индекс 3624

ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

ЕФИ-692(7)-84

ЦЕНТРАЛЬНЫЙ НАУЧНО-ИССЛЕДОВАТЕЛЬСКИЙ ИНСТИТУТ ИНФОРМАЦИИ И ТЕХНИКО-ЭКОНОМИЧЕСКИХ ИССЛЕДОВАНИЙ ПО АТОМНОЙ НАУКЕ И ТЕХНИКЕ

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ON PECULIARITIES OF FORMATION OF C-EVEN HEAVY QUARKONIA IN HADRON COLLISIONS

ЕРЕВАН-1984

1. Introduction.

The place occupied in quantum chromodynamics (QCD) by the physics of "quarkonia" - mesons composed of heavy quarks $Q\bar{Q}$ - is usually compared with the role of the hydrogen atom in the construction of quantum mechanics (see, e.g. review [1]). From this point of view, the family of P-wave levels of the $Q\bar{Q}$ system ($L = S = 1$) stands in a special position. The comparison of the connection of three quarkonia $\chi^{(J)}$ $(|\vec{J}| = |\vec{L} + S| = 0, 1, 2)$ with the world of usual light hadrons allows one to obtain a certain information on the nature of confinement. The point is that in the framework of "naive QCD" dealing with gluons as with real particles, decays $\chi^{(0)}$ and $\chi^{(2)}$ into light hadrons proceed mainly via annihilation of the $Q\bar{Q}$ pair into two gluons, whereas for the $\chi^{(1)}$ meson such decay is forbidden by the Landau-Yang theorem. According to this theorem, the axial particle cannot transform into two massless vector particles. This specific veto has absolutely different status than, for example, the veto $J/\psi \to 2g$: in the case of $\chi^{(1)}$ masslessness of gluons is essential, the presence of only two physical polarizations of the gluon field. Hence the study of the peculiarities of 1^{++} quarkonium is an important point in clearing up a delicate question: to what extent the "virtual" existence of gluons scattering from small distances $r \sim 1/M_{Q\bar{Q}}$ and transforming then into light hadrons is similar to real being of massless vector particles?

The present work deals with a discussion of specific features in cross sections of χ -particle production in hadron collisions, connected with the manifestation of the Landau-Yang rule and the nonrelativistic nature of quarkonia. We will consider m_Q a parametrically large quantity ($\alpha_s(m_Q^2)/\pi \ll 1$) attributing our treatment to particles of $b\bar{b}$ and future $t\bar{t}$ families, although, with certain reserve, qualitative consequences may manifest themselves also in the spectra of mesons with hidden charm ($c\bar{c}$), which are being studied experimentally.

2. Influence of Higher Corrections of Perturbation Theory on χ -Meson Spectra (the Gluon Formfactor).

From QCD viewpoint, hadroproduction of quarkonia is one of the hard processes, where applicability of perturbation theory is guaranteed by the smallness of the characteristic time τ needed for the conversion of partons from the composition of colliding hadrons into a pair of heavy quarks $Q\bar{Q}$: $\tau \sim 1/M_{Q\bar{Q}} \ll \mu^{-1}$, where μ is the characteristic scale of strong interactions - of the order of a few hundred MeV. Here the process of formation of C-even meson χ due to two gluon fusion $gg \to \chi$ (see Fig.1a,b) is in close analogy with the Drell-Yan process (Fig.1c). The total cross sections of these processes are determined mainly by the wide region of transverse momenta $\mu^2 \ll q_\perp^2 \ll q^2 (= M_{Q\bar{Q}}^2)$. where both spectra are proportional: $d\sigma/dq_\perp^2 \backsim \sigma \cdot (\bar{\alpha}_s/q_\perp^2)$. This simple result follows from the treatment of the process in the first order in $\bar{\alpha}_s$, when $q_\perp$ of the lepton pair or χ -meson is compensated

by a parton (e.g. by a gluon, as shown in Fig.1a,c). The logarithmic character of the spectrum is connected with the "quasi-reality" of the parton $K (|K^2| \sim q_\perp^2 \ll q^2)$. As was mentioned in [2] , this means that if $q_\perp$ is registered only, the quantity $q_\perp^2$ and not the total mass q^2 is the real measure of hardness of the process.

Two parametrically different scales of characteristic distances $1/q \ll 1/q_\perp$ arise in the problem, and as a consequence the spectrum of Drell-Yan pairs is essentially modified, when higher orders of perturbation theory are accounted. The differential cross section acquires a factor - square of the effective twice logarithmic quark formfactor (see, e.g. [3][5])

$$\frac{d\sigma}{dq_\perp^2} \backsim \sigma \cdot \frac{\alpha_s}{q_\perp^2} T_F^2 (q_\perp^2 , q^2), \tag{1}$$

whose origin is connected with the dynamic veto of soft gluon bremsstrahlung in the interval of transverse to the collision axis distances $\rho_\perp$ from $\rho_\perp \gtrsim 1/q$ to $\rho_\perp \lesssim 1/q_\perp$. In the range

$$\frac{C_F \alpha_s (q_\perp^2)}{\pi} \cdot \ln \frac{q^2}{q_\perp^2} < 1 \tag{2}$$

the following approximate formula

$$T_F^2 (q_\perp^2, q^2) = \exp\left\{-C_F \int_{q_\perp^2}^{q^2} \frac{d k_\perp^2}{k_\perp^2} \cdot \frac{\alpha_s (k_\perp^2)}{\pi} \ln \frac{q^2}{k_\perp^2} \right\} = \tag{3}$$
$$= \exp\left\{-\frac{4C_F}{b} \cdot \left[\ln \frac{q^2}{\Lambda^2} \cdot \ln \left(1 - \frac{\ln q^2/q_\perp^2}{\ln q^2/\Lambda^2}\right) - \ln \frac{q^2}{q_\perp^2}\right]\right\},$$

holds, where $b = 11 - \frac{2}{3} n_f$ enters into the expression for the running α_s via $\alpha_s (Q^2) = 4\pi/b \cdot \ln (Q^2/\Lambda^2)$; $C_F = 4/3$.

At decreasing $q_\perp^2$ the expression in the exponent (3) increases.

which results in a more smooth distribution $d\sigma/dq_\perp^2$ than in the first order of PT ($\backsim 1/q_\perp^2$). When $q_\perp^2$ decreases so much that inequality (2) turns into equality, the formfactor suppression compensates completely the pole growth of the Born cross section. This takes place, as one may readily see, at

$$q_\perp = q_{\perp 0} = \Lambda (q/\Lambda)^{\frac{c}{1+c}} \qquad \text{where} \quad c = \frac{4C_F}{b} \tag{4}$$

As a result of that, at $q_\perp < q_{\perp 0}$ the cross section becomes independent of $q_\perp$ [6] - a "plateau" arises whose width corresponds to the value of average (or, better to say, characteristic) transverse momenta of leptonic pairs with a given mass q^2 (for details, see [3]).

As was mentioned in Ref.[7] , a similar phenomenon takes place also for χ -meson production. An essential difference consists in the fact that now the problem involves a different twice logarithmic factor, namely the square of gluon formfactor T_G , the presence of which smears the peak in $d\sigma/dq_\perp^2$ much stronger. This is explained by the larger intensity of soft gluon bremsstrahlung "annihilating" by gg rather than by quarks $q\bar{q}$. For the spectrum of C-even quarkonia in the process $gg \to \chi \ldots$ one should in formulae (1)-(4), written for the quark case (Drell-Yan process $q\bar{q} \to \gamma^* + \ldots$) replace $C_F = 4/3$ by $C_V = 3$. The comparison of the shape of spectra for leptonic pairs and χ_b mesons with $q^2 = M_{bb}^2 \approx$ $\approx (10\ \text{GeV})^2$ is given in Fig.2.

Using relation (4) we establish that characteristic $q_\perp$ of χ_b mesons in gg and also in $q(\bar{q})g$ collisions must be about 2.5 times larger than $q_\perp$ of lepton pairs with the same invariant mass ($q \approx 10$ GeV)

$$\frac{(q_{\perp 0})^{\chi_b}}{(q_{\perp 0})^{D.Y.}} = \left(\frac{q}{\Lambda}\right)^{\frac{4(C_V - C_F)\cdot b}{(b+4C_V)(b+4C_F)}} \approx 2{,}5 \,.$$

Such a significant difference of spectra predicted by QCD gives a unique possibility to study with the use of χ -quarkonia the chromodynamic gluon self-action whose intensity determines the QCD-formfactor T_G .

This circumstance is highly important also from the practical viewpoint, since the wide distribution in $q_\perp$ can simplify the background conditions for detection of these particles in contemporary experiments searching for χ_b (and later χ_t) mesons in hadronic reactions.

3. A Phenomenological Approach to Description of the Amplitudes $\chi^{(1)} \to 2g$, $\chi^{(1)} \to 3g$.

The above given formulae correspond to the processes of two-gluon formation of only two of the three χ -particles, namely of $\chi^{(0)}$ and $\chi^{(2)}$ The point is that the spectrum of $\chi^{(1)}$ mesons in the region $q_\perp \ll M_\chi$ must not depend on $q_\perp$, contrary to the case of $\chi^{(0),(2)}$, whose spectra grow rapidly at $q_\perp \to 0$ and formally in the lower order of PT have a pole singularity $\backsim 1/q_\perp^2$ which smoothens, as was explained in sec.2, after accounting for higher gluon corrections. A flat spectrum $d\sigma/dq_\perp^2$ of axial meson $\chi^{(1)}$ is a consequence of the Landau-Yang theorem forbidding a real decay $\chi^{(1)} \to 2g$. In the crossing channel of χ production this veto manifests itself in the fact that vanishing of the amplitude $\chi^{(1)} \to 2g$ for physical polarizations of real gluons must compensate the pole $1/k^2$ in Fig.1a, b diagrams connected with the gluon propagator k which becomes quasi-real in the limit $q_\perp^2 \backsim |k^2| \to 0$. To demonstrate how this compensation is realized, we shall construct a vertex function $\Gamma_{\xi,\mu\nu}(k_1, k_2)$ of the transition $\chi^{(1)} \to 2g$, where ξ and $\mu\nu$ are vector symbols of χ -state and gluons with momenta K_1 and K_2 , respectively. Tensor Γ must satisfy the requirements

i) identity of gluons: $\Gamma_{\xi,\mu\nu}(K_1, K_2) = \Gamma_{\xi,\nu\mu}(K_2, K_1)$

ii) conservation of current: $K_{1\mu}\Gamma_{\xi,\mu\nu} = K_{2\nu}\Gamma_{\xi,\mu\nu} = 0$,

and must also include a completely antisymmetric tensor $\varepsilon_{\alpha\beta\gamma\delta}$ in order to ensure the pseudo-vector nature of the χ -state.

