Big], \tag{2}$$

$$\left(\frac{\partial}{\partial z}\pm\frac{1}{v_0}\frac{\partial}{\partial t}\right)C_{01,02}\pm\frac{i}{2K_0}\Delta_\perp C_{01,02}=-\frac{2\pi i\omega_0 N}{n_0 c}\Big[\frac{1}{2}(x^{mm}_0-x^{11}_0)C_{01,02}\langle\eta\rangle + d\exp(\pm iK_0 z)\langle 6\rangle\Big], \tag{3}$$

which are obtained when substituting the polarization of the atom, found by authors of [42,43] for resonance processes, into the Maxwell equation in parabolic approximation [44] . In (2), (3) n_j is the refraction index at the frequency ω_j ; x^{mm}_j and x^{11}_j are the polarizabilities at the frequency ω_j of the atom (molecule) in m-th and I energy states, respectively; x_{12} is the I$\leftrightarrow m$ two-photon transition polarizability, d is the dipole moment of the I$\leftrightarrow m$ transition, N is the density of the medium atoms (molecules);

v_0, v are the group velocities of resonance and transformable waves,

$$\Delta_\perp=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\,;\quad \langle(\ )\rangle=\int(\ )g(\nu)d\nu$$

where $g(\nu)$ is the renormalized-to-unity function of atom distribution in ν , characterizing inhomogeneous line broadening. We shall assume $g(\nu)$ to be an even function of ν and consider that $\omega_1+\omega_2$ coincides with the central frequency of transition $\overset{\circ}{\omega}_{m1}$ of the inhomogeneously broadened line. The population difference η and the non-diagonal element of the density matrix 6 satisfy equations of generalized two-level systems

$$\partial 6/\partial t+[T^{-1}-i(\Omega+\nu)]6=i\hbar^{-1}\gamma\eta \tag{4}$$

$$\partial\eta/\partial t+(\eta-\eta_p)\tau^{-1}=-4\hbar^{-1}\mathrm{Im}(6\gamma^*) \tag{5}$$

where

$$\gamma=-x_{12}C_1C_2\exp[-i(K_1+K_2)z]-dC_{01}\exp(-iK_0z)-dC_{02}\exp(iK_0z),$$

τ, T are the longitudinal and transverse relaxation times, $\Omega=\hbar^{-1}\sum_j(x^{mm}_j-x^{11}_j)|E_j|^2$ (summed over all fields); η_p is the equilibrium population difference. If there is a pumping source, using which one can produce inverse population between I and m levels, τ and η_p are defined via probabilities of the transition W_{ij} from i state to j state. For the three-level system (ruby type)

$$\eta_p=\frac{W_{13}W_{32}-W_{21}(W_{31}+W_{32})}{W_{13}W_{32}+W_{21}(W_{31}+W_{32})}\,,\qquad \tau=\frac{W_{31}+W_3}{W_{13}W_{32}+W_{21}(W_{31}+W_{32})}\,.$$

3. Plane Wave Approximation

Consider first the case, when the fields C_j vary along the transverse coordinates x, y slow enough, and one may neglect in eqs. (2), (3) the terms proportional to $\Delta_\perp C_j$. Turning from (2) to the coordinate system $t \to t - z/v$, $z \to z$, we shall obtain equations for the amplitudes $A_{1,2}$ and phases $\varphi_{1,2}$

$$\frac{\partial A_{1,2}}{\partial z} = -\frac{\pi N x_{12}\omega_{1,2}}{n_{1,2}c} A_{2,1}\langle R\rangle \quad (6)$$

$$\frac{\partial(\varphi_1+\varphi_2)}{\partial z} = \frac{\pi N}{c}\left\{x_{12}\left(\frac{A_2\omega_1}{A_1 n_1}+\frac{A_1\omega_2}{A_2 n_2}\right)\langle I\rangle + \langle\eta\rangle\left[(x_1^{mm} - x_1^{11})\omega_1 n_1^{-1} + (x_2^{mm} - x_2^{11})\omega_2 n_2^{-1}\right]\right\} \quad (7)$$

where $R + iI = 2i\sigma \exp i(\varphi_1+\varphi_2)$, $\Phi = \varphi_1 + \varphi_2$.

Let us integrate eqs. (4) and (5), and considering that $\tau_n \ll \tau, T, \Delta\nu^{-1}$ ($\Delta\nu$ - is the line breadth) we obtain

$$\sigma(0<t<\tau_n) = \exp\left[i\int_0^t(\Omega+\nu)dt\right]\left\{\sigma_0 + \frac{i}{\hbar}\int_0^t dt\,\gamma\eta\exp\left[-i\int_0^{t'}(\Omega+\nu)dt\right]\right\} \quad (8)$$

$$\eta(0<t<\tau_n) = \eta_0 - 4\hbar^{-1}\int_0^t \Im m(\sigma\gamma^*)dt,$$

where $\sigma_0 = \sigma(t=0)$, $\eta_0 = \eta(t=0)$ characterize the polarization and population difference determined by resonance field to the moment $t=0$ and are defined by solutions of eqs. (3)-(5), where $C_{1,2} = 0$. Let us assume that one of the fields of USP is supplied to the medium input ($C_1(z=0) \neq 0$, we shall call it the triggering field), whereas $C_2(z=0) = 0$ *. Consider the

* Such a state of things is typical for most problems on frequency transformation.

case, when at $0<t<\tau_n$ the resonance fields C_{01}, C_{02} are small, so that

$$\hbar^{-1}\left|\int_0^t \gamma_p\eta_0 dt\right| \ll |\sigma_0|, \quad \frac{4}{\hbar}\left|\int_0^t \Im m(\gamma_p^*\sigma_0)dt\right| \ll |\eta_0|, \quad \int_0^t \Omega_p dt \ll 1 \quad (9)$$

(γ_p, Ω_p are equal to γ and Ω, respectively, at $C_{1,2}=0$). In that case, at the initial stage (the field C_2 is still small and conditions (9) are valid, if γ_p and Ω_p are replaced by γ and Ω) one may assume

$$\sigma(0<t<\tau_n) \approx \sigma_0(z), \qquad \eta(0<t<\tau_n) \approx \eta_0(z) \quad (10)$$

Substituting (10) into the right hand side of (6) and integrating it, we obtain

$$a_{1,2} = \frac{a_{10}(t)}{2}\left\{\exp\left[-\frac{\gamma}{2}\int_0^z\langle R_0(z)\rangle dz\right] \pm \exp\left[\frac{\gamma}{2}\int_0^z\langle R_0(z)\rangle dz\right]\right\} \quad (11)$$

where $a_j = A_j\sqrt{n_j/\omega_j}$, $R_0 = R(t=0)$, $\gamma = 2\pi N C^{-1} x_{12}\sqrt{\omega_1\omega_2/n_1 n_2}$

Taking into account (10), one may see from eq. (7) that the phase $\varphi_1 + \varphi_2$ is practically instantaneously established in such a way that a_2 increases (see (11) *. It is seen that if (9) is satisfied, the presence of resonance field in the time interval $0<t<\tau_n$ at the initial stage does not affect the USP propagation. Let a_2 and a_1 increase at the initial stage so much that $x_{12}|C_1C_2| \gg d|C_{01}|$. Then at consequent stages, with the increase in a_2 and a_1, the resonance field C_{01} can substantially increase due to the presence

* Such a phenomenon is termed the phase capture of interacting waves [45].

