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ЦЕНТРАЛЬНЫЙ НАУЧНО-ИССЛЕДОВАТЕЛЬСКИЙ ИНСТИТУТ ИНФОРМАЦИИ И ТЕХНИКО-ЭКОНОМИЧЕСКИХ ИССЛЕДОВАНИЙ ПО АТОМНОЙ НАУКЕ И ТЕХНИКЕ

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SYMMETRIES OF RENORMALIZED THEORIES

I. NON-GAUGE THEORIES

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СИММЕТРИИ ПЕРЕНОРМИРОВАННЫХ ТЕОРИЙ

I. НЕКАЛИБРОВОЧНЫЕ ТЕОРИИ

Исследованы свойства симметрии перенормированных теорий поля с симметричным классическим действием в общем случае, когда не пренебрегается якобианом замены переменных в функциональном интеграле. Показано, что любой симметрии классического действия соответствует определенная симметрия перенормированного квантового действия и перенормированного производящего функционала вершинных функций.

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SYMMETRIES OF RENORMALIZED THEORIES

I. NON-GAUGE THEORIES

The symmetry properties of the renormalized field theories the classical actions of which have symmetry properties are studied in the general form, when the jacobian of change of variables in the functional integral is not ignored. It is shown that to any symmetry of classical action corresponds a certain symmetry of renormalized quantum action and renormalized generating functional of proper vertices.

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1. In Ref. [1] we have investigated the question about the properties of symmetry of the renormalized field theories the classical actions of which are symmetric under some transformations of the field variables

$$\phi^i \longrightarrow \phi^i + \xi R^i(\Phi) \tag{1}$$

where ξ is coordinate-independent infinitesimal parameter, $R^i(\Phi)$ is a local functional of fields Φ and their derivatives. It was shown that the renormalized quantum action $S_R(\Phi)$ of the theory and generating functional of proper vertices (GFPV) $\Gamma_R(\Phi)$ also has certain symmetry properties. In terms of the renormalized quantities the transformations were found, induced by transformation (1) and leaving $S_R(\Phi)$ and $\Gamma_R(\Phi)$ invariant The locality of functional $R^i(\Phi)$ together with the assumption on the presence of regularization (e.g. dimensional) conserving the theory symmetry properties ensured equality to unity of jacobian $\left|\delta\left(\phi^i + \xi R^i(\phi)\right) / \delta\phi^j\right|$ of change of variables (1) in the functional integral. The present work is devoted to the study of the same question in the most general form, when, in particular, the transformation jacobian is not ignored. Here, as in Ref. [1], being within the framework of perturbation theory (to be more precise, of loop-wise expansion), we do not restrict ourselves to the index-renormalized theories.

It is obvious that the presence of the nonunitary variable transformation jacobian which, generally speaking, does not possess the symmetry properties that the classical action $S_{cl}(\Phi)$ of the system possesses, violates the Ward identity standard form. In this work we shall formulate a number of verisimilar hypotheses with respect to the structure of the input quantum actions of theories, allowing to obtain the Ward identities in their usual form. We shall need a relation between the renormalized actions of two theories related to each other by change of variables (the exact formulation is given below) in the case when the change of variables jacobian differs from unity. This question is considered in Sec.2. It is shown that if the quantum theories are constructed by the actions related by the change of variable in the sense of (5), then also the renormalized actions are related to each other by some transformation of variables in the sense of (5), and GFPV are related to each other by some transformation in the usual sense.

In Sec.3 we have formulated the verisimilar hypotheses on the structure of the input quantum action of theory; for that, we have obtained the condition analogous to the symmetry condition of the classical action (see (20), (22)), which naturally can be called the quantum action symmetry condition. Sec.4 shows that if the input quantum action is symmetric in the sense of Sec.3, then also the renormalized action possesses some symmetry in the sense of Sec.3, while the renormalized GFPV possesses some symmetry already in the usual sense.