Let us consider as an example the following structure:

$$\Gamma_{\xi,\mu\nu} = A\left[\varepsilon^{\mu\alpha\beta\xi} K_{1\alpha}\left(K_2^2 g_{\beta\nu} - K_{2\beta}K_{2\nu}\right) + \varepsilon^{\nu\alpha\beta\xi} K_{2\alpha}\left(K_1^2 g_{\beta\mu} - K_{1\beta}K_{1\mu}\right)\right] \tag{5}$$

Expression (5), as can easily be verified, possesses all the required properties (a complete analysis of possible phenomenological vertices will be given in the sequel). The matrix element of two-gluon decay, which is proportional to $\Gamma_{\xi,\mu\nu}(K_1, K_2)e_{1\mu}e_{2\nu}$, vanishes at $K_1^2 = K_2^2 = 0$ for the physical polarizations of gluons $(e_1K_1) = (e_2K_2) = 0$, which agrees with the Landau-Yang theorem.

Substituting (5) as vertex functions into Fig.1a,b diagrams, we are convinced that in (5) works one of the two terms (e.g. only the first one), if we take $K_2 = K$, and identify K_1 with the external real gluon - parton. Further on, in virtue of current conservation, the term proportional to ($\sim K_\beta K_\nu$) also drops out, and hence each vertex Γ becomes effectively proportional to K^2 , which, as was discussed above, leads to the flat $q_\perp$ distribution.

For a complete solution of the problem of χ particle $q_\perp$ distribution, the knowledge of the two-gluon vertex $\chi \to 2g$ is insufficient, since in the same order of PT, vertices $\chi \to 3g$ are also essential as shown in Fig.3, where diagrams for cross sections of the process $gg \to \chi + g$.

If in the analysis of spectra $\chi^{(0)}$ and $\chi^{(2)}$ one could restrict himself only to the first two diagrams dominating in the region $q_\perp^2 \ll M_\chi^2$, then in case of $\chi^{(1)}$ all the contributions turn out to be of the same order.

The required information on the vertices can be derived from the knowledge of the internal structure of χ , decoding the coupling of χ with gluons via the amplitudes of annihilation of Q and $\bar{Q}$ nonrelativistic quarks into two and three gluons. This approach is however somewhat complicated technically: while the amplitude of $\chi \to$ two virtual gluons is known (see, e.g. [8]), the analysis of the $\chi = Q\bar{Q} \to 3g$ decay is absent so far in the literature. Below, we shall return back to the consideration of three-gluon decay of χ within the framework of nonrelativistic approach (whose amplitude for the $\chi^{(1)}$ case is calculated in the Appendix).

Here we shall discuss another, much more simpler semi-phenomenological approach, which uses the connection between two- $\Gamma_{\xi,\mu\nu}(K_1, K_2)$ and three- $\Gamma_{\xi,\mu\nu\lambda}(K_1, K_2, K_3)$ gluon vertices of χ . Its essence is as follows.

Let us consider one of the simplest $\chi \to 2g$ vertex, e.g. expression (5) for $\chi^{(1)}$, and take A = const in it as a phenomenological parameter whose value will aposteriori be defined from the hadronic decay width.

Then, let us construct the total amplitude $M(\chi \to 3g)$ containing both $\Gamma_{\xi,\mu\nu\lambda}$ and the products of $\chi^{(1)} \to 2g$ and $g \to gg$ vertices and make use of current conservation, for example

$$\ell_\lambda \cdot M_{\xi,\mu\nu\lambda}(K_1, K_2, \ell)e_{1\mu}e_{2\nu}\Big|_{\substack{K_1^2 = K_2^2 = 0 \\ \ell - \text{any}}} = 0 \tag{6}$$

The connection between $\Gamma^{(3)}$ and $\Gamma^{(2)}$ arising here together with the antisymmetry requirement $\Gamma_{\xi,\mu\nu\lambda}$ relative to simultaneous permutations of momenta and polarizations of any pair gluons * turn out sufficient to restore the form of straight $3g$ vertex of χ .

Combining the $\chi \to 2g$ vertex with the decay of virtual g into a $q\bar{q}$ pair, one can finally express the amplitudes and partial widths of three-body decays of $\chi^{(1)}$ into three real gluons or $gq\bar{q}$ via the phenomenological parameter $g_s \cdot A$

However such an approach, which completely proved its value in analyzing the spectra $\chi^{(0)}$ and $\chi^{(2)}$, fails, in some strange way, to work in the most interesting to us case of $\chi^{(1)}$ meson: the sum of Fig.3 diagrams (for the cross section of $\chi^{(1)}$ production in the gluon channel) calculated by using (5) as $\Gamma^{(2)}$ and reconstructed by current conservation $\Gamma^{(3)}$ turn out to be identically equal to zero. Another choice of $\Gamma^{(2)}$, not equivalent to (5), also leads to zero result for the cross section (as one can easily see, there exist only two independent amplitudes $\chi^{(1)} \to 2g$ corresponding to the projections ±1 and 0 of χ -spin on the decay axis). We shall discuss this negative result in more detail than it deserves, since the reason of the unexpected vanishing of the physical amplitude $\chi^{(1)} \to 3g$ is connected with the existence of a curious identity being of independent interest.

4. No-Go Theorem for a Minimum Local Effective Lagrangian $\chi^{(1)} \to$ Gluons.

The above-described procedure of restoring $\Gamma^{(3)}$ by $\Gamma^{(2)}$ is in fact equivalent to the description of the χ decay into gluons within the frame-

* Together with antisymmetry over colored indices (if_{abc}) (which is dictated by the requirement of C-evenness of χ) this agrees with the Boze statistics of gluons.

work of a local effective Lagrangian with the density

$$\mathcal{L}_{eff}(x) = \chi_\xi(x)\mathcal{M}_\xi(x) , \tag{7}$$

where $\mathcal{M}(x)$ is a colorless pseudovector gauge-invariant operator composed of gluon fields. The operator $\mathcal{M}$ must be quadratic in $G_{\alpha\beta}$ (in order to describe the virtual transition $\chi^{(1)} \to 2g$), must contain the ε -tensor or at least one derivative D_γ (as D_γ one should imply either a covariant derivative, if it acts on one of the two tensors G , or a usual derivative ∂_γ if a colorless expression is differentiated).

Let us consider all operators of the form

$$\mathcal{M}_\xi = \mathrm{Tr}\left\{ D_\sigma G_{\mu\nu} G_{\pi\rho} \varepsilon_{\alpha\beta\gamma\delta} \right\}_\xi , \tag{8}$$

where Tr denotes summation over colors and internal contraction over four pairs of vector indices at one external index ξ is implied. The structure (8) potentially contains vertices with any number of gluons, from two to five.

Let us show that despite this, a real decay of $\chi^{(1)}$ into gluons is absent: chromodynamical multiplication of gluons reduces the contribution of direct multigluon vertices defined by Lagrangian (7), (8) for any number of real gluons (and, in particular, for three, as shown in Fig.3). Let us list now the possible structures of operator (8). Since we are interested here in the coupling of $\chi^{(1)}$ with real gluons, we may omit the expressions

$$D_\mu G_{\alpha\beta} \varepsilon_{\mu\nu\alpha\beta} = D_\mu \tilde{G}_{\mu\nu} \qquad D_\mu G_{\mu\nu} ,$$

the first of which vanishes due to the equation of motion, and the second one reduces to quark current $J^a_\mu(x) = \bar{q}(x) t^a \gamma_\mu q(x)$ generating $q\bar{q}$ + gluons state. In virtue of this observation the only candidates for the role of $\mathcal{M}_\xi$ remain

1) $$\mathcal{M}^{(1)}_\xi = Tr\{G_{\alpha\beta}\cdot(D_\xi \tilde G_{\alpha\beta})\} = \frac{1}{2}\partial_\xi Tr\{G_{\alpha\beta}\tilde G_{\alpha\beta}\},$$

2) $$\mathcal{M}^{(2)}_\xi = Tr\{G_{\alpha\beta}\cdot(D_\mu G_{\beta\gamma})\}\varepsilon_{\mu\xi\alpha\gamma}, \tag{9}$$

3) $$\mathcal{M}^{(3)}_\xi = Tr\{G_{\mu\alpha}\cdot(D_\mu \tilde G_{\alpha\xi})\} \doteq Tr\{\tilde G_{\mu\alpha}(D_\mu G_{\alpha\xi})\} = \partial_\mu Tr\{G_{\mu\alpha}\tilde G_{\alpha\xi}\},$$

where the sign $\doteq$ in 3) denotes equivalence in the sense of equations of motion.

