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ЦЕНТРАЛЬНЫЙ НАУЧНО-ИССЛЕДОВАТЕЛЬСКИЙ ИНСТИТУТ ИНФОРМАЦИИ И ТЕХНИКО-ЭКОНОМИЧЕСКИХ ИССЛЕДОВАНИЙ ПО АТОМНОЙ НАУКЕ И ТЕХНИКЕ

ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

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PARTICLE MASSES AND MIXINGS IN GRAND UNIFIED MODEL SO(10)

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As is known, the SO(10) grand unified theory [1] contains, as an electroweak subgroup, the product $SU_L(2)xSU_R(2)xU(1)$ with a higher than in SU(5) [2] left-right symmetry. Therefore the introduction of several multiplets of Higgs bosons makes it possible to couple the mixing angles with fermion masses.

It is known that if only Higgs fields from the representation $\underline{10}$ contribute to the fermion masses in the SO(10) scheme, then masses of charged leptons and down quarks become equal (for the energy $\sim 10^{15}$ GeV), what agrees with experiment only for the third fermion generation. Therefore, as Higgs fields giving masses to fermions, we introduce, together with one field in the representation $\underline{10}$, one more Higgs field in the representation $\underline{126}$.

The introduction of several multiplets of Higgs fields may lead, generally speaking, to undesirable transitions with flavor nonconservation; however such transitions may be suppressed if all the Higgs doublets except one become superheavy [3,4].

An additional discrete symmetry we need for Yukawa couplings of fermions with Higgs fields leads to the mass relations for the three generations of fermions. In this case, the relations connecting the masses of light down quarks with those of leptons do not take place, and relation between the masses of b -quark and τ -lepton remains in somewhat modified form.

The relations obtained allow one also to express all mixing angles in fermion currents (in particular, Kobayami-Maskawa angles and mixing angles in currents connected with the proton decay) via the values of particles masses. Neglecting CP-violation one can obtain constraint for the t -quark mass. There is also a relation between the t -quark mass and the mixing angles in currents with baryon number nonconservation.

The mass operator of fermions in SO(10) has the form:

$$\underline{16} \times \underline{16} = \underline{10} + \underline{120} + \underline{126} \tag{1}$$

A complete form of Yukawa couplings of $\underline{10}$-plet and $\underline{126}$-plet of Higgs bosons with fermions involves many terms. It is natural therefore to introduce an additional condition for selecting these terms. Such condition may be a discrete symmetry:

$$\begin{gathered} \underline{10} \longrightarrow \underline{10}\ , \qquad \underline{126} \longrightarrow -\underline{126} \\ \Psi_1 \longrightarrow \Psi_1\ , \quad \Psi_2 \longrightarrow -\Psi_2\ , \quad \Psi_3 \longrightarrow \Psi_3 \end{gathered} \tag{2}$$

where Ψ_f is the f -th family of left fermions united into a 16-plet [5,6] :

$$\Psi_1^T = (\nu_e, u_1, u_2, u_3, e, d_1, d_2, d_3, -d_3^c, -d_2^c, d_1^c, -e^c, u_3^c, -u_2^c, -u_1^c, \nu^c)_L \tag{3}$$

and so on.

With respect to discrete symmetry (2), Yukawa couplings of fermions with Higgs fields are as follows:

$$\begin{gathered} A_{11} \bar{\Psi}_1^c \underset{\sim}{\Phi} \Psi_1 + A_{22} \bar{\Psi}_2^c \underset{\sim}{\Phi} \Psi_2 + A_{33} \bar{\Psi}_3^c \underset{\sim}{\Phi} \Psi_3 + A_{13} \bar{\Psi}_1^c \underset{\sim}{\Phi} \Psi_3 + \\ + A_{12} \bar{\Psi}_1^c \underset{\sim}{\mathfrak{X}} \Psi_2 + A_{23} \bar{\Psi}_2^c \underset{\sim}{\mathfrak{X}} \Psi_3 + h.c. \end{gathered} \tag{4}$$