Both boson and fermion fields are assumed present in the theory; the set of them we shall denote by Φ^i. We shall use condensed notations when all the particular indices (coordinate, Lorentz, isotopic, etc.) are combined into the general index. The derivatives over the fields are right, over the sources are left. Note, that here we shall consider theories without gauge symmetries (the case of gauge theories will be considered separately).

To conclude this section, let us introduce one definition. With that aim, let us consider the theory which at some choice of the field variables Φ^i is described by the action $S(\Phi)$. Write down the generating functional $Z(J)$ of the theory

$$Z(J)=\int d\Phi \exp\left\{\frac{i}{\eta}\left[S(\Phi)+J\Phi\right]\right\} \tag{2}$$

(η is the loop expansion parameter) and make in the functional integral change of variables

$$\Phi^i = F^i(\Phi', \lambda), \qquad F^i(\Phi, 0) = \Phi^i$$

where $\{\lambda^a\}$ is the set of parameters describing the parametrization (in the following, they will not be written down explicitly). We shall get (omitting the primes in Φ')

$$Z(J)=\int d\Phi \exp\left\{\frac{i}{\eta}\left[S(F(\Phi))+\mu(\Phi)+JF(\Phi)\right]\right\}= \tag{3}$$

$$=\int d\Phi \exp\left\{\frac{i}{\eta}\left[S'(\Phi)+JF(\Phi)\right]\right\}, \quad S'(\Phi)=S(F(\Phi))+\mu(\Phi)$$

where $\mu(\Phi)$ is defined by the relation

$$\mu(\Phi)=-i\eta \, \mathrm{str}\,\ln\left(\frac{\delta F^i(\Phi)}{\delta\Phi^j}\right) \tag{4}$$

("str" means supertrace; see the definition, e.g. in [2]). In what follows we shall speak about the transition from (2) to (3), (4) as about transition from Φ parametrization to Φ' parametrization of functional space. On the mass shell, the theory described by (3), (4) is equivalent to that with the generating functional

$$Z(y)=\int d\Phi \exp\left\{\frac{i}{\eta}\left[S'(\Phi)+y\Phi\right]\right\}$$

in virtue of the equivalence theory [3]. These considerations induce the reasons to introduce the following definition:

Definition.

Two quantum actions $S(\Phi)$ and $S'(\Phi')$ are related by the change of variables $\Phi \to F(\Phi)$ if the relation

$$S'(\Phi)=S(F(\Phi))-i\eta\, \mathrm{str}\, \ln\left(\frac{\delta F^{i}(\Phi)}{\delta\Phi^{j}}\right) \tag{5}$$

holds.

2. In this section, we deduce the relationship between the renormalized actions and GFPV of theory whose input quantum actions are connected by the change of variable in the sense of (5). We shall consider the infinitesimal transformations

$$\Phi^{i} \to F^{i}(\Phi)=\Phi^{i}+\varepsilon\Delta\Phi^{i}(\Phi) \tag{6}$$

where ε is the coordinate-independent infinitesimal Grassmann-even quantity, $\Delta\Phi(\Phi)$ is some functional of fields and their derivatives, which has the Grassmann parity of the field Φ.

Let $S(\Phi)$ and

$$\begin{aligned} S'(\Phi)&=S(\Phi+\varepsilon\Delta\Phi)-i\eta\, \mathrm{str}\, \ln\frac{\delta(\Phi^{i}+\varepsilon\Delta\Phi^{i})}{\delta\Phi^{j}}\approx \\ &\approx S(\Phi+\varepsilon\Delta\Phi)-i\eta\varepsilon\, \mathrm{str}\frac{\delta\Delta\Phi^{i}}{\delta\Phi^{j}}\equiv \bar{S}'(\Phi)-i\eta\varepsilon\, \mathrm{str}\frac{\delta\Delta\Phi^{i}}{\delta\Phi^{j}} \end{aligned} \tag{7}$$