In deriving the last equality, the property

$$Tr\{G_{\mu\alpha}\tilde G_{\alpha\xi}\} = Tr\{\tilde G_{\mu\alpha}G_{\alpha\xi}\},$$

is used, which follows from the relation

$$G^a_{\mu\alpha}\tilde G^b_{\alpha\xi} = G^a_{\xi\alpha}\tilde G^b_{\alpha\mu} + 2\varepsilon_{\mu\xi\alpha\gamma}G^a_{\alpha\beta}G^b_{\beta\gamma}, \tag{10}$$

and the observation that $Tr\{G_{\alpha\beta}G_{\beta\gamma}\}\cdot\varepsilon_{\xi\mu\alpha\gamma}=0$.

The relation (10), in turn, follows from a rather useful identity which we shall often exploit in what follows:

$$g_{\mu\nu}\varepsilon_{\alpha\beta\gamma\delta} = g_{\mu\alpha}\varepsilon_{\nu\beta\gamma\delta} + g_{\mu\beta}\varepsilon_{\alpha\nu\gamma\delta} + g_{\mu\gamma}\varepsilon_{\alpha\beta\nu\delta} + g_{\mu\delta}\varepsilon_{\alpha\beta\gamma\nu}. \tag{11}$$

We now present a simple proof of this identity and consider the derivation of eq.(10) as an example of its utilization.

Let a_μ and b_μ be 4-vectors of general position.

Consider a tensor

$$\Pi_{\alpha\alpha'} = g_{\alpha\alpha'} - \frac{b_\alpha a_{\alpha'}}{(ab)}, \quad a_\alpha\Pi_{\alpha\alpha'}=0 \tag{12}$$

and act with it as an operator on each of the indices of the :

$$\Pi_{\alpha\alpha'}\Pi_{\beta\beta'}\Pi_{\gamma\gamma'}\Pi_{\delta\delta'}\varepsilon_{\alpha'\beta'\gamma'\delta'}=0 \tag{13}$$

The right-hand side is zero, since we have arrived again at a completely antisymmetric tensor, each of the 4 indices of which now span the three-dimensional subspace, orthogonal to a_μ (see (12)). Using the explicit form of (12) we shall get

$$0=\frac{1}{(ab)}\cdot\left[(ab)\varepsilon_{\alpha\beta\gamma\delta} - (b_\alpha\varepsilon_{\nu\beta\gamma\delta} + b_\beta\varepsilon_{\alpha\beta\nu\delta} + b_\gamma\varepsilon_{\alpha\beta\nu\delta} + b_\delta\varepsilon_{\alpha\beta\gamma\nu})\cdot a_\nu\right] \tag{14}$$

and from this, due to the arbitrariness of a and b, the identity (11).

Let us apply (11) to the expression $G_{\mu\alpha}\cdot\tilde G_{\alpha\xi} = G_{\mu\alpha}\varepsilon_{\alpha\xi\beta\gamma}G_{\beta\gamma}$:

$$G_{\mu\alpha}\varepsilon_{\alpha\xi\beta\gamma} = G_{\alpha\alpha}\varepsilon_{\mu\xi\beta\gamma} + G_{\xi\alpha}\varepsilon_{\alpha\mu\beta\gamma} + G_{\beta\alpha}\varepsilon_{\alpha\xi\mu\gamma} + G_{\gamma\alpha}\varepsilon_{\alpha\xi\beta\mu}. \tag{15}$$

Contracting (15) with $G_{\beta\gamma}$ and after transformation of summation indices, we shall obtain relation (10).

Let us return now back to the discussion of structures (9).

The first of them, $\mathcal{M}^{(1)}_\xi$, evidently cannot be accepted, since the density $\mathcal{L}_{eff}(x) = \partial_\xi[\chi_\xi Tr\{GG\}]$ corresponding to it is a complete divergency on the physical polarizations of the field χ.

Further on,

$$\mathcal{M}^{(2)}_\xi \doteq \frac{1}{2}\mathcal{M}^{(3)}_\xi \tag{16}$$

Indeed, applying (11) to $\mathcal{M}^{(2)}$ from (9) we shall obtain

$$\begin{aligned}\mathcal{M}^{(2)}_{\xi} &= Tr\left\{G_{\alpha\beta} D_{\mu}\left(G_{\beta\gamma}\varepsilon_{\mu\xi\alpha\gamma}\right)\right\} = \\ &= Tr\left\{G_{\alpha\beta} D_{\mu}(G_{\mu\gamma}\varepsilon_{\beta\xi\alpha\gamma} + G_{\xi\gamma}\varepsilon_{\mu\beta\alpha\gamma} + \right. \\ &\left. + G_{\alpha\gamma}\varepsilon_{\mu\xi\beta\gamma} + G_{\gamma\gamma}\varepsilon_{\mu\xi\alpha\beta})\right\} \doteq \\ &\doteq Tr\left\{0 - \tilde{G}_{\mu\gamma} D_{\mu} G_{\xi\gamma} - G_{\beta\alpha} D_{\mu} G_{\alpha\gamma}\varepsilon_{\mu\xi\beta\gamma} + 0\right\} = \mathcal{M}^{(3)}_{\xi} - \mathcal{M}^{(2)}_{\xi}.\end{aligned} \tag{17}$$

The only structure remained is $\mathcal{M}^{(3)}_{\xi}$, but it also turns out to be a "fictitious" candidate, since it actually coincides with the already considered case of $\mathcal{M}^{(1)}_{\xi}$. This follows from the remarkable relation

$$G^{a}_{\mu\alpha}\tilde{G}^{a}_{\alpha\xi} = \frac{1}{4} g_{\mu\xi} G^{a}_{\beta\alpha}\tilde{G}^{a}_{\alpha\beta} , \tag{18}$$

which can readily be derived by a reader, familiar with the examples (15) and (17), utilizing the identity (11).

Thus, the "minimum" effective Lagrangian (8) reduces to $\mathcal{M}_{\xi} \backsim Tr\left\{J_{\mu}\tilde{G}_{\mu\xi}\right\}$ and does not lead to pure gluon decays of the axial meson.

5. Nonrelativistic Analysis. Infrared Singularity of Amplitudes for 3-Body Decays of P-Wave Quarkonia

Let us turn now to the analysis of the coupling of χ with gluons starting from the consideration of χ-meson as a system of weakly bound nonrelativistic quarks Q and $\bar{Q}$.

Under this consideration, as is known, particularly, the following ex-

pressions for the hadron widths of the χ-meson family decays in the leading approximation over α_s were obtained [1,9] :

$$\Gamma(\chi^{(0)} \to hadrons) \approx \Gamma(\chi^{(0)} \to 2g) = 96\pi N c_F \alpha_s^2 \Psi_1^2 M_\chi \tag{19a}$$

$$\Gamma(\chi^{(2)} \to hadrons) \approx \Gamma(\chi^{(2)} \to 2g) = \frac{128}{5}\pi N c_F \alpha_s^2 \Psi_1^2 M_\chi \tag{19b}$$

$$\Gamma(\chi^{(1)} \to hadrons) \approx \Gamma(\chi^{(1)} \to gq\bar{q}) = \frac{128}{9} n_f N c_F \alpha_s^3 \Psi_1^2 \times \times M_\chi \ln\frac{M_\chi}{\varepsilon} . \tag{19c}$$

Here, Ψ_1 is a dimensionless parameter, proportional to the derivative of the wave function of relative motion of quarks $Q\bar{Q}$ $\Psi'(r)$ at $r=0$. The last expression can be regarded only as an estimate of the hadron width, since the decay $\chi^{(1)} \to 3g$, not accounted here, proceeds in the same order α_s^3 and differs from (19c) by a factor $\ln(M_\chi/\varepsilon)$ which may be considered a large parameter only academically. The occurence of $\ln(M_\chi/\varepsilon)$ in expression (19c) is due to an interesting singularity of the three-body decays of P-wave $Q\bar{Q}$ states, i.e. with the infrared divergency of Γ in the approximation of free heavy quarks.

Let us consider in more detail the diagrams of Figs.4a and 4c corresponding to the double gluon annihilation of $Q\bar{Q}$. The amplitude $Q\bar{Q} \to 2g$ is determined by the expression

$$\tilde{M} = Sp\left[\hat{e}(m-\hat{p}+\hat{k})\frac{\gamma_\alpha(m+\hat{p}+\hat{k}-\hat{k}_1)\gamma_\mu}{2(p+k,k_1)-k_1^2}\cdot(m+\hat{p}+\hat{k}\right]g_s\left(\frac{\delta_{ab}}{2}\right) + + \left\{\begin{matrix}1 \leftrightarrow 2\\ \mu \leftrightarrow \nu\end{matrix}\right\}, \tag{20}$$

where e_α $(\hat{e} = e_\alpha\gamma_\alpha)$ is the vector of Q and $\bar{Q}$ total spin;

$2p=q$ is the total four-momentum of the meson. To extract the χ - states with $L=1$, one should retain in $\widetilde{M}$ only the linear term of the expansion over the small momentum of relative motion of quarks K_β. Adding together spin and orbital momentum we shall get amplitudes of annihilation for different χ.

For $\chi^{(1)}$, in particular, we have

$$M_\xi^{(1)}=\frac{i}{\sqrt{2}}\cdot\mathcal{E}^{\alpha\beta\gamma\xi}q_\gamma\frac{\delta}{\delta e_\alpha}\cdot\frac{\delta}{\delta K_\beta}\cdot\widetilde{M}(e,K)\Big|_{K=0}\cdot\Psi_1 \tag{21}$$

(ξ is the vector sign of the $\chi^{(1)}$ state).

The result of the calculations (see, e.g. [8]) by mesons of relation (11) can be written in the following compact form:

$$M^{\xi,\mu\nu}(\chi^{(1)}\to 2g)=\frac{i4g_s^2\Psi_1}{\sqrt{2}(K_1K_2)^2}\cdot\{\mathcal{E}^{\mu\alpha\beta\nu}K_{1\alpha}(K_2^2g_{\beta\nu}-K_{2\beta}K_{2\nu})+$$
$$+\mathcal{E}^{\nu\alpha\beta\gamma}K_{2\alpha}(K_1^2g_{\beta\mu}-K_{1\beta}K_{1\mu})\}(q^2g_{\gamma\xi}-q_\gamma q_\xi)\delta_{ab} \tag{22}$$

The structure in curly brackets coincides with expression (5); therefore the conclusions of sec.3 on the amplitude symmetry, current conservation and fulfilment of the Landau-Yang rule can be transported here as they are. The additional factor $(K_1K_2)^{-2}$ reflects the non-local nature of the annihilation.