of parametric coupling between $C_{1,2}$ and C_{01}. Let us show that despite this, one may ignore at subsequent stages the influence of the resonance field in the interval $0<t<\tau_n$. To do so let us assume that the parametrical coupling is provided in the best way, i.e. there occurs an exact phase synchronism. In this case, as is shown in [46] (in the approximation $x_j^{mm}=x_j^{11}$) the amplitudes a_{01}, a_1 and a_2 are connected by the relation

$$\beta\left[a_{01}(z)-a_{01}(z_0)\right]=\ln\frac{a_1(z)+a_2(z)}{a_1(z_0)+a_2(z_0)}, \tag{12}$$

where $\beta=d(\omega_0/n_0)^{1/2}x_{12}^{-1}(\omega_1\omega_2/n_1n_2)^{-1/2}$

As is seen from (12), if a_{01} increases according to the linear law, $a_{1,2}$ increases according to the exponent. Hence, if at the initial stage

$$x_{12}|C_1|\,|C_2|\gg d|C_{01}| \tag{13}$$

eq. (13) will be valid at the further increase in the fields C_j as well. Since there is no parametric coupling between inverse resonance wave C_{02} and fields $C_{1,2}$ (besides, due to small efficient length of their interaction $\Delta z=\vartheta\tau_n$) we consider C_{02} to change insignificantly with the change in $C_{1,2}$. Thus, we obtain that if the resonance field satisfies relation (9) in the interval $0<t<\tau_n$, one may neglect its influence on the evolution of USP of $C_{1,2}$ fields, and consider that

$$C_{01,02}(0<t<\tau_n)=0 \tag{14}$$

We shall further consider (14) satisfied. The complete set of equations for the fields $A_{1,2}$ of USP will be composed in

this case of (6), (7) and equations for medium

$$\dot{R}+I(\dot{\Phi}+\nu+\Omega_n)=2x_{12}\hbar^{-1}A_1A_2\eta \tag{15}$$

$$\dot{I}=(\dot{\Phi}+\nu+\Omega_n)R \tag{16}$$

$$\dot{\eta}=-2x_{12}\hbar^{-1}A_1A_2R \tag{17}$$

where $\Omega_n=\Omega(C_{01,02}=0)$. Let us find the solution for $A_{1,2}$ in the general form assuming $R_0(z), I_0(z)$ and $\eta_0(z)$ to be given functions of z. As seen from (11), at the initial stage a_1/a_2 is independent of t ($a_2\rightarrow a_1$ with the increase in z). If $a_2\approx a_1$ is established at a point z_0, then, as is seen from the Menly-Row relation ($a_1^2-a_2^2=a_{10}^2$), $a_2\approx a_1$ at a further increase in $a_{1,2}$. In the latter case we have proportionate regime of interaction. Let the proportionate regime start before the end of the initial stage of interaction. As is shown in [22] (for a similar problem) this occurs in all the cases, when the triggering field a_{10} is small enough. In this case a_1/a_2 may be considered t-independent in all cases. Differentiating eq. (7) with respect to t and combining it with (6), we shall obtain the integral of motion

$$f=\Phi+\Omega_n=\frac{\alpha a_{10}^4}{8}\left\{\left(\frac{2a_1^2}{a_{10}^2}-1\right)\left[\left(\frac{2a_1^2}{a_{10}^2}-1\right)^2-1\right]^{1/2}-\ln\left(\frac{2a_1^2}{a_{10}^2}-1+\left[\left(\frac{2a_1^2}{a_{10}^2}-1\right)^2-1\right]^{1/2}\right)\right\}\left[a_1^2(a_1^2-a_{10}^2)\right]^{-1/2} \tag{18}$$

where $\alpha=2\hbar^{-1}[(x_1^{mm}-x_1^{11})\omega_1/n_1+(x_2^{mm}-x_2^{11})\omega_2/n_2]$

Taking into account the fact that the function

$$F(z)=\hbar(2x_{12})^{-1}f/A_1A_2 \tag{19}$$

is t-independent as well (according to (18)), let us integrate

(15)-(16). We shall obtain

$$R = aq\sin\left[\varphi(z) + 2x_{12}a\hbar^{-1}\int_0^t A_1A_2\,dt\right]$$

$$\eta = b + q\cos\left[\varphi(z) + 2x_{12}a\hbar^{-1}\int_0^t A_1A_2\,dt\right] \qquad (20)$$

$$I = I_0 - F(\eta - \eta_0)$$

where $a = (1+F^2)^{1/2}$, $b = \frac{(I_0 + F\eta_0)F}{1+F^2}$, $q = \frac{\left[(1+F^2)\rho - (I_0 + F\eta_0)^2\right]^{1/2}}{1+F^2}$

$$\rho = R_0^2 + I_0^2 + \eta_0^2, \quad tg\,\varphi(z) = aR_0/(\eta_0 - I_0F). \qquad (21)$$

As is seen from (19), (18), $F(z)$ changes from zero at $a_2 = 0$ to $r/2 = \alpha\gamma^{-1}\pi NC^{-1}$ in the proportionate regime ($a_1 = a_2$). Substituting (20) into eq. (6) and averaging it, we obtain

$$\frac{\partial A_{1,2}}{\partial z} = -\frac{\pi N x_{12}\omega_{1,2}}{n_{1,2}c}A_{2,1}\,aq\sin\left[\varphi(z) + \frac{2x_{12}a}{\hbar}\int_0^t A_1A_2\,dt\right] \qquad (22)$$

where in the expressions a, q and φ one should replace R_0, I_0, η_0 by their mean values $\langle R_0\rangle, \langle I_0\rangle, \langle\eta_0\rangle$. Let us introduce the notations

$$\Psi = ctg\frac{1}{2}\left[\varphi(z) + \vartheta_t\right], \quad \vartheta_t = \frac{2x_{12}a}{\hbar}\int_0^t A_1A_2\,dt; \quad S(z) = \gamma aq.$$

We shall then obtain from (22), considering the proportionality condition ($a_1 = a_2$), a Rikkaty-type equation

$$2\frac{\partial\Psi}{\partial z} + (\Psi^2+1)\frac{\partial\varphi}{\partial z} + S(z)\left[\Psi^2 - 1 - (\Psi^2+1)\cos\varphi(z)\right] = 0 \qquad (23)$$

which has the partial solution $\Psi = ctg\frac{\varphi}{2}$. The general solution for (23) has the form

$$\Psi = ctg\frac{\varphi(z)}{2} + P(z)\left\{P(z_0)\left[\Psi(z_0) - ctg\frac{\varphi(z_0)}{2}\right]^{-1} + \right. \qquad (24)$$

$$\left. + \frac{1}{2}\int_{z_0}^{z}\left[\frac{d\varphi}{dz} + S(1-\cos\varphi)\right]P(z)\,dz\right\}^{-1}$$

where

$$P(z) = \exp\left\{-\int ctg\frac{\varphi}{2}\left[\frac{d\varphi}{dz} + S(1-\cos\varphi)\right]dz\right\}$$

The solution of (24) is, in fact, the generalized area theorem describing the interaction of USP with the two-photon-absorbing (enhancing) medium that is arbitrarily excited.

4. Stationary Excitation

In this section we consider the case when the amplitudes of the resonance field C_{01}, C_{02} are independent of t (stationary approximation). As is known, such a situation may occur when the duration of the exciting pulse is much more than the relaxation time τ, T [45]. Equations (4), (5), where give simple solutions

$$\sigma = \frac{i}{\hbar}\gamma_p T\eta_p\left[1 + i(\nu - \Omega_p)T\right]^{-1}\left\{1 + \frac{4|\gamma|^2\tau T}{\hbar^2}\left[1 + (\nu - \Omega_p)^2T^2\right]^{-1}\right\}^{-1} = \sigma_0 \qquad (25)$$

$$\eta = \eta_p\left\{1 + 4|\gamma_p|^2\tau T\hbar^{-2}\left[1 + (\nu - \Omega_p)^2T^2\right]^{-1}\right\}^{-1} = \eta_0 \qquad (26)$$