be two actions in Φ and Φ' parametrizations, and $Z(\mathcal{J})$ and $Z'(\mathcal{J})$ are the corresponding generating functionals (everywhere below, the functionals written in one or another parametrization will be supplied with the index to distinguish this parametrization between the others; **indices of** functionals' arguments **will as a rule** be omitted). Sign $\approx$ is used to denote equality to the accuracy up to terms of the order of ε^2. Let us make in the expression

$$Z'(\mathcal{J})=\int d\Phi \exp\left\{\frac{i}{\eta}\left[S(\Phi+\varepsilon\Delta\Phi)-i\eta\varepsilon\, str\,\frac{\delta\Delta\Phi^i}{\delta\Phi^j}+\mathcal{J}\Phi\right]\right\}$$

the change of variables $\Phi\to\Phi-\varepsilon\Delta\Phi(\Phi)$. Then

$$Z'(\mathcal{J})=\int d\Phi \exp\left\{\frac{i}{\eta}\left[S(\Phi)+\mathcal{J}\Phi-\varepsilon\mathcal{J}\Delta\Phi(\Phi)\right]\right\}\approx$$

$$\approx\int d\Phi\exp\left\{\frac{i}{\eta}\left[S(\Phi)+\mathcal{J}\Phi\right]\right\}\left(1-\frac{i\varepsilon}{\eta}\mathcal{J}\Delta\Phi(\Phi)\right)=Z(\mathcal{J})-\frac{i\varepsilon}{\eta}\mathcal{J}\Delta\Phi(\Phi)Z(\mathcal{J}) \tag{8}$$

In the last expression the substitution $\Phi^i\to -i\eta\delta/\delta\mathcal{J}_i$ is assumed. From (8) we obtain

$$\Delta Z(\mathcal{J})=Z'(\mathcal{J})-Z(\mathcal{J})\approx-\frac{i\varepsilon}{\eta}\mathcal{J}\Delta\Phi(\Phi)Z(\mathcal{J}) \tag{9}$$

Next, introduce in a usual way the GFPV $\Gamma(\Phi)$:

$$\Gamma(\Phi)=-i\eta\ln Z(\mathcal{J})-\mathcal{J}\Phi,\quad \Phi^i=-i\eta\frac{\delta}{\delta\mathcal{J}_i}\ln Z(\mathcal{J})$$

In terms of $\Gamma(\Phi)$, eq.(9) will be written in the form

$$\Delta\Gamma(\Phi)=\Gamma'(\Phi)-\Gamma(\Phi)\approx\varepsilon\frac{\delta\Gamma(\Phi)}{\delta\Phi^i}<\Delta\Phi^i(\Phi)>, \tag{10}$$

here $\Gamma(\phi)$ and $\Gamma'(\phi)$ are GFPV of theories with the actions $S(\phi)$ and $S'(\phi)$, respectively, and $\langle \Delta\phi(\phi) \rangle$ is the vacuum expectation value of operator $\Delta\phi(\phi)$ considered as a function of ϕ which is obtained from $\Delta\phi(\phi)$ by a substitution

$$\phi^i \to \phi^i + i\eta(-1)^{p_j(p_i+1)} \left[(\Gamma'')^{-1}\right]^{ij} \frac{\delta_L}{\delta\phi^j}$$

where $\delta_L/\delta\phi$ is the left derivative, p_i is the Grassmann parity of field ϕ^i and through $\left[(\Gamma'')^{-1}\right]^{ij}$ the matrix inverse to the $(\Gamma'')_{ij} = \delta^2\Gamma/\delta\phi^i\delta\phi^j$ one is denoted.