Here

$$\begin{gathered} \underset{\sim}{\Phi} = \gamma_0 \gamma_\alpha \Phi^\alpha \\ \underset{\sim}{\mathfrak{X}} = \frac{1}{5!} \gamma_0 \gamma_\alpha \gamma_\beta \gamma_\delta \gamma_\sigma \gamma_\varepsilon \mathfrak{X}^{\alpha\beta\delta\sigma\varepsilon} \qquad \alpha \neq \beta \neq \delta \neq \sigma \neq \varepsilon \end{gathered} \tag{5}$$

where Φ^α and $\mathfrak{X}^{\alpha\beta\delta\sigma\varepsilon}$ are $\underline{10}$-plet and $\underline{126}$-plet, respectively, and γ_α are generalized Dirac matrices

$$\{\gamma_\alpha, \gamma_\beta\} = 2\delta_{\alpha\beta}\ , \qquad \alpha, \beta = 0, 1, \ldots, 9 \tag{6}$$

Nonzero vacuum values of neutral colorless components of Higgs fields which give mass to fermions have the form:

$$\begin{gathered} \langle \underset{\sim}{\Phi} \rangle = s \cdot I + p \cdot \mu_{03} \\ \langle \underset{\sim}{\mathfrak{X}} \rangle = \sigma \cdot \mu_{12} \cdot \nu + \pi \cdot \mu_{03} \cdot \mu_{12} \cdot \nu \end{gathered} \tag{7}$$

Here $\mu_{\alpha\beta} = \frac{1}{2i} [\gamma_\alpha, \gamma_\beta]$ are generators of SO(10) group in spinor representation, ν is a singlet in color generator of SU(4), and s, p, σ, π are some parameters.

Mass matrix of fermions will take the form:

$$\begin{gathered} M_u = \kappa \cdot H + \frac{1}{3} g \cdot G \\ M_d = \ell \cdot H - \frac{1}{3} v \cdot G \\ M_e = \ell \cdot H + v \cdot G \end{gathered} \tag{8}$$

where

$$H = \begin{pmatrix} A_{11} & 0 & A_{13} \\ 0 & A_{22} & 0 \\ A_{13} & 0 & A_{33} \end{pmatrix}, \qquad G = \begin{pmatrix} 0 & A_{12} & 0 \\ A_{12} & 0 & A_{23} \\ 0 & A_{23} & 0 \end{pmatrix},$$

$$\kappa = s + p \quad , \qquad g = \sigma + \pi ,$$
$$\ell = s - p \quad , \qquad v = \sigma - \pi . \tag{9}$$

As one can see from (7)-(9), in our scheme the 10 Higgs field contributes only to the diagonal masses of fermions, and mixing between families occurs owing only to the 126 Higgs field contribution*.

In the following, we shall assume all the matrices $\mathcal{M}_u$, $\mathcal{M}_d$, $\mathcal{M}_e$ real, i.e. neglect CP-violation.

Then, matrices (8) can be diagonalized by orthogonal transformation

$$\mathcal{M}_f^{\circ} = O_f \cdot \mathcal{M}_f \cdot O_f^T \quad , \qquad f = u, d, \ell . \tag{10}$$

where $\mathcal{M}_f^{\circ}$ are diagonal matrices.

Matrices $\mathcal{M}_f$, generally speaking, are not positively defined ones, therefore $\mathcal{M}_f^{\circ}$ involves masses of particles with arbitrary signs. Those signs lead to some uncertainty in our predictions.

From (8)-(10), one can arrive at the relations:

$$\mathrm{Tr}\,\mathcal{M}_u^{\circ} = \mathrm{Tr}\,\mathcal{M}_u = \kappa\,\mathrm{Tr}\,H$$
$$\mathrm{Tr}(\mathcal{M}_u^{\circ})^2 = \mathrm{Tr}\,\mathcal{M}_u^2 = \kappa^2\,\mathrm{Tr}\,H^2 + \frac{1}{9} g^2\,\mathrm{Tr}\,G^2 \tag{11}$$
$$\mathrm{Tr}(\mathcal{M}_u^{\circ})^3 = \mathrm{Tr}\,\mathcal{M}_u^3 = \kappa^3\,\mathrm{Tr}\,H^3 + \frac{1}{3} g^2 \kappa\,\mathrm{Tr}(HG^2)$$

---

* Nondiagonal terms in H can be eliminated by the unitary transformation, the matrix G form being unchanged.