As the estimates in [45] show, for one-photon allowed transitions and fairly large fields (of the order of saturation fields)

$$|\Omega_p T| \ll 1 \qquad (27)$$

Substituting (25), (26), with account of (27), into the right-hand part of (3) and averaging both parts of equation over the segment equal to the wave length, we shall obtain an equation for the amplitudes A_{01}, A_{02} and phases φ_{01}, φ_{02} of the resonance field

$$\partial A_{01,02}/\partial z = \mp \pi N \omega_0 d \,(2 n_0 c)^{-1} \langle G_{1,2} \rangle \tag{28}$$

$$\frac{\partial \varphi_{01,02}}{\partial z} = \frac{\pi N \omega_0}{n_0 c}\left[(x_0^{mm} - x_0^{11})\langle G_0 \rangle \mp \frac{\nu T d}{2} \frac{\langle G_{1,2} \rangle}{A_{01,02}}\right] \tag{29}$$

where $G_{1,2} = \hbar \eta_p (2\tau d)^{-1}\left[8\tau T d^2 \hbar^{-2} A_{01,02} + A_{01,02}^{-1}\left(\sqrt{C^2-4B^2} - C\right)\right] \times$

$$\times (C^2-4B^2)^{-1/2},\quad G_0 = \eta_p (1+\nu^2 T^2)(C^2-4B^2)^{-1/2},\quad B = 4\tau T \hbar^{-2} A_{01} A_{02},$$

$$C = 1 + \nu^2 T^2 + 4\tau T \hbar^{-2} d^2 (A_{01}^2 + A_{02}^2).$$

Equations (28), (29) differ from corresponding equations derived by the authors of [47] for quantum generators only by the term proportional to $x_0^{mm} - x_0^{11}$. It is easy to find from (25), (26) the initial values R_0, I_0, η_0 averaged over the wave length

$$R_0 = G_1 (\cos \Delta\Psi_0 + \nu T \sin \Delta\Psi_0), \tag{30}$$

$$I_0 = G_1 (\sin \Delta\Psi_0 - \nu T \cos \Delta\Psi_0), \tag{31}$$

$$\eta_0 = G_0, \tag{32}$$

where $\Delta\Psi_0 = \Delta\Psi(t=0)\,(\Delta\Psi = \Phi - \varphi_0 + \delta K z,\ \delta K = K_1 + K_2 - K_0)$, in accord with (7), (29), satisfies equation

$$\partial \Delta\Psi_0/\partial z = \delta K + \frac{\pi N}{c}\left\{ x_{12}\left(\frac{A_2 \omega_1}{A_1 n_1} + \frac{A_1 \omega_2}{A_2 n_2}\right)\Big|_{t=0} \langle G_1 \rangle \sin \Delta\Psi_0 + \right.$$
$$\left. + \left[(x_1^{mm} - x_1^{11})\frac{\omega_1}{n_1} + (x_2^{mm} - x_2^{11})\frac{\omega_2}{n_2} - (x_0^{mm} - x_0^{11})\frac{\omega_0}{n_0}\right]\langle G_0 \rangle \right\}. \tag{33}$$

At the initial stage of transformation we shall obtain from (6), (30)

$$\partial A_{1,2}/\partial z = -\frac{\pi N x_{12} \omega_{1,2}}{n_{1,2}\, c} A_{2,1} \langle G_1 \rangle \cos \Delta\Psi_0. \tag{34}$$

Consider the case, when the resonance excitation is distributed uniformly along the medium:

$$G_{1,2} \neq G_{1,2}(z), \qquad G_0 \neq G_0(z). \tag{34*}$$

Such a situation occurs, for example, in media of two types:

1. Media with inverse population placed in the resonator and generating a resonance field. As is shown in [47], if reflection coefficients of mirrors approximate 100%, the distribution of the amplitudes A_{01} and A_{02} along the sample is close to the constant.

2. Media placed in an electrostatic field. Let $A_{20} = 0$ and the electrostatic field E_0 vary along z according to the law

$$d = d_{1m} + x_{20} E_0 = \frac{d(z=0)}{(1-\kappa z)^{1/2}} \tag{35}$$

where d_{1m} and $x_{20} E_0$ - are the intrinsic and induced dipole moments of the $1 \leftrightarrow m$ transitions, $K = \pi N \omega_0 \eta_p \hbar [n_0 c \tau A_{01}^2 (z=0)]^{-1} \times$ $\times [C(z=0) - 1]/C(z=0)$. In this case, as it follows from (28) (where $C_{02} = 0$), G_1, G_0 are constant along z.

Taking into account that $G_{1,2}, G_0$ are constant, it is not difficult to obtain the integral of motion of eqs. (33),(34)

$$\sin \Delta\Psi_0 = -\frac{\delta}{\gamma\langle G_1\rangle}\,\frac{a_2}{a_1} \tag{36}$$

where $\delta = \delta_K + \frac{\pi N}{c}\left[(x_1^{mm} - x_1^{11})\frac{\omega_1}{n_1} + (x_2^{mm} - x_2^{11})\frac{\omega_2}{n_2} - (x_0^{mm} - x_0^{11})\frac{\omega_0}{n_0}\right]$

Substituting (36) into (34) and integrating it, we shall obtain solutions for the initial stage of interaction

1) at $|\delta| < \gamma\langle G_1\rangle$

$$a_2 = \frac{a_{10}}{2}\left[1 - \delta/\gamma^2\langle G_1\rangle^2\right]^{-1/2}\left\{\exp\left[\tfrac{1}{2}(\gamma^2\langle G_1\rangle^2 - \delta^2)^{1/2} z\right] - \exp\left[-\tfrac{1}{2}(\gamma^2\langle G_1\rangle^2 - \delta^2)^{1/2} z\right]\right\} \tag{37}$$

$$a_1 = \sqrt{a_{10}^2 + a_2^2};$$

2) at $|\delta| > \gamma\langle G_1\rangle$

$$a_2 = a_{10}\gamma\langle G_1\rangle(\delta^2 - \gamma^2\langle G_1\rangle^2)^{-1/2}\sin\left[0{,}5\gamma\langle G_1\rangle(\delta^2 - \gamma^2\langle G_1\rangle^2)^{1/2} z\right], \tag{38}$$

$$a_1 = (a_{10}^2 + a_2^2)^{1/2}.$$

From (37),(38) one may see that if the efficient wave detuning δ is small $|\delta| < \gamma < G_1 >$ the rise of the fields $a_{1,2}$ is monotonic, whereas at $|\delta| > \gamma\langle G_1\rangle$ the fields $a_{1,2}$ oscillate according to the sinusoidal law; a_2 has a maximum $a_{2max} = a_{10}\gamma\langle G_1\rangle(\delta^2 - \gamma^2\langle G_1\rangle^2)^{-1/2}$. At $|\delta| < \gamma\langle G_1\rangle$ due to the unlimited rise of $a_{1,2}$ in (37), the conditions (10) will be sooner or later violated and the formula (37) will stop being valid. As is shown in [22] (for a similar problem), at a small enough a_{10} even before the violation of the conditions (10), a proportional regime of interaction ($a_1 \approx a_2$) is set. In connection with this, beginning with some point z_0 , where (10) and (37) are still valid, we shall assume that $a_1 = a_2$. Taking into account (34*) it is not difficult to obtain from (24) the area theorem

$$\Psi = \mathrm{ctg}\frac{\varphi}{2}\;\frac{1 + \zeta\exp[D(z - z_0)]}{1 - \zeta\exp[D(z - z_0)]} \tag{39}$$

where $\zeta = (\Psi_0 - \mathrm{ctg}\frac{\varphi}{2})/(\Psi_0 + \mathrm{ctg}\frac{\varphi}{2})$, $\Psi_0 = \Psi(z=0)$, $D = (\gamma^2\langle G_1\rangle^2 - \delta^2)^{1/2}$.