We represent the quantities entering into (10) in the form of the loop expansions:

$$\Gamma = S + \eta\left(\Gamma^{(1)}_{div} + \Gamma^{(1)}_{fin}\right) + O(\eta^2)$$
$$\Gamma' = \bar{S}' + \eta\left(\Gamma'^{(1)}_{div} + \Gamma'^{(1)}_{fin}\right) + O(\eta^2) \qquad (11)$$
$$\langle\Delta\phi\rangle = \Delta\phi + \eta\left(\langle\Delta\phi\rangle^{(1)}_{div} + \langle\Delta\phi\rangle^{(1)}_{fin}\right) + O(\eta^2)$$

With account of the written out expansions in the tree approximation from (10) we arrive at the relation

$$\Delta S = \bar{S}'(\phi) - S(\phi) = S(\phi + \varepsilon\Delta\phi) - S(\phi) \approx \varepsilon \frac{\delta S}{\delta\phi}\Delta\phi,$$

while in the one-loop approximation for the divergent and finite parts of $\Delta\Gamma$ we obtain

$$\Delta\Gamma^{(1)}_{div} = \Gamma'^{(1)}_{div} - \Gamma^{(1)}_{div} \approx \varepsilon \frac{\delta S}{\delta\phi}\langle\Delta\phi\rangle^{(1)}_{div} + \varepsilon \frac{\delta\Gamma^{(1)}_{div}}{\delta\phi}\Delta\phi \qquad (12)$$
$$\Delta\Gamma^{(1)}_{fin} = \Gamma'^{(1)}_{fin} - \Gamma^{(1)}_{fin} \approx \varepsilon \frac{\delta S}{\delta\phi}\langle\Delta\phi\rangle^{(1)}_{fin} + \varepsilon \frac{\delta\Gamma^{(1)}_{fin}}{\delta\phi}\Delta\phi$$

Choose now as a new action (in the Φ parametrization)

$$S_1(\Phi) = S(\Phi) - \eta \Gamma_{div}^{(1)} \equiv S_{1R}(\Phi)$$

Then with account of (7) and (12) for the one-loop renormalized action $S_1'(\Phi)$ of the theory in the Φ' parametrization (with the initial action $S'(\Phi)$) we have

$$S_1'(\Phi) = S'(\Phi) - \eta \Gamma_{div}^{\prime (1)}(\Phi) \approx$$

$$\approx S_{1R}(\Phi) + \varepsilon \frac{\delta S}{\delta \Phi} \Delta \Phi - i\eta\varepsilon \, str \frac{\delta \Delta \Phi^i}{\delta \Phi^j} - \varepsilon\eta \frac{\delta S}{\delta \Phi} \langle \Delta \Phi \rangle_{div}^{(1)} - \varepsilon\eta \frac{\delta \Gamma_{div}^{(1)}}{\delta \Phi} \Delta \Phi$$

One can readily see that $S_1'(\Phi)$ coincides with the one-loop approximation of the action

$$S_{1R}'(\Phi) \approx S_{1R}(\Phi + \varepsilon \Delta_1 \Phi) - i\eta\varepsilon \, str \frac{\delta \Delta_1 \Phi^i}{\delta \Phi^j} ,$$

$$\Delta_1 \Phi \equiv \Delta \Phi - \eta \langle \Delta \Phi \rangle_{div}^{(1)}$$

Applying the above consideration to the actions $S_{1R}(\Phi)$ and $S_{1R}'(\Phi)$ we obtain the equation

$$\Delta \Gamma_1(\Phi) = \Gamma_1'(\Phi) - \Gamma_1(\Phi) \approx \varepsilon \frac{\delta \Gamma_1(\Phi)}{\delta \Phi} \langle \Delta_1 \Phi \rangle , \qquad (13)$$

where $\Gamma_1(\Phi)$ and $\Gamma_1'(\Phi)$ are GFPV constructed by $S_{1R}(\Phi)$ and $S_{1R}'(\Phi)$, respectively, being finite in the one-loop approximation.

