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ЦЕНТРАЛЬНЫЙ НАУЧНО-ИССЛЕДОВАТЕЛЬСКИЙ ИНСТИТУТ
ИНФОРМАЦИИ И ТЕХНИКО-ЭКОНОМИЧЕСКИХ ИССЛЕДОВАНИЙ
ПО АТОМНОЙ НАУКЕ И ТЕХНИКЕ

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SYMMETRIES OF RENORMALIZED THEORIES

II. GAUGE THEORIES

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СИММЕТРИИ ПЕРЕНОРМИРОВАННЫХ ТЕОРИЙ
П. КАЛИБРОВОЧНЫЕ ТЕОРИИ

Исследованы симметричные свойства перенормированных калибровочных теорий общего вида (речь идет о симметриях, помимо калибровочной) в общем случае, когда не пренебрегается величинами типа якобианов замен переменных в функциональном интеграле. Показано, что если классическое действие калибровочной теории обладает дополнительной симметрией, то перенормированное эффективное действие и перенормированный производящий функционал вершинных функций также обладают определенной симметрией.

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SYMMETRIES OF RENORMALIZED THEORIES

II. GAUGE THEORIES

The symmetry properties of the renormalized general gauge theory (the symmetries besides the gauge one are implied) are studied in the general case, when the transformation jacobian-like quantities in the functional integral are not ignored. It is shown that if the classical action of gauge theory has additional symmetry, then the renormalized effective action and the renormalized generating functional of proper vertices also possess definite symmetry.

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1. The present work is devoted to the investigation of the symmetry properties of the renormalized general gauge theories (the symmetries besides the gauge one are implied), the classical action $\mathcal{S}_0(\varphi)$ of which is invariant under some transformations of field variables

$$\varphi^i \to \varphi^i + \xi R^i(\varphi) \tag{1}$$

Here $\varphi = \{\varphi^i\}$ symbolize all the fields of initial classical theory, ξ is the coordinate-independent infinitesimal Grassmann-even parameter, and $R^i(\varphi)$ is certain functional of fields φ and their derivatives, which has Grassmann parity of field φ^i. This work is the continuation of Part I [1] in which non-gauge theories were studied. Note, that in the mentioned aspect the symmetry properties of the renormalized theories - both gauge and non-gauge ones - were studied in Ref.[2]. However in the latter the jacobians of variable transformations in functional integral were considered equal to unity. Here, just as in Part I [1], the symmetries of renormalized gauge theory of the general form are studied in the general case, when the quantities of the type of jacobians of change of variables in functional integral are not ignored.

In Sec.2, the basic assumption on the relationship of two effective actions in different parametrizations is formulated. Sec.3 shows that if the initial classical action possesses symmetry in addition to the gauge one. then the effective action is symmetric under some canonical transformation. Finally, in Sec.4, these symmetry properties are being proved for the renormalized effective action and generating functional of proper vertices (GFPV) We use as before condensed notations (all indices - the coordinate, Lorentz, isotopic. etc. - are united into a general index). The derivatives over the fields are right, over the source - left.

2. It is well known that the total effective action $S_\Psi(K,\Phi)$ of the general gauge theory is constructed according to the rule [3]

$$S_\Psi(\Phi,K)=S_M(\Phi,K)\Big|_{K\to\delta\Psi(\Phi,K)/\delta\Phi} \tag{2}$$

where the modified action

$$S_M(\Phi,K)=S(\varphi,c,\kappa,\ell)+n\mathcal{T}$$

in case when the transformation jacobian-like quantities in the functional integral are not ignored, is determined from the equation

$$\frac{1}{2}(S_M,S_M)=i\eta\frac{\delta}{\delta\Phi^i}\frac{\delta}{\delta K_i}S_M \quad . \tag{3}$$

with the boundary condition at $\eta=0$

$$S_M(\Phi,K=0)\Big|_{\eta=0}=S_0(\varphi),$$

$S_0(\varphi)$ is the initial gauge-invariant action. Here [3]

$$(A,B)=\frac{\delta A}{\delta\Phi^i}\frac{\delta B}{\delta K_i}-(-1)^{(P_A+1)(P_B+1)}\frac{\delta B}{\delta\Phi^i}\frac{\delta A}{\delta K_i}$$

$_A$, P_B are Grassmann parities of functionals A and B; Φ^i is a t of all fields of theory: $\Phi=\{\varphi, c, \pi, \bar{c}\}$, where $\varphi=\{\varphi^\kappa\}$ re fields of initial classical theory with the action $S_0(\varphi)$; $c=\{c^\alpha\}$ $\bar{c}=\{\bar{c}^\alpha\}$ are the ghost fields, $\pi=\{\pi^\alpha\}$ are gauge introducing additional fields, $K_i=\{\kappa, \ell, m, n\}$ is a set of auxiliary sources conjugated to fields $\varphi, c, \pi, \bar{c}$, respectively; degeneration eliminating gauge fermion Ψ has the form

$$\Psi(\Phi,K)=K\Phi+\Psi(\Phi), \qquad (4)$$

η is the loop expansion parameter.