In accord with (39), the dependence of the fields $A_{1,2}$ of USP on the coordinates and time has the form

$$A_{1,2}^2(z,t) = \frac{A_{1,2}^2(z=0)\,4\,\mathrm{ctg}^2\frac{\varphi}{2}(1 + \Psi_0^2)e^{D(z - z_0)}}{(\Psi_0 + \mathrm{ctg}\frac{\varphi}{2})^2\left[\mathrm{ctg}^2\frac{\varphi}{2}(1 + \zeta e^{D(z - z_0)})^2 + (1 - \zeta e^{D(z - z_0)})^2\right]} \tag{40}$$

It is easy to notice that in the limit at $A_{01,02} \to 0$ (39) transforms into the known theorem of cotangents for two-photon absorption (TPA) of USP [16]. From the area theorem (39) follows that the pulses with the area

$$\vartheta = \frac{2x_{12}\sqrt{1 + r^2/4}}{\hbar}\int_0^\infty A_1A_2\,dt = \begin{cases} 2|\varphi| + 2\pi n & \text{(in noninverted medium } \eta_p > 0\text{)} \\ 2\pi(n+1) - 2(\varphi) & \\ 2\pi n & \text{(in inverted medium } \eta_p < 0\text{)} \end{cases} \tag{41}$$

(n is the integer number) propagate in the medium without the change in energy, i.e. they are pulses of SIT. From fig.3 one may see that the areas of SIT pulses $2|\varphi| + 2\pi n\,(\eta_p > 0)$ and $2\pi(n+1) - 2|\varphi|\,(\eta_p < 0)$ are stable unlike $2\pi n$ pulses. Figures 4 and 5, performed by the formula (40), illustrate the evolution of pulses in space and time. If the initial pulse area is small ($\vartheta_0 < 2|\varphi|$ at $\eta_p > 0$, $\vartheta_0 < 2\pi - 2|\varphi|$ at $\eta_p < 0$), then, as is seen from fig. 4 (we shall have a similar picture at $\eta_p > 0$ as well), the fields $A_{1,2}$ grow, the pulse area approximates $2|\varphi|\,(\eta_p > 0)$ or $2\pi - 2|\varphi|\,(\eta_p < 0)$ (see fig.3); the pulse is compressed and noticeably shifts forward (in other words, it travels at a velocity surpassing ϑ). If the initial pulse area is close to 2π , then, as is seen from fig.5, the pulse divides into two subpulses. The first, as in the previous case, propagates faster than ϑ , and its area approximates $2|\varphi|$ (at $\eta_p < 0$ $2\pi - 2|\varphi|$). The second subpulse propagates slower than ϑ (shifts to the opposite side). If the initial pulse area $\vartheta_0 < 2\pi$ (see fig.5c), the second subpulse is attenuated. If $\vartheta_0 = 2\pi$, the area of the second subpulse tends to $2\pi - 2|\varphi|$ (see fig.3,5a). At $\vartheta_0 > 2\pi$ (but $\vartheta_0 < 2\pi - 2|\varphi|$) the area of the second subpulse approximates 2π (fig.3,5c). The duration of both subpulses decreases with the propagation, whereas their energies (of areas) practically do not change. Thus, there occurs an efficient compression of pulses. Note that in the case of $\vartheta_0 > 2\pi$ the compression of the second subpulse proceeds faster than at $\vartheta_0 = 2\pi$ (see figs.5a,c). As is seen from (40), in this case the increase in the intensity and the decrease in the pulse duration with the distance takes place according to the exponential law. Let us remind that in the absence of a resonance field at coherent propagation of USP in a TPA medium the compression of pulses with the distances proceeds (e.g. for rectangular input pulses) according to the square law [16].

The results obtained from fig.5 and concerning the existence of "fast" and "slow" subpulses are confirmed when considering the possible stationary (automodel) solutions. Let us find such solutions. Let V be the group velocity of the stationary pulse. Let us pass on to the coordinate system moving with the pulse $(z - Vt \to \xi, z \to z)$. Assuming that $A_{1,2}$

as well as R, I, η are only ξ -dependent and making elementary computations from eqs. (6),(7),(15)-(17), we obtain

$$A_1 A_2 = \frac{4\langle R_0\rangle^2 \exp\left[\frac{\langle R_0\rangle \gamma \vartheta}{\vartheta - V}(\xi - \xi_0)\right]}{(b_1^2 - 4\langle R_0\rangle^2 b_2)^{1/2}\left(1 + \exp 2\left[\frac{\langle R_0\rangle \gamma \vartheta}{\vartheta - V}(\xi - \xi_0)\right]\right) - 2b_1 \exp\left[\frac{\langle R_0\rangle \gamma \vartheta}{\vartheta - V}(\xi - \xi_0)\right]} \quad (42)$$

where $b_1 = 2g(\langle \eta_0\rangle - \langle I_0\rangle F)$, $b_2 = -g^2(1+F^2)$, $g = 2\varkappa_{12}(\vartheta - V)/\gamma\hbar V\vartheta$

As is seen from (42), the distribution of fields in the stationary pulse as well as its area ϑ depend substantially on the parameters $\langle \eta_0 \rangle$, $\langle R_0 \rangle$ and the ratio between ϑ and V . In table 1 the possible values of the area (of energy) of the stationary pulse are plotted. It is easy to see that, e.g. in the absorbing medium ($\eta_0 > 0$) the area of the "fast" subpulse ($V > \vartheta$) is indeed set equal to $2|\varphi|$, and that of the "slow" subpulse ($V < \vartheta$) to $\vartheta = 2\pi - 2|\varphi|$ (in the enhancing medium $\eta_0 < 0$ an opposite situation occurs). Note an important peculiarity of $2|\varphi|$ pulses. If the beam of the exciting field E_0 is limited in the transverse cross section, then (see (21),(30)) on moving away from the beam center $\varphi \to 0$. Then $2|\varphi|$, the pulse of SIT is also limited in the transverse cross section. In fig.6 the distribution of the field is presented in the $2|\varphi|$ pulse from the transverse coordinate r and the time, when the exciting field E_0 in the transverse cross section has a Gaussian profile. Let us remind that the pulses of SIT obtained in 5,6 , at one- and two-photon resonances are the pulses of plane waves with infinite transverse dimensions. Unlike the latter, the $2|\varphi|$ pulse may be realized in practice and the SIT effect may be observed for it along the complete cross section of the pulse. It is seen from table 1 that the $2|\varphi|$ pulse of SIT may be both "fast" ($V > \vartheta$) and "slow" ($V < \vartheta$). The simplest way to practically realize the "fast" $2|\varphi|$ pulse of SIT is to supply a pulse of small area to the input of the absorbing medium ($\eta_p > 0$). With propagation its area will increase up to $2|\varphi|$ (see fig.3a). The slow" $2|\varphi|$- pulse of SIT is formed due to the 2π pulse separation in the enhancing medium ($\eta_p < 0$) (this process is similar to that of fig.5). Generally the "slow" $2|\varphi|$ pulse may exist isolated, if $R_0 > 0$ in the medium with $\eta_p < 0$ (it corresponds to the unstable solution (33): $\Delta\psi_0 = -\operatorname{Arcsin}[a_2 a_1^{-1} \delta/\gamma \langle G_1 \rangle]$ see (24),(39)). In the latter case the medium acts on the pulse as an absorbing one.

In this section we consider the case when φ =const. It follows from the generalized theorem of areas (24), that if φ varies along z sufficiently slowly (about its mean value), the pulse area ϑ will have enough time to follow adiabatically the variation of φ . In that case all the above results obtained for φ =const will remain valid. As is seen from (24), the condition of adiabaticity has the form

$$\left|\frac{d\varphi}{dz}\right| \ll S(1-\cos\varphi).$$

5. Account of Finiteness of Relaxation Time. Stability to Relaxation

It is easy to make sure from fig.3 that the stability of

$2|\varphi|(\eta_p > 0)$ and $2\pi - 2|\varphi|(\eta_p < 0)$ pulses of SIT to external perturbations is laid in the very area theorem. Consider as such an external perturbation the medium relaxation and study its effect on the coherent propagation of USP, regarding $\tau_n \tau^{-1}$, $\tau_n T^{-1}$ as small parameters. We shall seek for a solution of the set of equations (2)-(5) (in the approximation $\Delta_\perp C_j = 0$) in the form

$$C_{1,2} = C_{1,2}^{(0)} + \Delta C_{1,2}, \sigma = \sigma^{(0)} + \Delta\sigma, \quad \eta = \eta^{(0)} + \Delta\eta \tag{43}$$

where $C_{1,2}^{(0)}, \sigma^{(0)}, \eta^{(0)}$ - are the solutions for elements of density matrix obtained above in the assumption $\tau = T = \infty$; $\Delta C_{1,2}$, $\Delta\sigma$, $\Delta\eta$ are the small corrections $(|\Delta C_{1,2}| \ll |C_{1,2}^{(0)}|, |\Delta\sigma| \ll |\sigma^{(0)}|, |\Delta\eta| \ll |\eta^{(0)}|)$. Let us integrate both parts of eq.(6) and substitute η from (43) into the right hand part of the obtained expression, preserving only the terms of the first order of smallness. We shall obtain an equation for the pulse area in the first approximation

$$\frac{\partial\vartheta}{\partial z} = S[\cos(\varphi + \vartheta) - \cos\varphi + \delta\eta] \tag{44}$$

where $\delta\eta = \langle\Delta\eta\rangle + \tau^{-1}\int_0^t (\eta^{(0)} - \eta_0)\,dt$. The correction $\Delta\eta$ is easy to determine by substituting (43) into the right hand parts of (4),(5)

$$\Delta\eta = \int_0^t \frac{t'-t}{\tau}\dot{\eta}^{(0)}dt - \frac{2\varkappa_{12}}{\hbar}\int_0^t A_1 A_2 \operatorname{Re}\left\{e^{i\psi(t')}\left[(R_0 + iI_0)(i\nu t' - t'T^{-1}) + \frac{2\varkappa_{12}}{\hbar}\int_0^t A_1 A_2 \eta^{(0)} e^{-i\psi(t'')}\left[\frac{t''-t'}{T} + i\nu(t'-t'')\right]dt''\right\}dt' \tag{45}$$