With account of the relation

$$\langle \Delta_1 \Phi \rangle = \Delta \Phi - \eta \langle \Delta \Phi \rangle_{div}^{(1)} + \eta \langle \Delta \Phi \rangle^{(1)} + O(\eta^2) =$$

$$= \Delta \Phi + \eta \langle \Delta \Phi \rangle_{fin}^{(1)} + O(\eta^2)$$

from formula (13) it follows that

$$\Gamma_1'(\phi) \approx \Gamma_1\left(\phi + \varepsilon\Delta\phi + \varepsilon\eta < \Delta\phi >^{(1)}_{fin}\right)$$

i.e. $\Gamma_1'(\phi)$ comes from $\Gamma_1(\phi)$ by the variable replacement

$$\phi \to \phi + \varepsilon\Delta\phi + \varepsilon\eta < \Delta\phi >^{(1)}_{fin}$$

Applying further the mathematical induction method we finally obtain the relations

$$S_R'(\phi) \approx S_R\left(\phi + \varepsilon\Delta_R\phi(\phi)\right) - i\eta\varepsilon\, \mathrm{str}\left(\frac{\delta\Delta_R\phi^i(\phi)}{\delta\phi^j}\right)$$

$$\Gamma_R'(\phi) \approx \Gamma_R\left(\phi + \varepsilon < \Delta_R(\phi) >\right) \tag{14}$$

where

$$\Delta_R(\phi) = \Delta\phi - \sum_{n=1}^{\infty} \eta^n < \Delta\phi >^{(n)}_{div} \tag{15}$$

and $< \Delta\phi >^{(n)}_{div}$ is the divergent part of the n-loop approximation of the vacuum expectation value with subtracted subdivergences. Formulae (14), (15) express the relationship between the renormalized actions and GFPV of theories, the input actions of which are related by relation (7). Note that the relationship between the renormalized quantum actions in different parametrizations being inhomogeneous, the one between the renormalized GFPV is homogeneous.

3. Now turn to the consideration of the quantum theories whose classical actions have symmetry properties. Namely, formulate some natural **hypotheses** concerning the structure of the input quantum actions of field theories and

consider the consequences following from them.

So,

1) the effective quantum actions of one and the same classical theory in different parametrizations are related by the variable transformation in the sense of relation (5);

2) there exists a parametrization of the functional space ($\phi^{(0)}$ parametrization), such that the effective quantum action of theory $S^{(0)}(\Phi)$ in this parametrization has the form:

$$S^{(0)}(\Phi) = S_{cl}(\Phi) + \lambda(\Phi)$$

where $S_{cl}(\Phi)$ is classical action of theory, and functional $\lambda(\Phi)$ has the same symmetry properties as $S_{cl}(\Phi)$;

3) in the chosen $\phi^{(0)}$ parametrization the generators $R_{\alpha}^{(0)i}(\Phi)$ of quantum action $S_0(\Phi)$ **symmetry** have the property

$$\operatorname{str} \frac{\delta R_{\alpha}^{(0)i}(\Phi)}{\delta \Phi^{j}} = 0 \qquad (16)$$

Present some arguments in favour of the validity of these **hypotheses.** From the consideration at the end of Sec.1 and from the results of Sec.2, it follows that if the actions are related by relation (5), then they will describe one and the same quantum theory. It is evident that their classical parts are related simply by reparametrization. It seems quite natural that the opposite hypothesis, i.e. 1) also holds. In favour of hypothesis 3) speaks the following general result (see, e.g. [4]): by means of variable transformation and the transition to the linear combinations of transformation generators, the latters can be reduced to the shift generators. Of course, such transformation of variables is, generally speaking, nonlocal.

However this fact as well as many other specific examples speak undoubtedly in favour of hypothesis 3). The naturalness of hypothesis 2) also seems intuitively evident.