Considering gluon $K_1\equiv\ell$ as real $\ell^2=0\,(K_2\equiv\Delta;\ \Delta^2\neq 0)$ we transform (22) to (omitting inessential terms):

$$M^{\xi,\mu\nu}=\frac{i4g_s^2\Psi_1}{\sqrt{2}}\cdot\frac{1}{\omega^2}\cdot\mathcal{E}^{\mu\alpha\nu\gamma}\ell_\alpha\Delta^2(g_{\gamma\xi}-q_\gamma q_\xi)\delta_{ab} \tag{23}$$

where $\omega\equiv(\ell q)=(K_1K_2)$ is the gluon energy in the rest frame of the meson in units M_χ $(M_\chi^2=q^2=1)$

In the soft limit $\ell\sim\omega\to 0$, $\Delta^2=(q-\ell)^2\to 1$ the amplitude is singular

$$M\sim 1/\omega \qquad\qquad (\text{at } \omega\to 0),$$

which results in logarithmic divergency of the cross section $|M|^2\frac{d^3\ell}{\omega}\sim\frac{d\omega}{\omega}$ in the free quarks approximation.

The singular behaviour of the amplitude corresponds physically to soft gluon radiation from distances considerably exceeding the dimensions of the annihilation region $r_{anih}\sim(\Delta^2)^{-1/2}\approx 1$. Owing to the finite size of χ-meson, this singularity is cut off at frequencies ω of the order of the coupling energy ε of the $Q\bar{Q}$ system. Modifying formula (22) by the replacement

$$(K_1K_2)^{-2}=\omega^{-2}\to((K_1K_2)+M\varepsilon)^{-2}=(\omega+\varepsilon)^{-2} \tag{24}$$

we obtain the gauge-invariantly regularized amplitude of the annihilation

$$M_{reg}^{\xi,\mu\nu}(\chi^{(1)})=\left[\frac{(K_1K_2)}{(K_1K_2)+M\varepsilon}\right]\cdot M^{\xi,\mu\nu} \tag{25}$$

which at high frequencies $\omega>\varepsilon$ coincides with the original one, and in the limit $K_1=\ell=0$ vanishes. Multiplying (25) by the transition amplitude of the virtual gluon into a pair of light quarks $A_\nu(g(K_2)\to q\bar{q})$ we shall describe the decay $\chi^{(1)}\to gq\bar{q}$ - the logarithmical estimate of the width given by formula (19c). It should be noted that the discussed singularity is absent in the three-gluon channel of the decay, since in the soft limit the singularity of amplitude (23) in the diagram of Fig.4a and the corresponding contribution to the diagram of direct $3g$ transition (Fig.4b) cancel each other. One may readily be convinced in this in the gauge $q_\mu A^\alpha_\mu = 2P_\mu A^\alpha_\mu = 0$, where radiation of the soft gluon K_1 is factorized in the form of

$$M^{\mu\nu\lambda}(Q\bar{Q} \to ggg)_{K_1 \to 0} \approx \frac{2K_\mu}{(K_1 q)} \cdot \left[M^{\nu\lambda}(Q\bar{Q} \to gg)\Big|_{K=0} \right] \times \times \left(1 + O(K_1/M_\chi)\right). \tag{26}$$

The soft gluon "carries away" the orbital momentum of χ -meson quarks (and of course, color, too), not affecting, in other respects, the hard annihilation of $Q\bar{Q} \to gg$ described by a sum of diagrams in Fig.5.

Here, the system $Q\bar{Q}$ is in the octet state in color ${}^3S_1$ and possesses gluon quantum numbers and a finite mass $(q-K_1)^2 \approx q^2$.

As the calculation shows, the diagrams of Fig.5 cancel each other. The nature of this surprising cancellation is explained indirectly in Ref.[10]. Were the transition $Q\bar{Q}\,({}^3S_1) \to 2g$ possible, the scattering amplitude of longitudinally polarized "colored J/ψ " at high energies ($S \to \infty$) would have grown inadmissibly fast owing to gluon exchange, in formal contradiction with renormalization theory.

In the Appendix, we shall demonstrate explicitly the cancellation of infrared singularities and calculate precisely the amplitude of χ -meson gluon decay.

The amplitudes of three-body decays of $\chi^{(0)}$ and $\chi^{(2)}$ have the same behaviour. Here, the contribution of the channel $\Gamma^{gq\bar{q}}/M_\chi \sim \alpha_s^2 \ln(M_\chi/\varepsilon)$ is not usually considered when discussing total hadron widths $\Gamma^{tot}/M_\chi \sim \alpha_s^2$, which are defined mainly by the decay $\chi^{(0),(2)} \to 2g$ (see (19a, b)).

The situation is however different, if we are interested not in the total widths but in the cross section of χ -meson production in hadron collisions. In the quark channel ($q\bar{q} \to \chi + g$), mesons $\chi^{(0),(1),(2)}$ must be produced, roughly speaking, equally often. In this case, the infrared singularity of the transition amplitude caused by the weak coupling of the $Q\bar{Q}$ system, leads to an unexpected behaviour in the χ transverse momentum spectra.

6. Manifestation of Weak Coupling of $Q\bar{Q}$ in the $d\sigma/dq_\perp^2$ Spectra of χ -Mesons Produced in Light Quark Annihilation.

χ -meson production in light quark annihilation in the first non-vanishing order in α_s is determined by the diagram of Fig.6. The differential cross section of χ production with rapidity y and with transverse momentum $q_\perp$ has the form:

$$\frac{d\sigma}{dq_\perp^2\,dy} = \frac{\alpha_s}{4S}\left[\frac{1}{2}\cdot\left(\frac{1}{N}\right)^2\right] \iint \frac{dx_1}{x_1}\frac{dx_2}{x_2} D^q(x_1) D^{\bar{q}}(x_2)\,\delta_+(\ell^2) \times \times \left\{-\frac{1}{4} Sp\left(\hat{K}_1 \gamma_\nu \hat{K}_2 \gamma_{\nu'}\right)\right\} |M|^2_{\nu\nu'} \frac{1}{(\Delta^2)^2}, \tag{27}$$

where $|M|^2_{\nu\nu'}$ describes the probability of the transition of virtual gluon Δ into χ and a real gluon ℓ. Let us pass from the integration over parton momenta x_1, x_2 to the invariant energy $\Delta = \sqrt{2(K_1K_2)}$ and the full rapidity of the pair $Y = \frac{1}{2}\ln\frac{x_1}{x_2}$ and remove

$$\delta_+(\ell^2) = \delta\left((\Delta_\mu - q_\mu)^2\right):$$

$$2d\Delta\delta + (\ell^2) = 1\Big/\sqrt{(1+q_\perp^2)\,ch^2(Y-y)-1} = \omega^{-1}(q_\perp,\eta) \tag{28}$$

where $\eta = Y - y$ (note, that we have assumed $q^2 = M_\chi^2 = 1$).

We obtain

$$\frac{d\sigma}{dq_\perp^2\,dy} = \frac{\alpha_s}{16N^2 S}\int d\eta\,\frac{\Delta}{\omega}\,D^q(x_1)\,D^{\bar q}(x_2)\left(\frac{1}{\Delta^4}|M|^2_{\nu\nu'}\,g^\perp_{\nu\nu'}\right) \tag{29}$$

Here $g^\perp_{\nu\nu'} = \left[g_{\nu\nu'} - (K_{1\nu}K_{2\nu'} + K_{1\nu'}K_{2\nu})/(K_1K_2)\right]$ is the projection of the unit tensor on the plane transverse to the collision axis.

Momenta X_1 and X_2 in (25) are expressed through $q_\perp$ and η via the relations

$$X_{1(2)} = \Delta/\sqrt{S}\cdot e^{\pm(y+\eta)} \tag{30}$$

where $\Delta = 1+\omega+\omega^2$, and the quantity $\omega(q_\perp,\eta)$ is defined by (28).

For the case of $\chi^{(1)}$, for example, calculating the modulus square of the amplitude (23), summed over the polarizations of χ , and of the real gluon, we shall obtain

$$\sum_{\xi,\mu}\frac{1}{\Delta^4}M^{\xi,\mu\nu}M^{*\xi,\mu\nu'} = \frac{1}{\Delta^4}(\delta_{ab})^2\left|\frac{i\,4g_s^2\psi_1}{\sqrt{2}}\cdot\frac{\Delta}{(\ell q)^2}\right|^2\times$$

$$\times\,\varepsilon^{\mu\alpha\nu\gamma}\varepsilon^{\mu\beta\nu'\delta}\ell_\alpha\ell_\beta(q_{\gamma\delta} - q_\gamma q_\delta) = \tag{31}$$

$$= (N^2-1)\frac{8g_s^2\psi_1^2}{(\ell q)^4}\cdot\left[g_{\nu\nu'}(\ell q)^2 - \ell_\nu\ell_{\nu'} - (\ell q)(\ell_\nu q_{\nu'} + q_\nu\ell_{\nu'})\right]$$

Further on, contracting this tensor with $g^\perp_{\nu\nu'}$ we shall have

$$\left(\frac{1}{\Delta^4}|M|^2_{\nu\nu'}\,g^\perp_{\nu\nu'}\right) = 16(N^2-1)\,g_s^2\,\psi_1\cdot\left[\frac{2(\ell q)^2 + q_\perp^2(1-2(\ell q))}{2(\ell q)^4}\right] \tag{32}$$

This expression should be substituted into (29) making use of the relation between the product (ℓq) and the variables $q_\perp$, η :

$$(\ell_\mu q_\mu) = (\Delta_\mu q_\mu) - 1 = \omega(1+\omega+\omega^2) = \omega\cdot\Delta \tag{33}$$

The problem of spectrum description without concrete definition of the form of parton distributions $D(x_i)$ is settled, in principle, by formulae (29) and (32) together with a rather cumbersome set of relations (28), (30), (33). However, owing to specificity of P-wave χ -states, the integral in (29) in the region $q_\perp^2 \ll 1$ can be calculated explicitly.