Consider the obtained equation (44). As is shown in (44), the area ϑ is stable to the perturbations $\delta\eta$ related to relaxation. Small variations in $\delta\eta$ lead only to small variations in ϑ. The stability region of the pulse area in $\delta\eta$ is limited by inequalities $\cos\varphi - 1 < \delta\eta < \cos\varphi + 1$.

The solution for ϑ from (44) will not differ much from $\vartheta^{(o)}$ and, correspondingly, $A_{1,2}$ will not differ much from $A_{1,2}^{(o)}$ (and, hence, (43) will be valid), if

$$|\delta\eta| \ll |\cos(\vartheta + \varphi) - \cos\varphi| \tag{46}$$

At $z \to \infty$ the set value of the pulse area $\vartheta_{SET} \to \vartheta^{(o)}(z=\infty) + \Delta\vartheta$. One may see from (44) that the correction is defined by equation

$$\delta\eta\,(z=\infty) = \cos(\varphi + \vartheta_{SET}) - \cos\varphi \tag{47}$$

whence

$$|\Delta\vartheta| \approx \frac{\delta\eta\,(z=\infty)}{\sin\varphi} .$$

The principle possibility of the propagation of USP at large distances in the medium at the presence of relaxation consists in the following. Due to the parametric coupling between the resonance field and transformed fields of USP, there occurs an energy pumping from $A_{01,02}$ to $A_{1,2}$ and back. If $\delta\eta > 0$ (decrease in energy due to relaxation), the pulse area takes such a value that $\vartheta_{SET} < 2\pi n + 2|\varphi|$ (or $\vartheta_{SET} > 2\pi(n+1) - 2|\varphi|$ at $\eta_p < 0$). And if $\delta\eta < 0$, vice versa, $\vartheta_{SET} > 2\pi n + 2|\varphi|$ (or $\vartheta_{SET} < 2\pi(n+1) - 2|\varphi|$ at $\eta_p < 0$) will be set. In both cases ϑ_{SET} is determined by the relation (47). In other words, the decrease or increase in the pulse energy due to relaxation will be compensated by the increase in energy respectively via mutual energy pumping between the transformed fields and resonance field. As a result, the formation of an isolated pulse, that propagates in the medium without changing its energy and is stable to relaxation, turns possible.

The condition (46) will be undoubtedly fulfilled, if

$$\Delta_m \ll |\cos(\varphi + \vartheta) - \cos\varphi| , \tag{48}$$

where $\Delta_m = t\tau^{-1} q\, |\cos(\varphi + \vartheta^{(o)}) - \cos\varphi| + 2\vartheta^{(o)}\tau T^{-1}(1 + T\tau^{-1})^{1/2} > |\delta\eta|$.

Thus, the condition (48) is the condition of stability to relaxation at coherent propagation of USP.

In conclusion of this section consider for comparison the case of "pure" two-photon absorption (enhancement), when the resonance field is lacking ($\varphi = 0$). Integrating both parts of (6) and substituting $\eta^{(o)}$ from (20) into the right hand part of the obtained expression, we have

$$\frac{\partial\vartheta}{\partial z} = \frac{\gamma\eta_o}{a}\left(\cos\vartheta - 1 + \Delta\right) \tag{49}$$

where $\Delta = \tau^{-1}\int_0^t (\cos\vartheta - 1)\,dt < 0$. It is seen from (49), that the presence of the nonzero correction Δ due to relaxation leads to the violation of the coherence of the USP propagation. In the absorbing medium ($\eta_o > 0$) the pulse completely damps in the end. In the enhancing medium ($\eta_o < 0$) an unlimited rise of the pulse area (energy) is observed. In other words, when the resonance field is lacking there occurs an instability of the SIT pulses to relaxation.

6. Stability to Transverse Inhomogeneity of Light Fields

We shall now study the effect of diffraction on the coherent propagation of USP in the excited medium. In the process of propagation of pulses their transverse distribution will vary due to the dependence of the efficient refraction index

$$n_{\text{эфф}}(\omega_j) = \sqrt{1+4\pi N \operatorname{Re} \chi_j} \tag{50}$$

where $\chi_{1,2} = \frac{1}{2}\left\{x_{1,2}^{11} + x_{1,2} + (x_{1,2}^{11} - x_{1,2}^{mm})\eta\right\} + x_{12}\sigma \frac{C_{2,1}^{*}}{C_{1,2}} e^{i(k_1+k_2)z}$ depends on the coordinates of time and intensity (in the explicit form). It has been shown that at the coherent propagation of USP in one-photon [48] as well as two-photon [39] absorbing media there occurs an instability to the transverse inhomogeneity of light beams. The nonstationary self-focusing develops in the process of propagation. In the end the diffraction results in the complete damping of the pulse [23,48]. Let us show that when the medium is excited by the resonance field, the pulses of SIT are instable to the transverse inhomogeneity, and the propagation of USP without the change in energy to large distances is possible. The situation here is very similar to that considered in section 5.

Equations (2) for the amplitudes $A_{1,2}$ and phases $\varphi_{1,2}$ will be written as

$$\frac{\partial A_{1,2}}{\partial z} + \frac{1}{K_{1,2}}\left(\frac{\partial \varphi_{1,2}}{\partial x}\frac{\partial A_{1,2}}{\partial x} + \frac{1}{2}A_{1,2}\frac{\partial^2 \varphi_{1,2}}{\partial x^2} + \frac{\partial \varphi_{1,2}}{\partial y}\frac{\partial A_{1,2}}{\partial y} + \frac{1}{2}A_{1,2}\frac{\partial^2 \varphi_{1,2}}{\partial y^2}\right) = \tag{51}$$

$$= -\frac{\pi N x_{12}\omega_{1,2}}{n_{1,2}c} A_{2,1} R,$$

$$\frac{\partial \varphi_{1,2}}{\partial z} - \frac{1}{K_{1,2}}\left[\frac{1}{A_{1,2}}\left(\frac{\partial^2 A_{1,2}}{\partial x^2} + \frac{\partial^2 A_{1,2}}{\partial y^2}\right) - (\partial \varphi_{1,2}/\partial x)^2 - (\partial \varphi_{1,2}/\partial y)^2\right] = \tag{52}$$

$$= \pi N c^{-1}\left\{\omega_{1,2} n_{1,2}^{-1}(x_{1,2}^{mm} - x_{1,2}^{11})\langle\eta\rangle + x_{12}A_{2,1}\,\omega_{1,2}/A_{1,2}n_{1,2}\langle I\rangle\right\}.$$

