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ЦЕНТРАЛЬНЫЙ НАУЧНО-ИССЛЕДОВАТЕЛЬСКИЙ ИНСТИТУТ
ИНФОРМАЦИИ И ТЕХНИКО-ЭКОНОМИЧЕСКИХ ИССЛЕДОВАНИЙ
ПО АТОМНОЙ НАУКЕ И ТЕХНИКЕ

ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

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TOPOLOGICAL ASPECTS OF GAUGE AND SPIN POTTS MODELS

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ТОПОЛОГИЧЕСКИЕ АСПЕКТЫ КАЛИБРОВОЧНЫХ И СПИНОВЫХ МОДЕЛЕЙ ПОТТСА

Для d - мерной калибровочной модели Поттса получено представление для статистической суммы, в котором суммирование по полевым переменным заменено на суммирование по поверхностям (ориентируемым и неориентируемым), образованным плакетами решетки. При этом существенно используются топологические инварианты (числа Бетти) этих поверхностей. Обсуждаются некоторые возможные применения такого представления для статистической суммы.

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Spin [1] and gauge [2] Potts models are the natural generalization of the Ising spin and gauge models to more-than-two components. These models are interesting theoretical systems which undergo first- and second-order phase transitions.

It is rigorouly proved that two-dimensional Q-component spin Potts model has a first-order phase transition for $Q \geqslant 4$, and higher-order transition for $Q < 4$ [3]. As to the two-dimensional gauge Potts model, it is equivalent to the one-dimensional spin model and has no phase transition.

The three-dimensional gauge Potts model is dual to the spin one [4]. A number of approximate methods were used to establish that it has first-order phase-transition for $Q \geqslant 3$ [5].

d-dimensional Potts models were investigated with the help of $1/Q$ - expansion in Hamiltonian approach [6]. The Lagrangian version of $1/Q$-expansion for these models in four dimensions was investigated in [7], but since they calculated only lowest orders, the contribution of nonoriented manifolds was not noticed.

In this paper, we have found the representation for the partition function of the gauge Potts model in four dimensions, which explicitly takes into account both oriented and nonoriented manifolds. We shall essentially us modulo 2 homologies since they are irrelevant to the orientation of the manifold. The representation for the partition function, proposed here, will be useful for different exact and approximate (high-temperature, 1/Q-expansions etc.) schemes of the investigation of the lattice gauge Potts models in any dimension d. This representation is similar to the one proposed by Baxter for the spin systems [3] .

The Q-state spin Potts model [1] is described by the action

$$S = -\beta \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j} \tag{1}$$

where the site variables σ_i range over the discrete values 0,1,2,...Q-1, and the sum in (1) extends over nearest neighbours on a d-dimensional lattice.

The partition function, following Baxter, for this model can be written as

$$Z(Q,\beta) = \sum_{\{\sigma_i\}} \prod_{\langle ij \rangle} \left(1 + v\,\delta_{\sigma_i \sigma_j}\right) \tag{2}$$

where $v = e^{\beta} - 1$

The expression (2) takes the following simple form after carrying out the spin summations

$$Z_G^{spin}(Q,\beta) = \sum_{G' \subset G} v^{b(G')} Q^{n(G')} \tag{3}$$

where the summation goes over all subgraphs G' (i.e. ways of drawing lines on the edges of the lattice G), $b(G')$ is the number of lines in G' and $n(G')$ is the number of connected pieces.

Next, rewrite (3) in the form

Next, multiply out the product (6), and each term $\sum_{\{R_i\}} \upsilon^{f} \underbrace{\delta_{R_1R_2R_3R_4,1} \cdots \delta_{R_1R_2R_3R_4,1}}_{f}$ put in correspondence with a two-dimensional submanifold G_f of the lattice G, constructed from the plaquettes involved in δ-symbols. In the result of the summation over link variables each δ-symbol (plaquette) will impose one constraint, until the situation when the previous plaquettes form the boundary of the following plaquette, and while adding it one changes the second homology group $H_2(G_f)$ [9] of the submanifold. Thus we reduce the summation over link variables in (6) to the one over submanifolds of the lattice G

$$Z_G^{gauge} = \sum_{G_f \subset G} \tilde{\upsilon}^{f(G_f)} Q^{E - f(G_f) + R_2^{(2)}(G_f)} \tag{7}$$

$$Z_G^{gauge} = Q^E \sum_{G_f \subset G} \left(\frac{\tilde{\upsilon}}{Q}\right)^{f(G_f)} Q^{R_2^{(2)}(G_f)} \tag{7a}$$

The summation now goes in (7), (7a) over all two-dimensional submanifolds G_f (i.e. way of putting plaquettes (faces) on the hypercubic lattice G). $f(G_f)$ is the number of plaquettes in G_f. E is the full number of edges in the lattice G. $R_2^{(2)}(G_f)$ is modulo 2 second Betti number [9] of the submanifold G_f.

We used modulo 2 homologies with the purpose to treat nonorientable submanifolds too. It is necessary to point here that for the $d > 4$ case $R_2^{(2)}$-topological invariant might be not enough to treat all types of submanifolds G_f, constructed of plaquettes of the hypercubic lattice, since Z_p ($p > 2$) multiplier can be generated in the second homology group of G_f.

If we consider only $d \leqslant 4$-dimensional lattices, then modulo 2 second Betti number ($R_2^{(2)}$) will be enough for us to describe all submanifolds G_f. From this point of view, S^2-sphere (oriented) and Klein bottle (nonoriented) can be considered on the same grounds.

The partition function Z_G^{gauge} written in the form (7), (7a) can be used to derive duality relations in any dimension d.

It is known that three-dimensional gauge model is dual to the spin one. To show that, we use the following theorem from the theory of homologies [10] : if we have d-1-dimensional closed manifold P_{d-1} in d dimensional space, then the space is divided into $R_{d-1}(P) + 1$ -connected pieces where $R_{d-1}(P)$ is (d-1)-th Betti number of the manifold R_{d-1}. Hence $R_2(G_f) + 1 = n(G')$, where subgraph G' is dual to G_f. Since G' is dual to G_f $f(G_f) = E - \beta(G')$. Now the duality transformation looks like

$$Z_G^{gauge}(\tilde{\upsilon}, Q) = Q^E \sum_{G_f} \left(\frac{\tilde{\upsilon}}{Q}\right)^{f(G_f)} Q^{R_2(G_f)} = Q^E \sum_{G'} \left(\frac{\tilde{\upsilon}}{Q}\right)^{E - \beta(G')} Q^{n(G')} = \tag{8}$$

$$= \tilde{\upsilon}^E Z_G^{spin}(Q, \upsilon)$$

where $\upsilon \tilde{\upsilon} = Q$.

In the same way we can treat four-dimensional gauge model, which is self-dual. Let $\mathcal{D}_f$ be the submanifold dual to G_f; evidently $f(\mathcal{D}_f) = F - f(G_f)$, where F is the full number of plaquettes. Now using the relation $R_2^{(2)}(\mathcal{D}_f) = R_2^{(2)}(G_f) - f + \frac{E}{2}$ which is some generalization of the Poincaré's theorem [11] we have

The detailed investigations of these questions will be published elsewhere.

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