What must be the character of the $q_\perp$ distribution? One may assume, for general reasons, that according to the logarithmical width of the decay $\Gamma(\chi \to gq\bar q)$ this distribution must be of the form of $d\sigma/dq_\perp^2 \sim 1/q_\perp^2$. However, if for the Drell-Yan process (Fig.1c), where $q_\perp$ of the lepton pair is compensated by the radiation of a gluon in the initial state, such a spectrum is quite natural, then in the diagram of Fig.6 one cannot, at first sight, reveal a quasi-real intermediate state which could provide an anomalously large cross section at $q_\perp \to 0$. But actually it is there, being concealed in the amplitude $\chi \to 2g$ whose infrared singularity corresponds to large formation lengths of radiation in the final state.

The main contribution to the integral (29) at small $q_\perp$ comes from a narrow region of rapidities $|\eta| \sim q_\perp \ll 1$. Here the quantity $\omega(q_\perp,\eta)$, introduced above, coincides with the energy of gluon in the rest frame of χ :

$$(\ell q) \approx \omega = \sqrt{q_\perp^2 + (1+q_\perp^2)\mathrm{sh}^2\eta} \approx \sqrt{q_\perp^2 + \eta^2} \sim q_\perp^2 \ll 1$$

Let us introduce the variable $v = \eta/q_\perp$ and, omitting relatively small contributions $\sim O(q_\perp)$, present the cross section in the form:

$$\frac{d\sigma}{dq_\perp^2 dy} = \frac{32\pi^2}{S}\cdot\frac{C_F}{N}\cdot\alpha_s^3\psi_1^2\int\frac{dv}{\sqrt{1+v^2}}\cdot\left[\frac{2(1+v^2)+1}{q_\perp^2\cdot 2(1+v^2)^2}\right]\times$$

$$\times D^q(x_1)\, D^{\bar q}(x_2)(1+O(q_\perp)) \cdot \tag{34}$$

Here x_1 , x_2 are determined by the mass and the rapidity of χ ($\eta = 0$, $\Delta \approx 1$ in (30)). The integral over v converges at $v \sim 1$, which justifies the assumption of smallness of essential η . This is caused by the singularity of the amplitude over ω :
$|M|^2 \backsim (2\omega^2 + q_\perp^2)/\omega^4 \sim 1/\omega^2$.

The approximate expression (34) can be derived directly from the soft amplitude in the gauge $q_\mu A_\mu^a = 0$

$$M_{ij}^{\mu\nu} = \frac{\delta}{\delta\kappa_i}\cdot\frac{\delta}{\delta e_j}\left[4\,\frac{\kappa_\mu e_\nu}{(\ell q)}\right]. \tag{35}$$

using in calculating the cross section summed over polarizations of $\chi^{(1)}$ the projection operator

$$\Pi(1) = \frac{1}{2}(\delta_{ii'}\delta_{jj'} - \delta_{ij'}\delta_{i'j}) .$$

Analogously, using projection operators

$$\Pi(0) = \frac{1}{3}\delta_{ii'}\delta_{jj'}$$

$$\Pi(2) = \frac{1}{2}(\delta_{ii'}\delta_{jj'} + \delta_{ij'}\delta_{i'j} - \frac{2}{3}\delta_{ij}\delta_{i'j'}) ,$$

'e shall obtain spectra of $\chi^{(0)}$ and $\chi^{(2)}$ mesons.

Thus, in the free $Q\bar{Q}$ approximation, the spectrum actually is proportional to $q_\perp^{-2}$. The growth at small $q_\perp$ cuts off, however, at accounting the finite size of the meson. Modifying, in accordance with (24), the denominator of the soft radiation amplitude ($(\ell q)^{-4} \approx \omega^{-4} \to (\omega+\varepsilon)^{-4}$ in formula (32)) we get, finally,

$$\frac{d\sigma^{(J)}}{dq_\perp^2\, dy} = \frac{32\pi^2}{S}\cdot\frac{C_F}{N}\cdot\alpha_s^3\psi_1^2 D^q(x_1) D^{\bar q}(x_2)\times$$

$$\times\left[\frac{1}{q_\perp^2}\int_{-\infty}^{\infty}\frac{dx}{(\mathrm{ch}x+a)^4}B^{(J)}(x)\right] \tag{36}$$

where $a \equiv \varepsilon/q_\perp$, and the factors $B^{(J)}$ for χ over spin J are as follows:

$$B^{(0)} = \frac{1}{3}(2\mathrm{ch}^2x - 1);\; B^{(1)} = \frac{1}{2}(2\mathrm{ch}^2x + 1);\; B^{(2)} = \frac{1}{2}\left(\frac{14}{3}\mathrm{ch}^2x - \frac{1}{3}\right) \tag{37}$$

The form of the distributions $d\sigma^{(J)}/dq_\perp^2$ derived from here is shown in Fig.7.

We now present expressions for the integrals entering into

$$\rho^{(J)}(q_\perp)=\frac{1}{q_\perp^2}\cdot\int_{-\infty}^{\infty}dx\,\frac{1}{(\mathrm{ch}x+a)^4}\,B^{(J)}(x):$$

$$I_1(a)\equiv\int_{-\infty}^{\infty}\frac{dx\,\mathrm{ch}^2x}{(\mathrm{ch}x+a)^4}=\frac{1}{3(a^2-1)^3}\cdot\left[a^4-10a^2-6+3a(a^2+4)\cdot\varphi(a)\right]$$

$$I_2(a)\equiv\int_{-\infty}^{\infty}\frac{dx}{(\mathrm{ch}x+a)^4}=\frac{1}{3(a^2-1)^3}\cdot\left[-11a^2-4+3a(2a^2+3)\cdot\varphi(a)\right], \tag{38}$$

where

$$\varphi(a)=\begin{cases}\dfrac{\arccos a}{\sqrt{1-a^2}} & \text{at } a<1 \quad (\text{recall that } a=\varepsilon/q_\perp\ )\\[2ex] \dfrac{\ln(a+\sqrt{a^2-1})}{\sqrt{a^2-1}} & \text{at } a>1\end{cases}$$

At $q_\perp \gg \varepsilon\ (a\approx 0)$ we have $I_1(0)=2$, $I_2(0)=4/3$.
From here

$$\rho^{(0)}(q_\perp)=\frac{1}{q_\perp^2}\cdot\frac{8}{9};\quad \rho^{(1)}=\frac{1}{q_\perp^2}\cdot\frac{8}{9};\quad \rho^{(2)}=\frac{1}{q_\perp^2}\cdot\frac{40}{9}, \tag{39}$$

i.e. at the spectrum edge the yields of $\chi^{(J)}$ are proportional to the statistical weights ($2J+1$):

$$\chi^{(0)};\ \chi^{(1)},\ \chi^{(2)}\ = 1:3:5 \tag{40}$$

At small $q_\perp \lesssim \varepsilon$ the cross sections flatten. The details of the amplitude's polarization structure become essential here, and the simple re-

lation (40) does not hold (see Fig.7). In particular, at $q_\perp \to 0\ (a\to\infty)$ we get

$$\rho^{(0)}(0)=\frac{1}{\varepsilon^2}\cdot\frac{2}{9};\quad \rho^{(1)}=\frac{1}{\varepsilon^2}\cdot\frac{1}{3};\quad \rho^{(2)}=\frac{1}{\varepsilon^2}\cdot\frac{7}{9}, \tag{41}$$

i.e.

$$\chi^{(0)}:\ \chi^{(1)}:\ \chi^{(2)} = 2:3:7.$$

7. Conclusion.

We have considered the spectra $d\sigma/dq_\perp^2$ of heavy C-even quarkonia $\chi^{(J)}$ produced in hadron collisions in the gg and $q\bar{q}$ channels:

$$g+g\to\chi^{(J)}+g \tag{39a}$$

$$q+\bar{q}\to\chi^{(J)}+g \tag{39b}$$

In the quark channel (39b) the distribution in $q_\perp$ of three $\chi^{(J)}$ (J = 0, 1, 2) proved to be similar, the shape of the spectra in the region $q_\perp < M_\chi$ being defined by the parameter ε/M, whose smallness corresponds to the nonrelativistic nature of $Q\bar{Q}$ quarks in the meson. At the same time, the distributions in the gluon channel (39a) are qualitatively different. For $\chi^{(0)}$, $\chi^{(2)}$ the spectrum behaves as $d\sigma/dq_\perp^2 \backsim 1/q_\perp^2$ whose singularity at $q_\perp\to 0$ smoothens due to the gluon formfactor T_a^2, arising at accounting multiple bremsstrahlung of gluons. In the case of $\chi^{(1)}$ the spectrum is essentially smoother, since the gluon bremsstrahlung by primary partons from relatively large distances $\rho_\perp \sim 1/q_\perp \gg 1/M_{Q\bar{Q}}$ is suppressed in virtue of the Landau-Yang veto, and infrared singularity of the radiation in the final state (accounting of it has lead to the growth with decreasing $q_\perp$ of the spectrum in the channel (39b)) is absent in the gluon channel (39a).