As in the preceding section, we shall search for a solution

for the set of equations (51),(52) in the form

$$C_{1,2} = C_{1,2}^{n} + \Delta C_{1,2} \tag{53}$$

where $C_{1,2}^{n}$ is the solution of (51),(52) in the plane wave approximation (at $\tau = T = \infty$), $\Delta C_{1,2}$ is the small addition; $|\Delta C_{1,2}| \ll |C_{1,2}^{n}|$. Let us integrate eq. (2) using (53) and we shall obtain

$$\frac{\partial \vartheta}{\partial z} = S\left[\cos(\vartheta + \varphi) - \cos\varphi - \mathcal{D}\right] \tag{54}$$

where

$$\mathcal{D} = \frac{1}{SK_1}\int_0^t \left(\frac{\partial \varphi_1}{\partial x}\frac{\partial \dot{\vartheta}_n}{\partial x} + \dot{\vartheta}_n\frac{\partial^2 \varphi_1}{\partial x^2} + \frac{\partial \varphi_1}{\partial y}\frac{\partial \dot{\vartheta}_n}{\partial y} + \dot{\vartheta}_n\frac{\partial^2 \varphi_1}{\partial y^2}\right)dt,$$
$$\vartheta_n = \vartheta(A_{1,2} \longrightarrow A_{1,2}^{n}). \tag{55}$$

Consider the obtained eq.(54). The solution for ϑ from (54) will not differ much from ϑ_n (respectively, $A_{1,2}$ will not differ much from $A_{1,2}^{n}$), if

$$|\mathcal{D}| \ll |\cos(\vartheta + \varphi) - \cos\varphi| \tag{56}$$

At $z \to \infty$ the set pulse area $\vartheta_{SET} \to \vartheta_n(z=\infty) + \Delta\vartheta$, where, as it is seen from (54), ϑ_{SET} is defined by the equality

$$\mathcal{D}(z=\infty) = \cos(\vartheta_{\text{уст}} + \varphi) - \cos\varphi \cdot \tag{57}$$

whence $|\Delta\vartheta| \approx |\mathcal{D}(z=\infty)/\sin\varphi|$. The physical condition (57) implies that the change in the pulse energy connected with diffraction is compensated by the mutual pumping of energy between the fields $A_{1,2}$ of USP and resonance field. Let us estimate the diffraction term $\mathcal{D}$ substituting the solution for the plane wave into the right hand part of (55). It has been shown in the preceding section that if the exciting field is limited in the transverse cross section, the pulses of SIT are also limited in the transverse cross section (see fig.6); their area is $\vartheta_{SET}=2|\varphi|$. Since the light pulse is always limited in cross section, the problem on the stability of pulses of SIT is of the most practical interest. Thus, estimating $\mathcal{D}$, we shall make use of the distribution of fields (40) for $2|\varphi|$ pulses. Let us assume that

$$|\langle R_o\rangle|\,,\ |\langle I_o\rangle|\ll 1 \tag{58}$$

Then $|tg\varphi|\ll 1$ (see (21)) and, as is seen from (20), R, I, η change slightly in t round their initial values R_o, I_o, η_o. If we reject the small term in the right hand part of (52) that is proportional to $\langle I\rangle$ and neglect the weak dependence of η on t , equation for φ_1 (52) will become similar to that for the phase of the wave propagation in the medium, where the refraction index is distributed according to the law

$$n_{эфф}\approx n_o(x,y)=\left(1+2\pi N Re\left[x_1^{11}+x_1^{mm}+(x_1^{11}-x_1^{mm})\eta_o(x,y)\right]\right)^{1/2} \tag{59}$$

Let us find the solution for $\partial\varphi_1/\partial r$ using the approximation of geometrical optics ($K_1\to\infty$). In this case the change of the field phase is determined by the eikonal equation [50]

$$(\vec{\nabla}S)^2=n_o^2(x,y,z) \tag{60}$$

This equation may also be deduced directly from the Maxwell equations with a refraction index (50). In (60) the eikonal is $S=c\omega^{-1}(\varphi+Kz)$. The eikonal equation is similar to the ray equation [50]

$$\frac{d}{ds}\left(n_o\frac{d\vec{r}}{ds}\right)=\vec{\nabla}n_o \tag{61}$$

where $\vec{r}$ is the radius-vector of the point on the ray, S is the distance measured along the ray. The relation between $\vec{r}$ and S is given by $n_o d\vec{r}/ds=\vec{\nabla}S$. Equation (61) has been investigated in [51] in the case when $n=n(r)$. Assuming that while entering the medium the ray is directed along the Z-axis, let us write the sought expression (omitting intermediate calculations)

$$\left|\frac{\partial\varphi}{\partial r}\right|=\frac{\omega}{c}\,\frac{n_o(r)}{n_o(r_o)}\sqrt{n_o^2(r)-n_o^2(r_o)}=\frac{\omega}{c}\,\frac{n_o(r)}{n_o(r_o)}\left(2\pi N|x^{11}-x^{mm}|\left[\eta(r)-\eta(r_o)\right]\right)^{1/2}, \tag{62}$$

where $r=r_o$ at $z=0$. If $x^{11}-x^{mm}$ is close to zero (or r is large, see fig.6), one cannot neglect the summand $(2K_1A_1)^{-1}\Delta_\perp A_1$ in eq.(52) as compared to the term proportional to $x^{11}-x^{mm}$. In this case the application of equations of geometrical optics is not consistent. The estimates obtained using typical parameters of laser pulses and gaseous media, show that (at $|x^{11}-x^{mm}|\sim 10^{-24}$ cm^3) the main contribution into the change of φ_1 in r in (52) is made by the term proportional to $x^{11}-x^{mm}$, and one may neglect the summand $\Delta_\perp A_1/2K_1A_1$ practically on the complete transverse cross section of the pulse ($r<3$).

Utilizing the above solution (39), we find

$$\frac{\partial\vartheta}{\partial r}=\frac{X\partial(ctg\frac{\varphi}{2})/\partial z+2ctg\frac{\varphi}{2}W/(X^2-1)}{1+ctg^2\frac{\varphi}{2}X^2} \tag{63}$$

where $W=(4a_+a_-)^{-1}\{a_+[\partial a_-/\partial r+za-\partial\mathcal{D}/\partial r]-a-\frac{\partial a_+}{\partial r}\}$,

$a_+=\psi_0+ctg\,\varphi/2;\ a_-=\psi_0-ctg\,\varphi/2;\ X=[a_++a_-\exp(\mathcal{D}z)]/[a_+-a_-\exp(\mathcal{D}z)]$.

Substituting the obtained expressions (62) and (63) into (55), one may easily estimate

$$|\mathcal{D}|<\mathcal{D}_m=S^{-1}\sqrt{N|x^{11}-x^{mm}V2\pi}\ r_0^{-1}\left(\tfrac{1}{2}|\partial\varphi/\partial r|+2|\varphi|\right), \qquad (64)$$

where is the characteristic radius of the beam of the exciting resonance field (taking into account that). If we assume that the diffraction factor weakly affects the pulse propagation, when its area is small *( ), then to fulfil the stability conditions (56),(57) of the plane wave approximation it would be sufficient to require that

$$\mathcal{D}_m\ll 1-\cos\varphi \qquad (65)$$

It is most interesting to find out, when the pulse inhomogeneous in the transverse cross section (in the form shown in fig. 6), will propagate for large distances without changing its form. Let us carry out numerical estimations. Let the resonance exciting field propagate in a medium with the induced dipole moment (the constant electric field changes according to the law (35)). Let us assume the transverse distribution of the intensity in the beam of the field A_{01} to be Gaussian. At $2\hbar^{-1}\sqrt{\tau T}\,dA_{01}(r=0)=1$ we shall obtain from (30)-(32)

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* It is shown in refs. [49,52] that at two-photon absorption the influence of the diffraction on the USP propagation, where the pulse area is small, is weak, and in the SIT 2π - pulse generation region it is essential.

$$\langle R_0\rangle=-2(T\tau^{-1})^{1/2}e^{-\pi r^2/2}/(1+e^{-\pi r^2});\ \langle\eta_0\rangle=(1+e^{-\pi r^2})^{-1}; \qquad (66)$$
$$\langle I_0\rangle=0$$

(for simplicity we assume $\delta=0, g(\nu)=\delta(\nu)$). At $\tau\sim10^{-7}$sec, $T\sim10^{-9}$sec (the times characteristic for gaseous media), we obtain from (64),(65)

$$(r_0\gamma)^{-1}\sqrt{N|x^{11}-x^{mm}V2\pi}\,(\pi r+4)\,\mathrm{ch}(\pi r^2/2)\ll 0{,}1 \qquad (67)$$

From (67) as well as from (64) one may see that far enough from the beam axis, where r is not small, the conditions (67) as well as (64) will be violated. The physical essence of this fact is the following. The resonance field is small far from the axis. It already can not compensate the diffraction so, that the distribution of fields remain close to (42). Thus inequalities (67),(64) determine the region round the axis of the beam of radius r_n, where the pulse envelope will be close to (42) (obtained for the plane wave). Beyond the limits of this region ($r>r_n$) will, apparently, also be established the stationary field distribution satisfying the condition of the type (57), which may considerably differ from (42). If we take the typical parameters $r_0=3$ mm, $x_{12}\sim10^{-22}$ cm^3, $|x^{11}-x^{mm}|\sim10^{-22}$ cm^3, $\gamma\approx5$ cm^{-1}, then $r_n\approx\sqrt{2}$. That is, the established field distribution, considering diffraction, will be close to (42) practically for the complete pulse, excluding the lateral "tails", where the intensity is hundred times less than at the maximum.