Consider now the generating functional

$$Z(J) = \int d\Phi \exp\left\{\frac{i}{\eta}\left[S(\Phi) + J\Phi\right]\right\}$$

and make in it the change of variables

$$\phi^i \longrightarrow \phi^i + \xi R^i(\Phi)$$

where ξ is coordinate-independent infinitesimal parameter, and $R^i(\Phi)$ are the generators of the classical action symmetry transformation. We have arrived at

$$Z(J) = \int d\Phi \exp\left\{\frac{i}{\eta}\left[S\big(\Phi + \xi R(\Phi)\big) - i\eta\xi \,\mathrm{str}\,\frac{\delta R^i(\Phi)}{\delta\phi^j} + J\Phi + J\xi R(\Phi)\right]\right\} \approx$$

$$\approx \int d\Phi \exp\left\{\frac{i}{\eta}\left[S(\Phi) + \xi\frac{\delta S}{\delta\Phi}R(\Phi) - i\eta\xi \,\mathrm{str}\,\frac{\delta R^i(\Phi)}{\delta\phi^j} + J\Phi + J\xi R(\Phi)\right]\right\},$$

whence follows the relation

$$\int d\Phi \exp\left\{\frac{i}{\eta}\left[S(\Phi) + J\Phi\right]\right\}\big(\nu(\Phi) + JR(\Phi)\big) = 0, \qquad (17)$$

where $\nu(\Phi)$ is given by the expression

$$\nu(\Phi) = \frac{\delta S}{\delta\phi^i}R^i(\Phi) - i\eta\,\mathrm{str}\,\frac{\delta R^i(\Phi)}{\delta\phi^j}\,. \qquad (18)$$

One can see from relation (17) that if $\nu(\Phi)$ were zero, then (17) would be the Ward identity in its standard form.

Let us investigate how $\nu(\Phi)$ changes when passing to another parametrization (in a new Φ' parametrization denote it $\nu'(\Phi')$). Note that generators $R^i(\Phi)$ and $R'^i(\Phi')$ of the classical action symmetry transformations in a Φ and Φ' parametrizations ($\Phi = F(\Phi')$) are related by the expression

$$R^i(\Phi) = \frac{\delta F^i(\Phi')}{\delta \Phi'^\ell} R'^\ell(\Phi') \tag{19}$$

Expressing $\nu(\Phi)$ in (18) through Φ' and taking into account (19), we obtain

$$\nu(\Phi) = \frac{\delta}{\delta\Phi'^i}\left[S(F(\Phi')) - i\eta\,\mathrm{str}\,\ln\frac{\delta F^\kappa(\Phi')}{\delta\Phi'^\ell}\right]R'^i(\Phi') - i\eta\,\mathrm{str}\,\frac{\delta R'^i(\Phi)}{\delta\Phi'^j} =$$

$$= \frac{\delta S'(\Phi')}{\delta\Phi'^i} R'^i(\Phi') - i\eta\,\mathrm{str}\,\frac{\delta R'^i(\Phi)}{\delta\Phi'^j} \equiv \nu'(\Phi')$$

Thence it follows that functional $\nu(\Phi)$ is a scalar in the variable transformation, and therefore, if in any one parametrization $\nu(\Phi)$ is zero, it will be zero in all the other parametrizations, too.

Consider functional $\nu^{(0)}(\Phi)$ (in the $\Phi^{(0)}$ parametrization):

$$\nu^{(0)}(\Phi) = \frac{\delta S^{(0)}}{\delta\Phi^i} R^{(0)i}(\Phi) - i\eta\,\mathrm{str}\,\frac{\delta R^{(0)i}(\Phi)}{\delta\Phi^j}$$

The first term is zero by hypothesis 2), and the second one is zero by hypothesis 3). Thus $\nu^{(0)}(\Phi) = 0$; hence in any parametrization we have

$$\nu(\Phi) = 0 \tag{20}$$

Let now the action $S(\Phi)$ be given. Consider the action $\tilde{S}(\Phi)$ constructed of $S(\Phi)$ by the variable transformation $\Phi^i \to \Phi^i + \xi R^i(\Phi)$

in the sense of (5):

$$\tilde{S}(\Phi) \approx S(\Phi + \xi R(\Phi)) - i\eta \xi \,\mathrm{str}\, \frac{\delta R^i(\Phi)}{\delta \Phi^j} \tag{21}$$