The properties of eq.(3) solutions were studied in Ref.[illegible]. It was shown, in particular, that the solution $S_M(\Phi,K)$ of eq.(3) under the canonical transformation of variables $\Phi,K\to\Phi_x(\Phi,K)$, $K_x(\Phi,K)$ with a generating functional $X(\Phi,K_x)$ transforms into expression $S_{MX}(\Phi,K)$ defined by the relation

$$S_{MX}(\Phi,K)=S_M(\Phi_x,K_x)-i\eta\ln\left(\mathrm{sdet}\,\frac{\delta\Phi_x^i(\Phi,K)}{\delta\Phi^j}\right) \qquad (5)$$

which is also a solution of eq.(3). Here, construction (5) has a group property with respect to the group of canonical transformations.

Note that in [5] they studied in detail the properties of solutions of the Zinn-Justin equation [6]

$$(S_M, S_M)=0 \qquad (6)$$

to which eq.(3) reduces at $\eta=0$. It was found, in particular, that any two solutions of this equation are related by canonical transformation of variables, therefore the general solution can be obtained by a canonical transformation of variables in some particular solution. This property of eq.(6) solutions together with the S-matrix invariance under arbitrary canonical transformations of the action is the basis of the statement that the S-matrix is completely defined by classical action S_0 . In the case considered here, when the action S_M is defined from (3), its solutions with boundary condition $S_M\big|_{K=\ell=n=\eta=0}=S_0(\varphi)$ may differ not only by canonical transformation, but also by additional to $S_0(\varphi)$ summands of the $\eta\Delta S(\varphi;\eta)$ type. The presence of such summands, first, does not allow one to prove that the S-matrix is independent of the choice of the solution, and second, means apparently the transition to the other theory with "classical" action $S_0(\varphi)+\eta\Delta S(\varphi;\eta)$. It seems natural that the effective actions of one and the same classical theory define one and the same quantum theory, so we shall assume that actions S_M corresponding to one and the same classical theory satisfy (3) and are related to each other by canonical replacement of variables in the sense of (5).

3. Turn now to the consideration of gauge theory whose initial gauge-invariant action $S_0(\varphi)$ has also symmetry with respect to transformations (1):

$$S_0(\varphi+\xi R(\varphi))\approx S_0(\varphi) \tag{7}$$

(sign $\approx$ denotes equality up to terms ξ^2).

Let $S_M(\Phi,K)$ be a solution of eq.(4) with boundary condition at $\eta=0$

$$S_M(\Phi, K=0)\Big|_{\eta=0} = S_0(\varphi) \tag{8}$$

Using transformation (1), construct canonical transformation of variables $\Phi, K \longrightarrow \Phi_X, K_X$, defined by the relations

$$\Phi^i_X = \frac{\delta X}{\delta K_{Xi}}\ , \qquad K_i = \frac{\delta X}{\delta \Phi^i} \tag{9}$$

with generating functional

$$X(\Phi, K_X) = K_X\left(\varphi + \xi R(\varphi)\right) + \ell_X c + m_X \pi \tag{10}$$

As a result of transformations (9), (10), the solution $S_M(\Phi, K)$ of eq.(3) transforms into $S_{MX}(\Phi, K)$ according to formula (5). As was mentioned above, $S_{MX}(\Phi, K)$ also is a solution of eq.(3) and in virtue of symmetry (7) is defined by the same classical action, i.e. satisfies the same boundary condition (8). Then, according to the assumption made in Sec.2, $S_M(\Phi, K)$ and $S_{MX}(\Phi, K)$ are related by canonical transformation in the sense of (5) with some generating functional Y :

$$S_{MX}(\Phi,K) \approx S_{MY}(\Phi,K) = S_M(\Phi_Y, K_Y) - i\eta \ln\left(\mathrm{sdet}\ \frac{\delta \Phi^i_Y(\Phi,K)}{\delta \Phi^j}\right) . \tag{11}$$

From (11), with account of the group property of eq.(4) solutions under the canonical transformations we have

$$S_M(\Phi,K) \approx S_M(\Phi_{X^{-1}Y}, K_{X^{-1}Y}) - i\eta \ln\left(\mathrm{sdet}\ \frac{\delta \Phi^i_{X^{-1}Y}(\Phi,K)}{\delta \Phi^j}\right) . \tag{12}$$

Relation (12) is an expression of symmetry of modified action $S_M(\Phi, K)$ under the canonical transformations with generating functional $X^{-1}Y$. Emphasize, that these transformations are not identical.