Note also, that in one more channel, i.e. in the quark-gluon channel, of χ production

$$q(\bar{q}) + q \to \chi^{(J)} + q(\bar{q}) \qquad \text{(39c}$$

accounting of matrix element singularity connected with the soft gluon radiation by nonrelativistic $Q\bar{Q}$ in the P-wave leads to a specific prediction on χ 's rapidity distribution, namely to the preferable formation of χ in the region of colliding hadron fragmentation.

The present study is aimed at clarifying the possibility of experimental observation of the double-logarithmic formfactor T_G^2 which is displayed in χ -meson spectra formed in fusion of two gluons (an analog of quark formfactor T_F^2 in the Drell-Yan process). To solve this problem, one should extract contributions of $\chi^{(0)}$ and $\chi^{(2)}$ (contribution of $\chi^{(1)}$ leads to a "non-formfactor" flattening of the spectrum $d\sigma/dq_\perp^2$) and take into account the background channels (39b) and (39c).

It should be emphasized that the study of χ yields in the quark channel (39b), which can relatively easily be extracted experimentally, is of independent interest: at moderate energies ($S/M^2 \gtrsim 1$), this channel dominates, e.g. in $p\bar{p}$, πp and $K^- p$ reactions (valence quarks annihilation), and also determines (at any S) the differences between cross sections $[d\sigma(ab \to \chi + \ldots) - d\sigma(\bar{a}b \to \chi + \ldots)]$ The checking of simple predictions of perturbation theory is particularly interesting here due to the fact that the expected shape of spectra, as was already stated, depends essentially on the dynamics of bound state, heavy $Q\bar{Q}$ in the $\chi^{(J)}$ meson.

It is worth noting at the end, that although the above-given analysis was based on the approximation $M_{Q\bar{Q}}/(300\ \text{MeV}) \to \infty$, a number of qualitative consequences (see, e.g. relations (39)-(42)) may be displayed already in the spectra of known χ -mesons of the ($b\bar{b}$) family.

The authors are thankful to A.A.Anselm, Ya.I.Azimov, Ed.Sh.Egoryan, B.L.Ioffe, V.A.Khoze, A.G.Sedrakyan, S.I.Troyan for helpful discussions; to V.A.Khoze for constant technical assistance. One of the authors (R.Sh. Egoryan) is indebted to A.Ts.Amatuni, S.G.Matinyan and E.A.Mamidjanyan for their constant interest in the work and support, and also to the LNPI staff and all his colleagues from the LNPI TP lab for their hospitality and support.

APPENDIX

Let us describe briefly the procedure and give the result of calculating the amplitude $\chi^{(1)} \to 3g$. Let the gluon momenta be $\{K_1, K_2, K_3\}$ and corresponding to them vector and colored indices $\{\mu, \nu, \lambda\}$ and $\{a, b, c\}$

One of the six diagrams for a direct transition $\chi^{(1)} \to 3g$ is shown in Fig.8a. Six more contributions correspond to diagrams of a "fork" type (Fig.8b). It is convenient to combine them in the following way:

$$M_{(1)} \equiv (\text{Fig.8a}) + \frac{1}{2}\left[(\text{Fig.8b}) + (\text{Fig.8c})\right] \tag{A.1}$$

Then, the total amplitude M will take the form

$$M = \left[M_{(1)} + M_{(1)}\begin{pmatrix} K_1 \leftrightarrow K_3 \\ \mu \leftrightarrow \lambda \\ a \leftrightarrow c \end{pmatrix}\right] + \text{cycle} \tag{A.2}$$

The "cycle" operation here and in the following adds to the right-hand side two more terms of analogous structure, obtained by means of the cyclical permutation $K_1 \to K_2 \to K_3 \to K_1$; $\mu \to \nu \to \lambda \to \mu$; $a \to b \to c \to a$

The representation (A.2) allows one to reduce the calculations leaving the possibility of controlling their correctness using current conservation.

To the diagram of Fig.8a corresponds the color factor

$$\mathrm{Tr}(t^a t^b t^c) \to \frac{1}{2} \cdot \frac{i}{2} \cdot f^{abc} = \frac{i}{4} f^{abc} \tag{A.3}$$

(the antisymmetric combination is chosen by the C-evenness requirement); and to diagrams of Fig.8b,c corresponds a twice larger color factor

$$\mathrm{Tr}(t^d t^c) \cdot i f^{abd} = \frac{i}{2} f^{abc}$$

Thus, to the amplitude (A.1) corresponds a sum of "Lorentz" parts of diagrams of Fig.8a,b,c (i.e. with no account of colors) with the same factor (A.3).

The sum of amplitudes of Fig.8b and c for the $\chi^{(1)}$ case (to which we here restrict ourselves), can easily be obtained by combining the transition $\chi^{(1)} \to 2g$ (22) with the $3g$ vertex. The result is as follows:

$$V = \frac{1}{\omega} \cdot \Big\{ -g_{\nu\lambda}[\mu, K_2 - K_3, 1] + 2K_{2\lambda} \cdot [\mu, \nu, 1] - 2K_{3\nu}[\mu, \lambda, 1] \Big\} - \left\{ \begin{matrix} 1 \leftrightarrow 3 \\ \mu \leftrightarrow \lambda \end{matrix} \right\} \tag{A.4}$$

Here we have introduced a convenient symbol

$$[abc] \equiv \varepsilon^{\alpha\beta\gamma\delta} a_\alpha b_\beta c_\gamma (g_{\delta\xi} - q_\delta q_\xi) \tag{A.5}$$

and "1" in this symbol denotes the vector of <u>4-velocity</u> of gluon 1:

$v_{1\gamma} = K_{1\gamma}/(K_1 q) \equiv K_{1\gamma}/\omega_1$, i.e.

$$[\mu, K_2 - K_3, 1] \equiv \varepsilon^{\mu\beta\gamma\delta} (K_2 - K_3)_\beta \frac{K_{1\gamma}}{(K_1 q)} \cdot (g_{\delta\xi} - q_\delta q_\xi)$$

For the contribution of Fig.8a we have

$$A = \mathbf{P} \cdot \frac{1}{4} \mathrm{Sp}\left[\hat{e}(m - \hat{P} + \hat{K}) \gamma_\lambda (m - \hat{P} + \hat{K} + \hat{K}_3) \gamma_\nu (m + \hat{P} + \hat{K} - \hat{K}_1) \times \gamma_\mu (m + \hat{P} + \hat{K})\right] \frac{1}{2(K_3, P-K)} \cdot \frac{1}{2(K_1, P+K)}, \tag{A.6}$$

where the operator $\mathbf{P}$, encountered before,

$$\mathbf{P}\tilde{A} \equiv \varepsilon^{\alpha\beta\gamma\xi} q_{\gamma} \frac{\delta}{\delta K_{\alpha}} \cdot \frac{\delta}{\delta e_{\beta}} \cdot \tilde{A}(K)\Big|_{K=0} \,, \tag{A.7}$$

extracts from the amplitude A the terms linear in K (P -wave!).

Three sources of such terms

1) $\hat{K}$ in "external" structure $(m+\hat{p}+\hat{K})\hat{e}(m-\hat{p}+\hat{K})$,

2) $\hat{K}$ in the numerator, and

3) $(KK_i$ in the expansion of denominators (propagators)

of the annihilation diagram correspond to the three types of structures in

$A = A_1 + A_2 + A_3$

Taking into account $m^2-(p+K)^2 \approx m^2-(p-K)^2 = O(\varepsilon \sim \alpha_6) \approx 0$ and replacing (A.8) by an equivalent expression *, we present (A.6) in the form $(m^2=(q/2)^2=1/4)$:

$$\tilde{A} = \frac{1}{4} S_p \Big\{ \big(\tfrac{1}{2}\hat{e} + \hat{e}\hat{K}\hat{q}\big)\big[(-q_{\lambda}+\gamma_{\lambda}\hat{K}_3)\gamma_{\nu}(q_{\mu}-\hat{K}_1\gamma_{\mu}) + 2K_{\lambda}\gamma_{\nu}(q_{\mu}-\hat{K}_1\gamma_{\mu}) + 2K_{\mu}(-q_{\lambda}+\gamma_{\lambda}\hat{K}_3)\gamma_{\nu}\big]\Big\} \frac{1}{(\omega_3-2(KK_3))(\omega_1+2(KK_1))} \tag{A.9}$$

The terms singular in the limit $\omega_1 \to 0$ (or $\omega_3 \to 0$) can readily be analyzed:

$$\tilde{A}^{sing} = \tilde{A}_1^{sing} + \tilde{A}_2^{sing} + \tilde{A}_3^{sing}$$

---

* In the following, when replacing an expression by an equivalent one we shall use the sign $\doteq$ (in particular, we shall omit everywhere inessential terms in $M \sim K_{1\mu}$, $K_{2\nu}$, $K_{3\lambda}$).

Acting by the operator (A.7) on them we obtain (using (11))

$$A^{sing} = \mathbf{P}\tilde{A}^{sing} \doteq (-1/\omega_1\omega_3)\cdot\{g_{\nu\lambda}[\mu, K_3, 1] + q_{\lambda}(1-2\omega_1)[\mu,\nu,1] - K_{3\nu}[\mu,\lambda,1]\} - \left\{ {1 \leftrightarrow 3 \atop \mu \leftrightarrow \lambda} \right\} \tag{A.10}$$

The contribution of "forks" (A.4) as well as that of direct diagram (A.10) are singular at $\omega_1 \to 0\ (\omega_3 \to 0)$. However in the total amplitude the infrared singularities cancel (see sec.5).