In a similar way, as is done in sections 5 and 6, it is, apparently, possible to prove the stability of SIT pulses to the inhomogeneous line broadening.

7. Description of Experiment

Let us take as a resonance medium the vapor of Cs; the operation levels are $6^2S_{1/2}-9^2D_{3/2}$. The frequency of the transition ω_{m1} = 28818.90 cm^{-1} coincides with the doubled frequency of the ruby laser, if the ruby sample is frozen to $T \approx 148^0$ K. Thus, the second harmonic of the ruby laser may be used as the resonance exciting field. The layout of the experimental setup is given in fig.7. As a pulse triggering source both picosecond solid state lasers (and parametric light generators constructed on their basis) and lasers on crystals may be used. In order to increase the polarizability of two-photon transition, the frequency ω_1 of the triggering field may be chosen close to that of the intermediate transition $6^2S_{1/2}-6^2P_{3/2}$ (or $6^2P_{3/2}-9^2D_{3/2}$). The phase synchronism in the system is provided by introducing buffer gas in the cell with the vapor of Cs [53]. The cell with cesium vapor is placed in the electric field; the induced dipole moment is $d=\varkappa_{20}\mathcal{E}_0 \approx 10^{-21}$ CGSE ($\varkappa_{20}\approx 10^{-23}$ cm^3, $\mathcal{E}_0 \approx 100$ kV/cm). The polarizability of TPA is $\varkappa_{12}\approx 10^{-22}$ cm^3 at frequencies $\omega_1 \approx 17000$ cm^{-1}, $\omega_2 \approx 11819$ cm^{-1}. Relaxation times are $\tau \sim 10^{-7}$ sec, $T \sim 10^{-8}$ sec; the density of vapor is N=10^{16}cm^{-3}, $K \approx 0.06$ cm^{-1}, $D \approx 1.6$ cm^{-1}, $\gamma \approx 5.4$ cm^{-1}. At the power of the exciting resonance field 0.31 MW/cm^2 ($A^2_{01}(z=0) \approx 625$ CGSE) $\varphi \approx -0.25\pi$. Let a triggering pulse be supplied to the cell input at the frequency ω_1 of the power 155 kW/cm^2 ($A^2_{10} \approx 300$ CGSE) and duration $\tau_n \approx 30$ psec. In that case, on the

length ≈ 3.3 cm a proportional field pulse is generated at frequencies ω_1, ω_2 . The area of such a pulse ϑ approximates $2|\varphi|$ ($\vartheta \approx 0.95\ 2|\varphi|$) on the length ≈ 5.3 cm, i.e. an interaction regime is established close to SIT. With the further propagation the pulse compresses: on the length 13.3 cm the pulse duration diminishes by a factor of 5. The estimates show that noticeable variations in the pulse energy and duration occur on the lengths, where the resonance field varies slightly. In that case the constant along Z electrostatic field $\mathcal{E}_0$ may be used. In order to fulfil the condition $\varphi = const$ on large lengths the $\mathcal{E}_0$ should increase in z in accordance with (35) ($K \approx 0.06$ cm^{-1}). In order to compensate the decrease in the resonance field A_{01} from z , one may also make use of the focusing of the ruby laser radiation into the cell (at a constant $\mathcal{E}_0$). The estimates show that for the pulse with the Gaussian distribution of intensity in transverse cross section and characteristic radius $\tau_0 \approx 3$ mm ($|\varkappa^{11}-\varkappa^{mm}| \sim 10^{-22}$cm^3, the other parameters are presented above in this section), the condition of stability to the transverse inhomogeneity of light beams (65) is fulfilled practically for the complete pulse, excluding the regions far from the beam axis ($\tau \geqslant \sqrt{3}$), where the intensity is four orders lower than in the maximum (see (42)). One may easily see that (at $\tau \sim 10^{-7}$sec, $T \sim 10^{-8}$sec, $\tau_n \approx 30$ psec) in this very region ($\tau \leqslant \sqrt{3}$) is fulfilled the condition of stability (48) of pulses of SIT to the medium relaxation.

8. Conclusion

In the present paper we have studied the resonance interaction of ultrashort pulses of light in the medium with two-photon transitions in the presence of an exciting field with a frequency resonant to that of transition.

1. The area (energy) of the pulse of self-induced transparency in such systems is shown to be $\vartheta = 2\pi n + 2|\varphi|$ (n = 0,1,2,...), where φ is determined by the polarization and difference in populations, induced in the medium with an exciting field.

2. In the process of propagation in the medium the 2π pulse divides into two subpulses: the first ("fast") travels at a group velocity $V > \upsilon \approx c/n$ and has an area equal to $2|\varphi|$, and the second ("slow") has the group velocity $V \leqslant \upsilon \approx \frac{c}{n}$ and area $2\pi - 2|\varphi|$. In the general case the $2\pi n$ pulse divides into n + 1 subpulses, whose duration decreases and the strength increases with the distance.

3. The compression rate of such subpulses, determined by the dependence of the pulse duration and maximum intensity on the distance, is in the considered interaction higher than that of analogous subpulses at TPA in the absence of the resonance field (for example, for rectangular input pulses, in the first case the subpulses compress with the distance according to the exponential law, whereas in the second case, according to the square law).

4. It is shown that in the presence of the exciting field the pulses of SIT may be stable to the medium relaxation and transverse inhomogeneity of light fields. The stability regions are found.

5. It has been found out that the $2|\varphi|$ pulses of SIT may be generated from the small triggering field, independent of the phase modulation of triggering field. The energy of the $2|\varphi|$ pulse may be continuously varied with the variation of the exciting field. The aperture of the $2|\varphi|$ pulse is limited at the limited aperture of the exciting field. With propagation the $2|\varphi|$ pulse compresses. The $2|\varphi|$ pulses of SIT (in the form (42)) are shown to be stable to the relaxation and transverse inhomogeneity of light fields for the typical radiation characteristics and medium parameters.

Thus, the considered interaction may be used for the compression of ultrashort pulses as well as to observe the SIT at two-photon absorption (and related coherent phenomena) from pulses with small power and phase modulation, for which, in the ordinary case of TPA, no coherent phenomena are observed.

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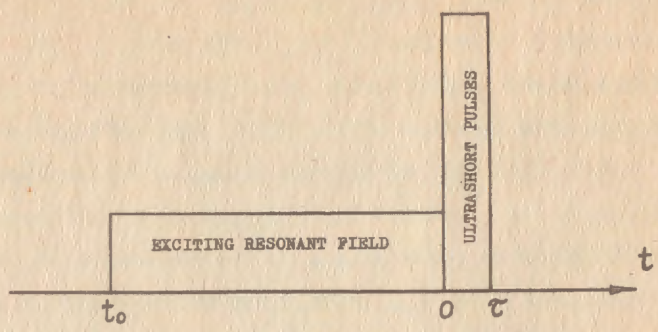

Fig.1 Time diagram of interaction.

