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ЦЕНТРАЛЬНЫЙ НАУЧНО-ИССЛЕДОВАТЕЛЬСКИЙ ИНСТИТУ
ИНФОРМАЦИИ И ТЕХНИКО-ЭКОНОМИЧЕСКИХ ИССЛЕДОВАНИ
ПО АТОМНОЙ НАУКЕ И ТЕХНИКЕ

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# QUANTIZATION OF SUPERPARTICLES BY MEANS OF A PATH INTEGRAL

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# QUANTIZATION OF SUPERPARTICLES BY MEANS OF A PATH INTEGRAL

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Recently a number of attempts have been made to extend the framework of classical mechanics so that it describes the dynamics of particles with both integral and half-integral spin. The most natural method is connected with the introduction of additional Grassman degrees of freedom (here there are at least two possible versions, see [1] and [2]).

The action of the scalar relativistic particle coincides with the length of its world line (below all the calculations are made in the Euclidean space). If we replace the length with a superlength in superspace, as is done in [1,3], we shall obtain a new dynamic system, "superparticle", with an action

$$m\int d\tau \sqrt{(\dot{x}_\mu + i\bar{\theta}\gamma_\mu\dot{\theta})^2 + \alpha\dot{\bar{\theta}}\dot{\theta}} \qquad (1)$$

whose configuration space is described by the bosonic coordinate $x_\mu$ and the Grassman spinor $\theta_\alpha$. Later on we shall consider $\theta$ to be a Majorana spinor, though it is not obligatory. The parameter $\alpha$ has a dimension of length.

The action (1) is invariant with respect to reparametrizations, Lorentz-transformations and translations in superspace

$$\theta_\alpha \to \theta_\alpha + \varepsilon_\alpha$$
$$x_\mu \to x_\mu + c_\mu + i\bar{\varepsilon}\gamma_\mu\theta \qquad (2)$$

The canonical quantization of superparticle according to Dirac is carried out in [3] and [4], where the space of states of the system is shown to coincide with the one-particle sector of the quantized field with the same supersymmetry. In this formalism the physical degrees of freedom are connected rather intricately with the initial variables $X$ and $\theta$ . In order to reveal the physical degree of freedom, one should either abandon the relativistic invariant description or introduce additional ghost multiplets which vanish in the limit $\to 0$ only.

In this paper we propose a simpler method of quantization, completely based on the formalism of path integrals.

Following Polyakov [5], we shall proceed from the equivalent at a classical level action that contains an intrinsic metric along the path

$$A[x(\tau),\theta(\tau)] = \frac{1}{2}\int_0^1 d\tau\left[\frac{(x_\mu + i\theta\gamma_\mu\dot{\theta})^2 + a\dot{\bar{\theta}}\dot{\theta}}{e(\tau)} + m^2 e(\tau)\right]. \qquad (3)$$

The particle propagation amplitude is then expressed via following path integral

$$K(x_1\theta_1\,;\,x_2\theta_2) = \int_{x(0)=x_1}^{x(1)=x_2} \mathcal{D}x(\tau)\int_{\theta(0)=\theta_1}^{\theta(1)=\theta_2}\mathcal{D}\theta(\tau)\; e^{-A[x(\tau),\theta(\tau)]} \qquad (4)$$

After imposing the gauge $\dot{e} = 0$ the right-hand side takes the form

$$\int_0^\infty dT\int_{x(0)=x_1}^{x(T)=x_2}\mathcal{D}x(\tau)\int_{\theta(0)=\theta_1}^{\theta(T)=\theta_2}\exp\left\{-\frac{1}{2}\int_0^T\left[(\dot{x}_\mu + i\bar{\theta}\gamma_\mu\dot{\theta})^2 + a\dot{\bar{\theta}}\dot{\theta} + m^2\right]\right\} \qquad (5)$$

Using the properties of Gaussian integrals, we turn to the integration in the phase space

$$K(x,\theta_1;x_2\theta_2) = \int_0^\infty dT\int_{x(0)=x_1}^{x(T)=x_2}\mathcal{D}x(\tau)\int_{\theta(0)=\theta_1}^{\theta(T)=\theta_2}\mathcal{D}\theta\int\mathcal{D}p\int\mathcal{D}\pi \qquad (6)$$
$$\exp\left\{-\frac{1}{2}\int_0^T\left[p_\mu^2 + \frac{1}{a}\bar{\pi}\pi + i\bar{\pi}\dot{\theta} + ip_\mu(\dot{x}_\mu + i\bar{\theta}\gamma_\mu\dot{\theta})\right]d\tau\right\}$$

After the displacement $\pi \to \pi - i\bar{\theta}$ the exponent takes the form

$$p_\mu^2 + \frac{1}{a}(\bar{\pi} - i\bar{\theta})(\pi - i\theta) + ip_\mu\dot{x}_\mu + i\bar{\pi}_\alpha\dot{\theta}_\alpha + m^2 \qquad (7)$$

It is now easy to recognize in (6) the representation in the form of the path integral of the operator equation

$$K(x_1\theta_1\,;\,x_2\theta_2) = \left\langle x_1\theta_1\right|\frac{1}{\partial^2 + \frac{1}{a}\bar{D}D + m^2}\left|x_2\theta_2\right\rangle;\; D = \frac{\partial}{\partial\theta} - i\bar{\theta}\,. \qquad (8)$$

In order to deduce the relation (8) we have used the direct generalization of the Feynman formula [6] for the case of configuration space described by means of commutative and Grassman coordinates.

Consider in detail the simplest case of two-dimensional space. The Lagrangian that results in the propagator (8) reads

$$\mathcal{L} = \int d^2x\,d^2\theta\left[\bar{\Phi}\bar{D}D\Phi + \bar{\Phi}(m^2+\partial^2)a\Phi\right] =$$
$$= \int d^2x\left\{\varphi(\partial^2 + a^2(m^2+\partial^2)^2)\varphi + \bar{\psi}(\quad + a(m^2+\partial^2))\psi\right. \qquad (9)$$

where we have used the expansion $\Phi = \varphi + \bar{\theta}\psi + \bar{\psi}\theta + \bar{\theta}\theta F$ and eliminated the field $F$ . The poles corresponding to physical particles are obtained from equation

$$p^2 = M^2 \; ; \qquad a(m^2 + M^2) = M \tag{10}$$

Thus, we have two scalar super-multiplets with masses $M_{1,2} = -\frac{1}{2a} \pm \sqrt{\frac{1}{2a^2} - m^2}$ , respectively. If we turn to the limit $a \to 0$ at the fixed value of the mass $M_1$ , the mass $M_2$ will go into infinity, and only one super-multiplet will survive.

Thus, we have shown that the path integral

$$\int \mathcal{D}x(\tau) \int \mathcal{D}\theta(\tau)\, e^{-m\int d\tau \sqrt{(\dot{x}_\mu + i\bar{\theta}\gamma_\mu\dot{\theta})^2}} \tag{11}$$

coincides with the propagator of the appropriate quantum-field theory (with a renormalized mass).

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