Indeed, adding to (A.4) the term

$$\frac{1}{\omega_1} g_{\nu\lambda}[\mu, q, 1] - \frac{1}{\omega_3}[\lambda, q, 3] \,,$$

whose contribution drops out in the "cycle" operation in (A.2), we shall arrive at

$$V \doteq (2/\omega_1)\cdot(g_{\nu\lambda}[\mu, K_3, 1] - K_{3\nu}[\mu,\lambda,1] + K_{2\lambda}[\mu,\nu,1]),$$

$$V + A^{sing} \doteq (-1/\omega_1\omega_3)\{(1-2\omega_3)(g_{\nu\lambda}[\mu,K_3,1] - K_{3\nu}[\mu,\lambda,1]) + (-q_{\lambda}(1-2\omega_2) + 2K_{1\lambda}\omega_3)[\mu,\nu,1]\} - \left\{ {1 \leftrightarrow 3 \atop \mu \leftrightarrow \nu} \right\} \tag{A.11}$$

The latter expression is nonsingular, since

$$(1-2\omega_3) = (2K_1K_2) \xrightarrow[\omega_1 \to 0]{} 0\,, \quad (1-2\omega_2) = (2K_1K_3) \to 0;$$

One should add to (A.11) the regular contributions of (A.9) from

$$\tilde{A}_1^{reg} = (1/\omega_1\omega_3)\frac{1}{4} Sp(\hat{q}\hat{e}\hat{K}\gamma_{\mu}\hat{K}_1\gamma_{\nu}\hat{K}_3\gamma_{\lambda}) \tag{A.12a}$$

$$\tilde{A}_3^{reg} = (1/\omega_1\omega_3)\cdot\frac{1}{4}Sp\left(\hat{e}\gamma_\mu\hat{K}_1\gamma_\nu\hat{K}_3\gamma_\lambda\right)\left(K, v_1 - v_3\right) \qquad \text{(A.12b)}$$

In calculating the contribution to $M_{(1)}$ from (A.12a) we use a simple relation

$$P\{\hat{q}\hat{e}\hat{K}\hat{a}\hat{b}\hat{c}\} = 2[a,b,c] \qquad \text{(A.13)}$$

where the symbol $\{\dots\}$ means $\frac{1}{4}Sp\{\dots\}$

The difficulty here is not the calculation itself but guessing the form of the answer for a total sum of (A.11) and (A.12). It is convenient to reduce all the contributions to the following structures:

$$M_1 = [\nu,2,3-1]B_{\lambda\mu} + [\mu,1,3]C_{\nu\lambda} + [\lambda,1,3]C'_{\mu\nu} + \\ + [1,2,3]D_{\mu\nu\lambda}\,, \qquad \text{(A.14)}$$

where $B_{\mu\nu}$ and $D_{\mu\nu\lambda}$ are invariant under replacement $\{1 \leftrightarrow 3, \mu \leftrightarrow \lambda\}$ and C and C' transform into each other.

Symbols $[\dots]$ in (A.14) together with the explicit form of B, C and D tensors ensure current conservation over each of the three gluons. Moreover, B and C are closely connected structurally, this enabling one to write a compact expression for the amplitude M. Applying the "cycle" operation in (A.12) we get (replacement $\{1 \leftrightarrow 3\}$ doubles M_1):

$$M = 2\big([\mu,1,2-3]B_{\nu\lambda} + [\mu,1,3]C_{\nu\lambda} + [\mu,2,1]C'_{\nu\lambda} + \\ + [1,2,3]D_{\mu\nu\lambda} + \text{cycle}. \qquad \text{(A.15)}$$

after which it turns out that

$$(\omega_1/\omega_2)C_{\nu\lambda} \doteq (\omega_1/\omega_3)C'_{\nu\lambda} \doteq B_{\nu\lambda} = \omega_1^2 Q_{\nu\lambda}\,, \qquad \text{(A.16)}$$

where the universal symmetrical tensor Q is

$$Q_{\nu\lambda} \equiv \big((v_1v_2)v_{3\nu}v_{1\lambda} + (v_1v_3)v_{1\nu}v_{2\lambda} - \\ - (v_2v_3)v_{1\nu}v_{1\lambda} - (v_1v_2)(v_1v_3)g_{\nu\lambda}\big)\cdot 2 \qquad \text{(A.17)}$$

The following representation

$$Q_{\nu\lambda} \doteq \{12\nu 1\lambda 3\} \equiv \frac{1}{4}Sp\{\hat{v}_1\hat{v}_2\gamma_\nu\hat{v}_1\gamma_\lambda\hat{v}_3\} \qquad \text{(A.18)}$$

demonstrates explicitly current conservation $K_{2\nu}Q_{\nu\lambda} = K_{3\lambda}Q_{\nu\lambda} = 0$ (recall that we assumed $K_i^2 = 0$ everywhere).

Using (A.16) we finally obtain for the amplitude $\chi^{(1)} \to 3g$ (restoring all the coefficients):

$$M = if^{abc}\frac{i\Psi_1}{\sqrt{2}}\cdot 2\big(Q_{\nu\lambda}[\mu,\hat{K}_1,\vec{\xi}\,] - \\ - \omega_1^2(\{\mu 2\nu 1\lambda 3\} - q_\mu Q_{\nu\lambda})[1,2,3] + \text{cycle}\big) \qquad \text{(A.19)}$$

where $\vec{\xi}$ is a vector (antisymmetric with respect to the permutation of any pair of gluon momenta) which lies in the decay plane

$$\xi_\alpha = v_{1\alpha}(\omega_2 - \omega_3) + v_{2\alpha}(\omega_3 - \omega_1) + v_{3\alpha}(\omega_1 - \omega_2)\,; \\ (\xi q) = 0\,. \qquad \text{(A.20)}$$

The symbols entering into (A.19) are defined in (A.5), (A.17); tensor

$$\{\mu 2 \nu 1 \lambda 3\} \equiv \frac{1}{4} Sp \{ \gamma_\mu \hat{v}_2 \gamma_\nu \hat{v}_1 \gamma_\lambda \hat{v}_3 \}$$

The amplitude (A.19) can be simplified essentially by a suitable choice of gauge (e.g. $q_\mu A^a_\mu = 0$); however we failed to think of a compact analytical representation for the probability of transition $\chi^{(1)} \to 3g$.

Figure Captions

Fig.1. Lower order diagrams for χ production in gg (a) and gq (b) channels. (c) - Drell-Yan process of production of massive lepton pairs.

Fig.2. Comparison of $q_\perp$ spectra: a) lepton pairs and b) $\chi_b^{(0),(2)}$ mesons ($q^2 = M_\chi^2 \approx (10\ \text{GeV})^2$).

Fig.3. Total set of Feynmann diagrams for the cross section of

$$gg \to g + \chi$$

Fig.4. Diagrams for the amplitudes of 3-body decays of χ

Fig.5. Sum of the effective diagrams for decay of colored $^3S_1$ state of $Q\bar{Q}$ into two real gluons, obtained as a result of factorization of soft gluon radiation $(K_1 \to 0)$ in the diagrams of Fig.4a,b.

Fig.6. Lower order diagram for χ production in the quark channel.

Fig.7. Comparison of shapes of spectra $\frac{1}{\sigma} \cdot \frac{d\sigma}{dq_\perp^2}$ $\chi^{(J)}$ $(J=0,1,2)$ in the channel $q\bar{q} \to \chi^{(J)} + g$ for $q_\perp < M_\chi$ (ε is the coupling energy of $Q\bar{Q}$ quarks in χ -meson):

a) absolute yields;

b) normalized spectra

$$\frac{1}{2J+1} \cdot \frac{1}{\sigma} \cdot \frac{d\sigma}{dq_\perp^2}$$

Fig.8. Sum of diagrams for the amplitude M_1 (see the text).

