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ЦЕНТРАЛЬНЫЙ НАУЧНО-ИССЛЕДОВАТЕЛЬСКИЙ ИНСТИТУТ ИНФОРМАЦИИ И ТЕХНИКО-ЭКОНОМИЧЕСКИХ ИССЛЕДОВАНИЙ ПО АТОМНОЙ НАУКЕ И ТЕХНИКЕ

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Quantum effects undoubtedly had played crucial role at the earliest phases of evolution of the Universe. In particular, they could remove singularities inherent in the cosmological models. Such nonsingular solution was obtained when considering the Einstein equations with one-loop corrections describing the polarization of the vacuum of physical fields in classical gravitational field [1]. Later on, it was shown [2] that de Sitter type solution is unstable with respect to the metric fluctuations.

In this paper, we show that the Einstein equations with such polarized vacuum possess also stable Friedmann type solution for the radiation dominated Universe.

Consider the Einstein equations with one-loop corrections describing the contribution to the vacuum polarization from the massless physical fields:

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \ell^2 \langle T^{\mu\nu} \rangle ; \qquad \ell^2 = 8\pi G \quad (\hbar = c = 1) \tag{1}$$

for the Friedmann-Robertson-Walker metric

$$ds^2 = -(dt)^2 + a^2(t)\left\{(d\chi)^2 + f^2(\chi)\left[(d\vartheta)^2 + \sin^2\vartheta (d\varphi)^2\right]\right\} \tag{2}$$

$$f(x) = \begin{cases} x\ , & \kappa = 0 \\ \sin x, & \kappa = 1 \\ \operatorname{sh} x, & \kappa = -1\ , \end{cases}$$

where $\langle T^{\mu\nu} \rangle$ is given by the following expression [3] (for completeness, see also [4]):

$$\langle T^{\mu\nu} \rangle = \alpha \left(R^{\mu\rho} R^{\nu}_{\rho} - \frac{2}{3} R R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R^{\rho\lambda} R_{\rho\lambda} + \frac{1}{4} g^{\mu\nu} R^2 \right) + \tag{3}$$

$$+ \frac{\beta}{6} \left(2R^{;\mu\nu} - 2g^{\mu\nu} R^{;\rho}_{\ \rho} - 2RR^{\mu\nu} + \frac{1}{2} g^{\mu\nu} R^2 \right).$$

The coefficients α , β arise as a result of summation of contributions of quantum fields of different spins. The condition of applicability of the one-loop approximation with the classical gravitational field is

$$| R^{\mu\nu\rho\lambda} R_{\mu\nu\rho\lambda} | << \ell^{-4}.$$

First, note an important circumstance connected with the structure of $\langle T^{\mu\nu} \rangle$. One can readily check that solutions of the equation

$$R^{\mu\nu} - \frac{1}{4} g^{\mu\nu} R = 0 \tag{4}$$

at $R = -12/\alpha\ell^2$ and $R = 0$ satisfy the initial equations (1), (3).

Here, the first of the conditions $R = -12/\alpha\ell^2$ corresponds to the above-mentioned de Sitter solution [1] .

In the most interesting case

$$|\beta| >> \max \left\{ |\alpha|,\ t^2/\ell^2 \right\}$$

for metric (2) equations (1), (3) can be rewritten in the form

$$(\dot{a}^2 + \kappa)^2 - a^2 \ddot{a}^2 + c\dot{a} = 0 \tag{5}$$

$$a^3 \frac{d}{dt} \left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2 + \kappa}{a^2} \right) = \frac{c}{2}\ ,$$

where C is a constant. If $C = 0$, eqs.(5) have only two types of solutions - de Sitter (R = const $\neq 0$) and Friedmann ones ($R = 0$, $k = 1$, $k = -1$) coinciding with the solution of Einstein equations together with the equation of photon gas state ($P = \varepsilon/3$).

For $C \neq 0$, eqs.(5) turn into one equation

$$c\sqrt{u} + (u + \kappa)^2 = \frac{a^2 u'^2}{4}\ , \quad \sqrt{u(a)} = \dot{a} \tag{6}$$

which is integrable at $k = 0$:

$$\int_{t_0}^{t} dt = (4A)^{1/3} \int_{a_0}^{a(t)} \frac{a^{\sigma} da}{(a^{3\sigma} - Ac)^{2/3}}\ ; \quad \begin{matrix} \sigma = \pm 1 \\ A > 0 \end{matrix} \tag{7}$$

At $a << 1$, from (7) we get a single solution in a form

$$a(t) \sim \text{const} \cdot t^{1/2} \tag{8}$$

corresponding to the above-mentioned case with $R = 0$. * As is seen from (5), conditions $k = 0$ and $a << 1, (\dot{a} > 0)$ are boundary ones for definition of C i.e. the solution (8) does not depend on C .

Thus, for the case of early and flat universe, (8) is the <u>only</u> solution of eqs. (5).

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* An analogous Friedmann solution was obtained in [8] from initial equations with $\alpha = 0$ and material tensor $T^{\mu\nu} = (p + \varepsilon) u^{\mu} u^{\nu} - p g^{\mu\nu}$, $(p = \varepsilon/3)$.

It can also be shown that the partial solution of the initial equations (1), (3) can be represented as the expansion in α/β . ($\beta \gg t^2/\ell^2$), $a \ll 1$:

$$a(t) = \sqrt{Mt} + \frac{\alpha}{\beta}\sqrt{Nt}\,\ell n(St) + O\left(\frac{\alpha}{\beta}\right);$$
$$M, N, S = const$$

which means an effective renormalization of (8) for small t : $t^{1/2 + O(\alpha/\beta)}$

This points out the correctness of the assumptions made when deriving solution (8).

One can readily be convinced that solution (8) is stable relative to small perturbations of metric: in the case we are interested in they decrease with time as $t^{-1/2}$

Thus the Einstein equations with one-loop quantum corrections besides the solutions of de Sitter type ($R = -12/\alpha\ell^2$) have, as applied to the earliest Universe ($a \ll 1$), stable solutions of Friedmann type, and one can say that polarized vacuum takes the role of the photon gas.

Such a scenario of evolution of the earliest Universe corresponds to zero value of the cosmological constant $\Lambda = R - \ell^2(3p - \varepsilon)$

In other words, $\Lambda = 0$ holds not only for the flat radiation dominated Universe [5] , but also for the "empty" Universe.

The "Friedmann" solution found here may be of interest as concerning the ideas of the inflationary Universe [6, 7].

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