Further on, taking into account that the effective action $S_\psi(\Phi, K)$ is derived from $S_M(\Phi, K)$ by means of canonical change of variables $\Phi, K \to \Phi_Z, K_Z$ with generating functional

$$Z(\Phi_Z, K) = K\Phi_Z + \Psi(\Phi_Z)$$

$$\Phi = \frac{\delta Z}{\delta K} = \Phi_Z \quad , \quad K_Z = \frac{\delta Z}{\delta \Phi_Z} = K + \frac{\delta \Psi}{\delta \Phi_Z}$$

(see (2) and (4)) and using relation (12) we find

$$S_\psi(\Phi, K) = S_{MZ}(\Phi, K) \approx S_{MX^{-1}YZ}(\Phi, K) = S_{\psi Z^{-1}X^{-1}YZ}(\Phi, K)$$

or

$$S_\psi(\Phi, K) \approx S_\psi(\Phi_T, K_T) - i\eta \ln\left(\operatorname{sdet} \frac{\delta \Phi^i_T(\Phi, K)}{\delta \Phi^j}\right), \qquad (13)$$

$$T = Z^{-1}X^{-1}YZ$$

Generating functional $T(\Phi, K_T)$ being presented in the form of

$$T(\Phi, K_T) \approx K_T\Phi + \xi \Delta T(\Phi, K_T)$$

(ξ is infinitesimal quantity), formula (13) can be written **in equivalent form** (14

$$(S_\psi, \Delta T(\varphi, K_T)) - i\eta(-1)^{P_i} \frac{\delta_L}{\delta \Phi^i} \frac{\delta}{\delta K_i} \Delta T(\Phi, K_T(\Phi, K)) = 0 ,$$

where P_i is Grassmann parity of field Φ^i , $\delta_L/\delta\Phi$ is the left derivative.

For the GFPV of theory constructed by effective action $S_\psi(\Phi, K)$ we obtain the relation [4]

$$(\Gamma(\Phi,K),<\Delta T(\Phi,K_T)>)=0, \tag{15}$$

where

$$<\Delta T(\Phi,K_T)>=\Delta T\left(\Phi^i+i\eta(-1)^{P_j(P_i+1)}\left[(\Gamma'')^{-1}\right]^{ij}\frac{\delta L}{\delta\Phi^j}\cdot K_T\right) \tag{16}$$

and $\left[(\Gamma'')^{-1}\right]^{ij}$ is the matrix inverse to the $(\Gamma'')_{ij}=\delta^2\Gamma/\delta\Phi^i\,\delta\Phi^j$.

Thus, we have arrived at the conclusion that if the initial classical action $S_0(\varphi)$ of gauge theory has symmetry (besides the gauge one), then the modified action $S_M(\Phi,K)$, the effective action $S_\Psi(\Phi,K)$ and the GFPV also have some symmetry.

4. In this section we shall derive the analogs of formulae (14), (15), (16) for renormalized effective action $S_{\Psi R}(\Phi,K)$ and GFPV $\Gamma_R(\Phi,K)$. They are derived on the basis of the property that if the effective actions are related by canonical change of variables, then the renormalized actions and the GFPV are also related by some canonical change of variables [4]; in all other respects we go through steps completely analogous to those in [1] which led to the derivation of the symmetry properties of renormalized non-gauge theories. The technical details being omitted, for $S_{\Psi R}(\Phi,K)$ we obtain

$$(S_{\Psi R},\Delta T(\Phi,K_T))-i\eta(-1)^{P_i}\frac{\delta L}{\delta\Phi^i}\frac{\delta}{\delta K_i}\Delta_R T(\Phi,K_T(\Phi,K))=0 \tag{17}$$

where $\Delta_R T$ is given by the expression

$$\Delta_R T(\Phi,K_T)=\Delta T(\Phi,K_T)-\sum_{n=1}^{\infty}\eta^n<\Delta T(\Phi,K_T)>^{(n)}_{div} \tag{18}$$

and $\langle \Delta T \rangle^{(n)}_{div}$ is the divergent part of n-loop approximation of vacuum expectation value ΔT with subtracted subdivergences. The analog of formula (15) for the renormalized quantities is given by the relation

$$\left(\Gamma_R(\Phi,K),\ \langle \Delta_R T(\Phi,K_T)\rangle\right)=0 \tag{19}$$

where

$$\langle \Delta_R T(\Phi,K_T)\rangle = \Delta T(\Phi,K_T)+\sum_{n=1}^{\infty}\eta^n\langle \Delta T(\Phi,K_T)\rangle^{(n)}_{fin}$$

Thus formulae (17), (18), (19) express the symmetry properties of the renormalized effective action $S_{\psi R}$ and GFPV of gauge theory. They can be written down in the equivalent form as follows:

$$S_{\psi R}(\Phi,K)\approx S_{\psi R}(\Phi_{T_1},K_{T_1})-i\eta \ln\left(\operatorname{sdet}\frac{\delta\Phi^i_{T_1}(\Phi,K)}{\delta\Phi^j}\right)$$

$$\Gamma_R(\Phi,K)\approx\Gamma_R(\Phi_{T_2},K_{T_2})$$

where T_1 and T_2 are generating functionals of canonical transformations given by the expressions

$$T_1(\Phi,K_T)\approx K_T\Phi+\xi\Delta_R T(\Phi,K_T)$$

$$T_2(\Phi,K_T)\approx K_T\Phi+\xi\langle\Delta_R T(\Phi,K_T)\rangle$$

Summarize the result obtained: if the classical action $S_0(\varphi)$ of gauge theory has additional symmetry, then the renormalized effective action $S_{\psi R}(\Phi,K)$ and GFPV $\Gamma_R(\Phi,K)$ also possess definite symmetries induced by symmetry of classical action.

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