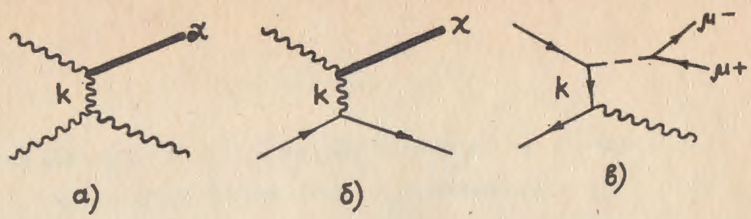


Fig.1

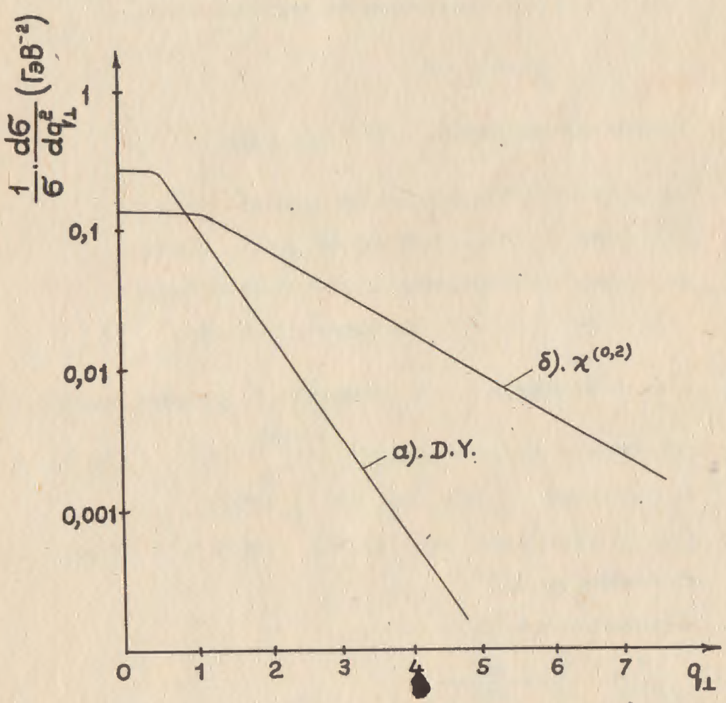


Fig.2

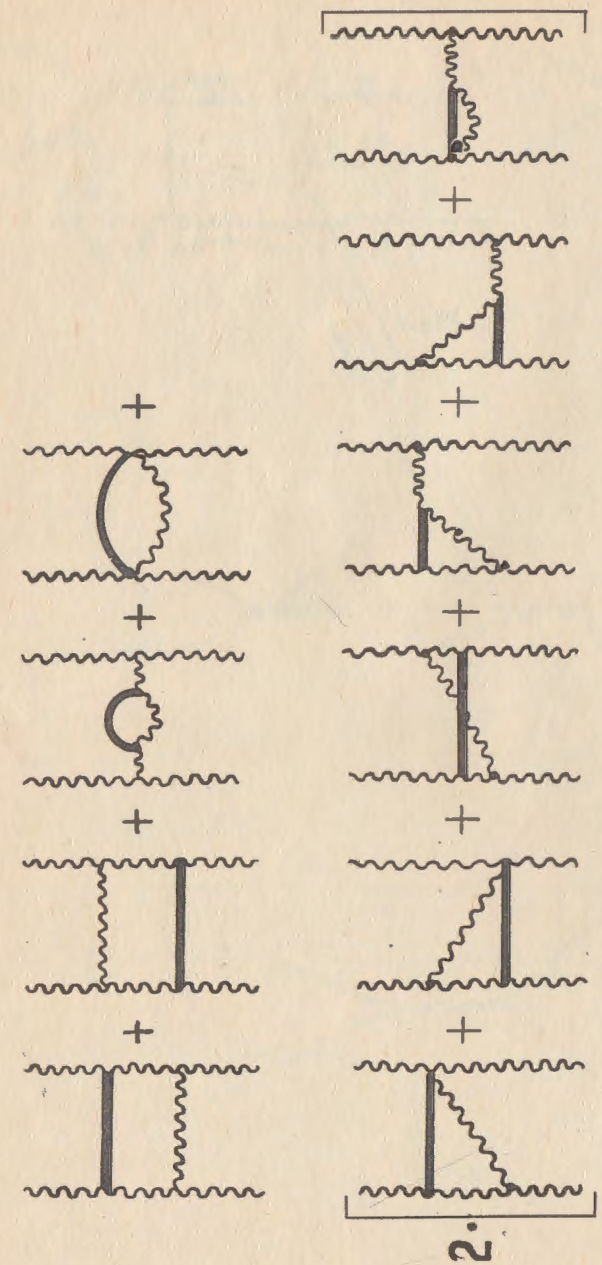


Fig.3

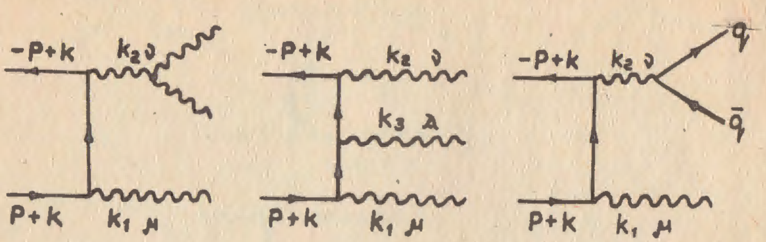


Fig.4

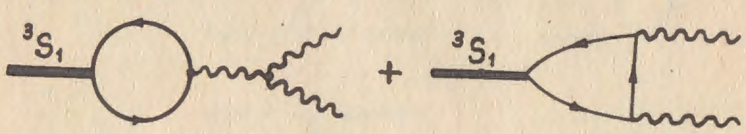


Fig.5

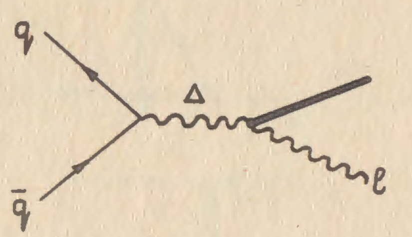
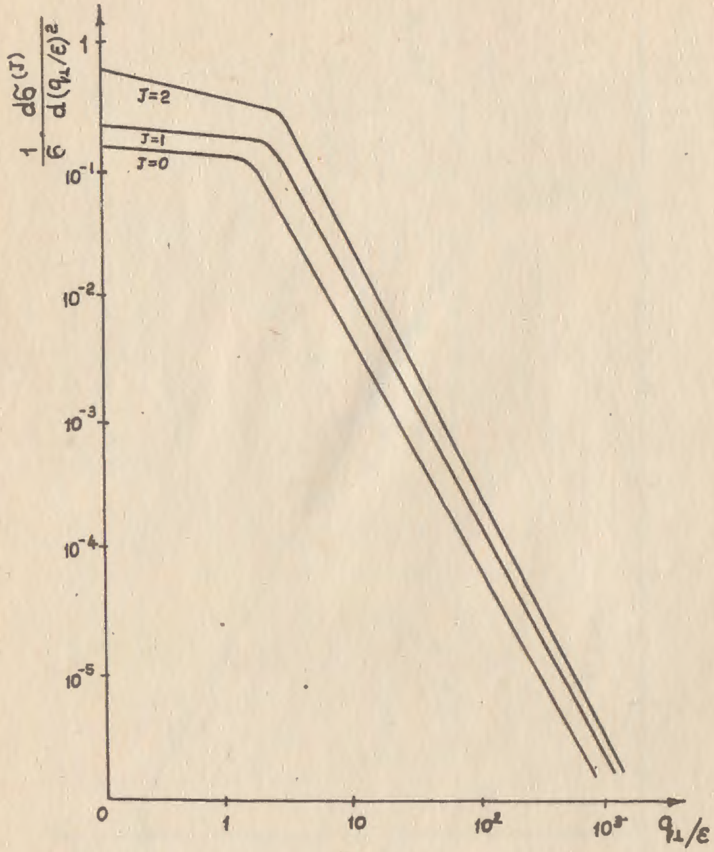
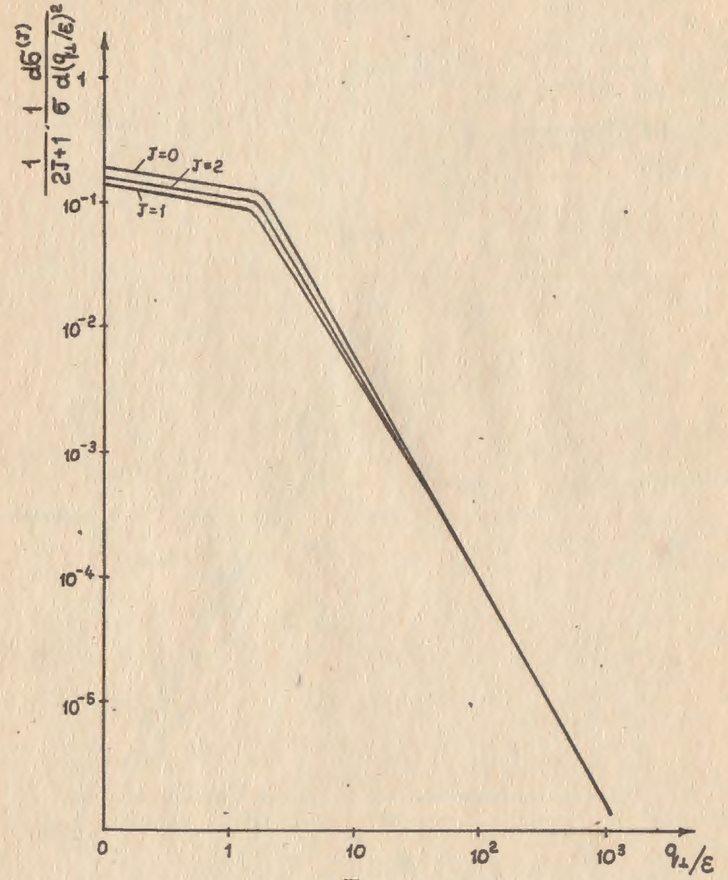


Fig.6



A

Fig.7


$\frac{1}{2J+1}\cdot\frac{1}{\sigma}\frac{d\sigma^{(J)}}{d(q_\perp/\varepsilon)^2}$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
J=0
J=2
J=1
0
1
10
10^2
10^3
$q_\perp/\varepsilon$

Б

Fig. 7

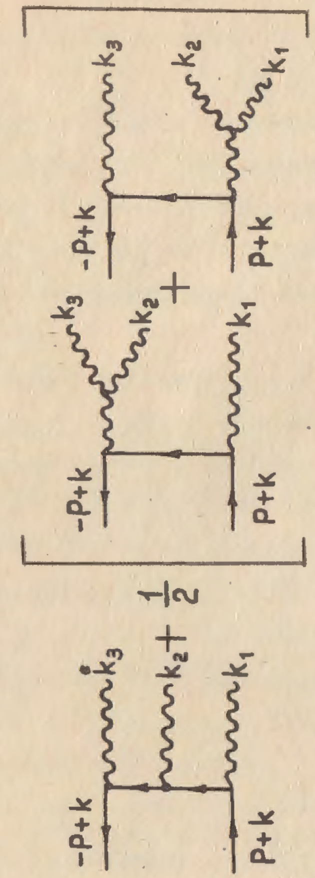

-P+k
P+k
k_3
k_2
k_1
+ $\frac{1}{2}$
+

Fig.8

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The manuscript was received 20 September 1983

Ю.Л.ДОКШИЦЕР, Р.Ш.ЕГОРЯН

ОБ ОСОБЕННОСТЯХ ОБРАЗОВАНИЯ C -ЧЕТНЫХ ТЯЖЕЛЫХ КВАРКОНИЕВ В АДРОННЫХ СОУДАРЕНИЯХ

(на английском языке, перевод З.Н.Асланян)

Ереванский физический институт

Редактор Л.П.Мукаян

Тех.редактор А.С.Абрамян

Заказ556 ВФ-00050 Тираж 299

Индекс 3624 Печать овсетная Формат издания 60x84/16

Подписано к печати 30/Ш-84г.2 уч.изд.л. Ц. 30 к.

Издано Отделом научно-технической информации Ереванского физического института, Ереван 36, Маркаряна 2