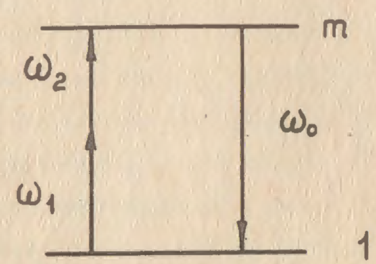

Fig.2 Frequency diagram of interaction.

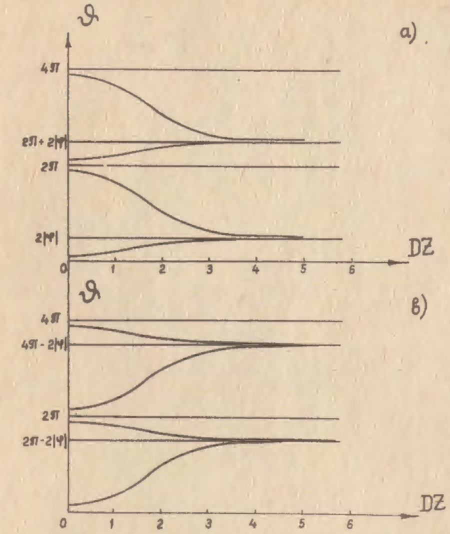

Fig.3 Dependence of the pulse area ϑ on the distance calculated by the formula (39): a) in noninverted medium ($\eta_p > 0$); b) in inverted medium ($\eta_p < 0$); $|\varphi| = 0.25\pi$

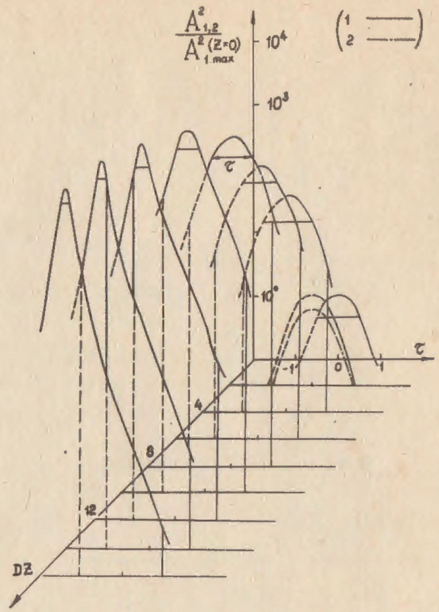

Fig.4 Evolution of the pulse of transformed fields $\dot{A}_1$, A_2 (at $\eta_p < 0$), if a triggering pulse with the Gaussian distribution of intensity is supplied to the medium input: $A_1^2(z=0)=A_{1m}^2(z=0)\exp(-\pi\tau^2)$. The field $\dot{A}_2$ at the input into the medium is lacking. In calculation the following parameters have been used: $\varphi = 0.36\pi$, $\tau\sim T$, $d = 3\ 10^{-21}$ CGSE, $\varkappa_{12}\sim 10^{-24}$ cm^3, $\varkappa^{mm}-\varkappa^{11}\sim 10^{-24}$ cm^3, $\omega_1\simeq\omega_2$, $\delta = 0$, the exciting field power is ~ 500 W/cm^2; the triggering pulse power is 0.5 MW/cm^2 at the pulse duration $\tau_n\sim 30$ psec.

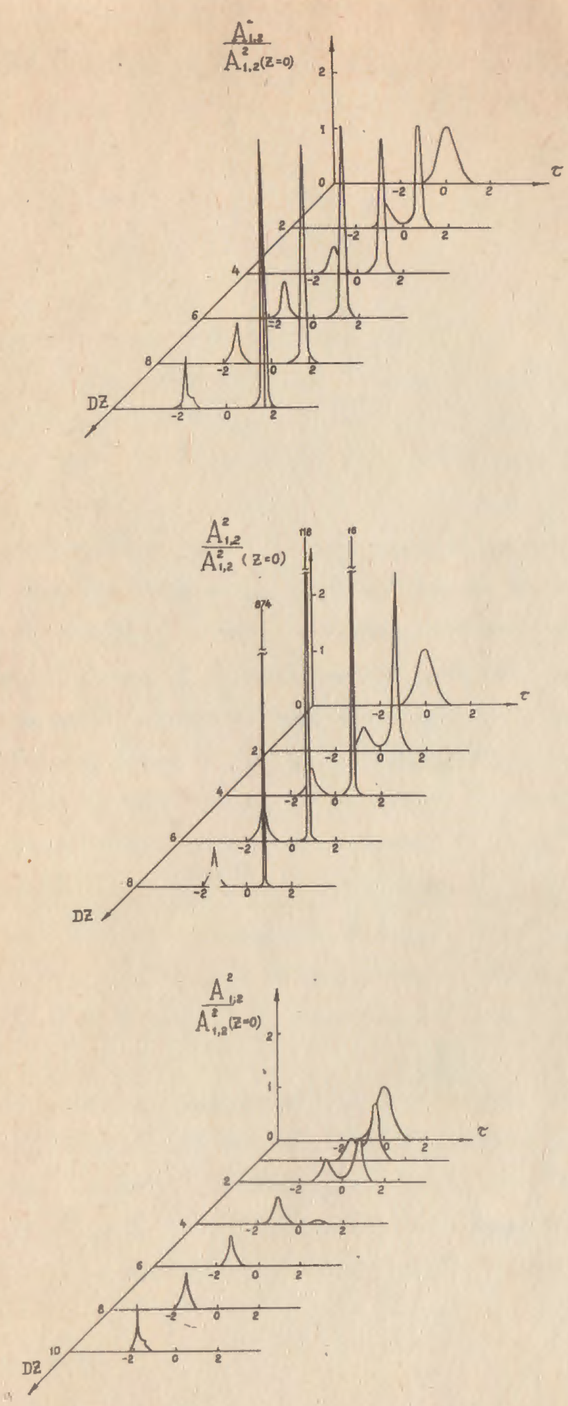

Fig.5 Evolution in space and time of the pulse with t. Gaussian envelope in the absorbing medium ($\eta_p > 0$);

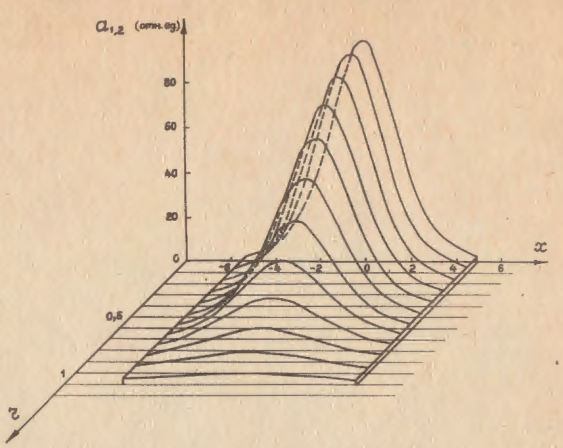

Fig.6 Distribution of the fields $a_{1,2}$ in the $2|\Psi|$ pulse in the transverse coordinate τ and the time ($x=\gamma\vartheta(\xi-\xi_0)/(\vartheta-V)$), when the exciting field beam has the Gaussian distribution of intensity in the transverse cross section ($E_0=E_0^m\exp(-\frac{\pi\tau^2}{2})^2$).

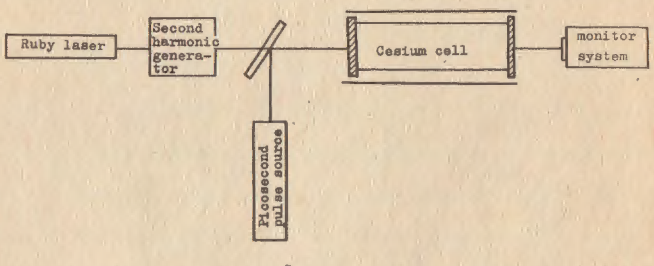

Fig.7 Proposed layout of the experimental setup.

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В.С.ГРИГОРЬЯН

ВЗАИМОДЕЙСТВИЕ УЛЬТРАКОРОТКИХ ИМПУЛЬСОВ СВЕТА В СРЕДЕ С ДВУХФОТОННЫМИ ПЕРЕХОДАМИ В ПРИСУТСТВИИ РЕЗОНАНСНОГО ПОЛЯ

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