$$\mathrm{Tr}\,\mathcal{M}_d^{\circ} = \mathrm{Tr}\,\mathcal{M}_d = \ell\,\mathrm{Tr}\,H$$
$$\mathrm{Tr}(\mathcal{M}_d^{\circ})^2 = \mathrm{Tr}\,\mathcal{M}_d^2 = \ell^2\,\mathrm{Tr}\,H^2 + \frac{1}{9} v^2\,\mathrm{Tr}\,G^2$$
$$\mathrm{Tr}(\mathcal{M}_d^{\circ})^3 = \mathrm{Tr}\,\mathcal{M}_d^3 = \ell^3\,\mathrm{Tr}\,H^3 + \frac{1}{3} v^2 \ell\,\mathrm{Tr}(HG^2)$$
$$\mathrm{Tr}\,\mathcal{M}_e^{\circ} = \mathrm{Tr}\,\mathcal{M}_e = \ell\,\mathrm{Tr}\,H$$
$$\mathrm{Tr}(\mathcal{M}_e^{\circ})^2 = \mathrm{Tr}\,\mathcal{M}_e^2 = \ell^2\,\mathrm{Tr}\,H^2 + v^2\,\mathrm{Tr}\,G^2$$
$$\mathrm{Tr}(\mathcal{M}_e^{\circ})^3 = \mathrm{Tr}\,\mathcal{M}_e^3 = \ell^3\,\mathrm{Tr}\,H^3 + 3v^2 \ell\,\mathrm{Tr}(HG^2)$$

In fact, we have obtained 9 equations with 9 unknowns, two of which are masses of t and b quarks.

All relations given here correspond to energies $\sim 10^{15}$ GeV. They involve the particle masses which are related to the masses at our energies by the normalization coefficients [7]

$$\frac{m_d(\mu)}{m_e(\mu)} = \frac{m_d(M)}{m_e(M)} = \left(\frac{\alpha_s(\mu)}{\alpha(M)}\right)^{\frac{4}{11-\frac{2}{3}f}} \cdot \left(\frac{\alpha\cdot(\mu)}{\alpha(M)}\right)^{\frac{3}{27}}$$
$$\frac{m_u(\mu)}{m_e(\mu)} = \frac{m_u(M)}{m_e(M)} = \left(\frac{\alpha_3(\mu)}{\alpha(M)}\right)^{\frac{4}{11-\frac{2}{3}f}} \cdot \left(\frac{\alpha_1(\mu)}{\alpha(M)}\right)^{\frac{21}{207}} \tag{12}$$

where f is the number of quark flavors.

The energy of 1 GeV is chosen as a normalization point μ for masses of u, d, s quarks. For the rest of quarks it is taken from the condition

$$\mu = 2 \cdot m(\mu) \tag{13}$$

From (11), there follows the relation

$$\mathrm{Tr}\,\mathcal{M}_d^{\circ} = \mathrm{Tr}\,\mathcal{M}_e^{\circ} \tag{14}$$

From this relation, with respect to stated above, one can define the mass of b -quark. However, because of phases, we can get only the upper and lower limits for m_b . The result depends also on the parameter of quantum chromodynamics.

$$4.6 \text{ GeV} \leqslant m_b \leqslant 5.5 \text{ GeV} \qquad \text{for } \Lambda^2 = 0.03 \text{ GeV}^2$$
$$4.1 \text{ GeV} \leqslant m_b \leqslant 5.0 \text{ GeV} \qquad \text{for } \Lambda^2 = 0.005 \text{ GeV}^2 \tag{15}$$

One can see that the mass of b quark is within quite reasonable limits.

Chosen by us form of interaction of fermions with Higgs fields allows to get additional relations.

Let us consider traces of products of different powers of matrices

$$\mathrm{Tr}(\mathcal{M}_u \cdot \mathcal{M}_d) = \kappa \cdot \ell \cdot \mathrm{Tr} H^2 - \frac{1}{9} g \cdot v \cdot \mathrm{Tr}\, G^2$$
$$\mathrm{Tr}(\mathcal{M}_u \cdot \mathcal{M}_d^2) = \kappa \cdot \ell^2 \cdot \mathrm{Tr} H^3 + \left(\frac{\kappa \cdot v^2}{9} - \frac{2 \cdot g \cdot \ell \cdot v}{9}\right) \mathrm{Tr}(HG^2) \tag{16}$$
$$\mathrm{Tr}(\mathcal{M}_u^2 \cdot \mathcal{M}_d) = \kappa^2 \cdot \ell \cdot \mathrm{Tr} H^3 + \left(\frac{\ell \cdot g^2}{9} - \frac{2 \cdot g \cdot \kappa \cdot v}{9}\right) \mathrm{Tr}(HG^2)$$

As is seen, these traces are expressed by the same quantities $\mathrm{Tr} H^2$, $\mathrm{Tr} H^3$, $\mathrm{Tr} G^2$, $\mathrm{Tr}(HG^2)$, κ, ℓ, g, v entering into eqs.(12).