Taking into account (19) and (20) we have

$$\tilde{S}(\Phi) \approx S(\Phi) \tag{22}$$

Thus, condition (20) generalizes the symmetry condition of classical action and it is natural to call it the symmetry condition of quantum action. It follows from (17) that for the symmetric (in the sense of (20)) quantum action the Ward identity for nonrenormalized theory has a standard form

$$\mathcal{J}_i R^i(\Phi) Z(\mathcal{J}) = 0 \tag{23}$$

(the substitution $\Phi^i \to -i\eta\delta/\delta\mathcal{J}_i$ is assumed).

4. Turn now to the discussion of the symmetry properties of renormalized theory, the input quantum action of which has symmetry (20), (22). For that, we use the results of Sec.2. Since $\tilde{S}(\Phi)$ is obtained from $S(\Phi)$ by the change of variables $\Phi \to \Phi + \xi R(\Phi)$ (see (21)), then according to (14) for the renormalized quantum actions $\tilde{S}_R(\Phi)$ and $S_R(\Phi)$ we have the relation

$$\tilde{S}_R(\Phi) \approx S_R(\Phi + \xi R_R(\Phi)) - i\eta\xi \,\mathrm{str}\, \frac{\delta R^i_R(\Phi)}{\delta\Phi^j},$$

where

$$R^i_R(\Phi) = R^i(\Phi) - \sum_{n=1}^{\infty} \eta^n \langle R^i(\Phi) \rangle^{(n)}_{div}$$

(see the explanation to formula (15)).

On the other hand, in virtue of (22), the equality $\tilde{S}_R(\Phi) = S_R(\Phi)$ also holds. Hence the renormalized quantum action $S_R(\Phi)$ satisfies the condition

$$S_R(\Phi + \xi R_R(\Phi) - i\eta\xi \,\mathrm{str}\, \frac{\delta R^i_R(\Phi)}{\delta \Phi^j} \approx S_R(\Phi) \tag{24}$$

or, what is the same,

$$\mathcal{V}_R(\Phi) \equiv \frac{\delta S_R(\Phi)}{\delta \Phi^i} R^i_R(\Phi) - i\eta \,\mathrm{str}\, \frac{\delta R^i_R(\Phi)}{\delta \Phi^j} = 0$$

Thus, also the renormalized quantum action has some symmetry.

By analogy with the Ward identity (23) for the renormalized generating functional of Green functions $Z_R(\mathcal{J})$ we obtain the identity

$$\mathcal{J}_i R^i_R(\Phi) Z_R(\mathcal{J}) = 0,$$

whence for the renormalized GFPV we have

$$\frac{\delta \Gamma_R(\Phi)}{\delta \Phi^i} \mathcal{R}^i(\Phi) = 0 \ ,$$

$$\mathcal{R}^i(\Phi) \equiv \langle R^i_R(\Phi) \rangle = \langle R^i(\Phi) \rangle - \sum_{n=1}^{\infty} \eta^n \langle R^i(\Phi) \rangle^{(n)}_{div} = \tag{25}$$

$$= R^i(\Phi) + \sum_{n=1}^{\infty} \eta^n \langle R^i(\Phi) \rangle^{(n)}_{fin} \ .$$

In the equivalent form the relation (25) can be rewritten as

$$\Gamma_R(\Phi) \approx \Gamma_R\left(\Phi + \xi \mathcal{R}(\Phi)\right).$$

Thus we can see that if the input quantum action of theory has symmetry in the sense of (20), then the renormalized quantum action and the GFPV also have certain symmetry.

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СИММЕТРИИ ПЕРЕНОРМИРОВАННЫХ ТЕОРИЙ

I.НЕКАЛИБРОВОЧНЫЕ ТЕОРИИ

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