On the other hand,

$$\mathrm{Tr}(\mathcal{M}_u \cdot \mathcal{M}_d) = \mathrm{Tr}(O_d^T \cdot O_u \cdot \mathcal{M}_u^0 \cdot O_u^T \cdot O_d \cdot \mathcal{M}_d^0) =$$
$$= \mathrm{Tr}(K^{-1} \mathcal{M}_u^0 \cdot K \cdot \mathcal{M}_d^0) \tag{17}$$
$$\mathrm{Tr}(\mathcal{M}_u \cdot \mathcal{M}_d^2) = \mathrm{Tr}(K^{-1} \mathcal{M}_u^0 \cdot K \cdot \mathcal{M}_d^{02})$$
$$\mathrm{Tr}(\mathcal{M}_u^2 \cdot \mathcal{M}_d) = \mathrm{Tr}(K^{-1} \cdot \mathcal{M}_u^{02} \cdot K \cdot \mathcal{M}_d^0)$$

where K is Kabayasha-Maskawa (KM) matrix.

Making use of (11), (16), (17) one can obtain relations which connect the mixing angles of KM matrix with the masses of six quarks and charged leptons:

$$\mathrm{Tr}(K^{-1} \cdot \mathcal{M}_u^0 \cdot K \cdot \mathcal{M}_d^0) = \tau_1 \cdot \tau_2 + \frac{1}{8} \cdot \tau_3 \cdot \tau_8$$
$$\mathrm{Tr}(K^{-1} \cdot \mathcal{M}_u^0 \cdot K \cdot \mathcal{M}_d^{02}) = \tau_1^2 \cdot \tau_4 + \frac{\tau_5}{24}(2 \cdot \tau_8 - \tau_1) \tag{18}$$
$$\mathrm{Tr}(K^{-1} \cdot \mathcal{M}_u^{02} \cdot K \cdot \mathcal{M}_d^0) = \tau_1^2 \cdot \tau_4 + \frac{\tau_5}{24}(2 \cdot \tau_1 \cdot \tau_8 - \tau_7)$$

$$\tau_1 = \frac{\mathrm{Tr}\, \mathcal{M}_u^0}{\mathrm{Tr}\, \mathcal{M}_e^0}$$
$$\tau_2 = \frac{1}{8}\left(9\, \mathrm{Tr}(\mathcal{M}_d^0)^2 - \mathrm{Tr}(\mathcal{M}_e^0)^2\right)$$
$$\tau_3 = \mathrm{Tr}\, \mathcal{M}_d^{02} - \mathrm{Tr}\, \mathcal{M}_e^{02}$$
$$\tau_4 = \frac{1}{8}\left(9 \cdot \mathrm{Tr}\, \mathcal{M}_d^{03} - \mathrm{Tr}\, \mathcal{M}_e^{03}\right) \tag{19}$$
$$\tau_5 = \mathrm{Tr}\, \mathcal{M}_d^{03} - \mathrm{Tr}\, \mathcal{M}_e^{03}$$
$$\tau_6 = \mathrm{Tr}\, \mathcal{M}_u^{02}$$
$$\tau_7 = \frac{8}{\tau_3}\left[\tau_2 \cdot \tau_1^2 - \tau_6\right]$$
$$\tau_8 = \sqrt{\tau_7}$$

Similarly, one can obtain the relation for the other mixing angles connected with fermion currents with baryon number nonconservation.

To find out mixing angles in $\bar{d}e$ current, consider traces of products of different powers of matrices $\mathcal{M}_e$, $\mathcal{M}_d$, and introducing matrix K' with respect to their turn, in diagonalization we shall have

$$
\begin{aligned}
&Tr\,(K'^{-1}\cdot \mathcal{M}_d^{0}\cdot K'\cdot \mathcal{M}_e^{0}) = \tau_2 + \tfrac{3}{8}\cdot\tau_3 \\
&Tr\,(K'^{-1}\cdot \mathcal{M}_d^{0}\cdot K'\cdot \mathcal{M}_e^{02}) = \tau_4 - \tfrac{1}{6}\,\tau_6 \\
&Tr\,(K^{-1}\cdot \mathcal{M}_d^{02}\cdot K'\cdot \mathcal{M}_e^{0}) = \tau_4 + \tfrac{5}{24}\cdot\tau_5
\end{aligned}
\tag{20}
$$

Matrix K'' which describes mixing in $\bar{u}e$ current is expressed by K and K' :

$$K'' = K\cdot K' \tag{21}$$

From the derived relations (19), (21), (22) we can, in principle, express mixing angles in weak charged current, and also in currents related to proton decay via particle masses $m_e, m_\mu, m_\tau, m_d, m_s, m_b, m_u, m_c, m_t$.

From (12), one can obtain also a relation connecting the mass of t - quark with ones of the remaining quarks and charged leptons:

$$
\begin{aligned}
u^3+c^3+t^3 &= \frac{1}{8}\left(\frac{u+c+t}{e+\mu+\tau}\right)^3\left[9(d^3+s^3+b^3)-(e^3+\mu^3+\tau^3)\right]+ \\
&+\frac{u+c+t}{e+\mu+\tau}\left[u^2+c^2+t^2-(u+c+t)^2\right]\times \\
&\times\frac{e^3+\mu^3+\tau^3-(d^3+s^3+b^3)}{e^2+\mu^2+\tau^2-(d^2+s^2+b^2)}
\end{aligned}
\tag{22}
$$

Here, it is assumed that relation (23) involves particle masses with arbitrary signs (+ and -), i.e. $\mathcal{M}_d^0$, $\mathcal{M}_u^0$, $\mathcal{M}_e^0$ involve only real quantities.

The solution of eqs.(18), (20), (22) yields the following results.

Mass renormalization depends on QCD dimensional parameter Λ. We shall consider two possibilities: $\Lambda^2 = 0.03$ GeV^2 and $\Lambda^2 = 0.005$ GeV^2. For each value of Λ we shall get a few solutions of eqs.(23) for m_t, and hence for mixing angles. Here we considered solutions for which $m_t > 18.8$ GeV.

First, note, that there is no solution with $m_t > 40$ GeV for $\Lambda^2 = 0.03$ GeV^2 and with $m_t > 36.5$ GeV for $\Lambda^2 = 0.005$ GeV^2, i.e. one may claim that the mass of $t\bar{t}$ bound state is less than 80 GeV.

An interesting relation takes place between the mass of t quark and mixing angles in currents connected with the proton decay.

Namely, if

$$26.5 \text{ GeV} \leqslant m_t \leqslant 36.4 \text{ GeV} \qquad \text{for } \Lambda^2 = 0.005 \text{ GeV}^2$$

$$28.6 \text{ GeV} \leqslant m_t \leqslant 39.3 \text{ GeV} \qquad \text{for } \Lambda^2 = 0.03 \text{ GeV}^2,$$

then mixing matrices in $\bar{d}e$ and $\bar{u}e$ currents are nearly diagonal, the coefficient in currents $\bar{u}\mu$, $\bar{d}\mu$, $\bar{s}e$ $\leqslant 0.1$. This means that if the t -quark mass is in the limits mentioned, then the proton decays mainly by modes $e^+\pi^0$, μ^+K^0 etc.

If

$$m_t < 26.5 \text{ GeV} \qquad \text{for} \quad \Lambda^2 = 0.005 \text{ GeV}^2$$

and

$$m_t < 28.6 \text{ GeV} \qquad \text{for} \quad \Lambda^2 = 0.003 \text{ GeV}^2,$$

then the above situation, generally speaking, doesn't take place, and the dominant decay modes might be e^+K^0, $\mu^+\pi^0$ etc.

Note also, that in all the cases considered, the Cabbibo and the other angles in KM matrix are small, of the order of 0.2-0.3 *.

In conclusion, we can say that our scheme allows to obtain interesting mass relations for charged leptons and quarks. All mixing angles in fermion currents are expressed through particle masses. Reasonable limits for b - quark mass are derived. Interesting predictions are obtained for t -quark mass, dependent on the probability of various modes of proton decay.

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* Ambiguities in phases do not allow one to obtain exact predictions for those angles.

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